

Operator-based nonlinear control using
extended robust right coprime
factorization

March, 2017

Fazhan Tao

The Graduate School of Engineering
(Doctor's Course)
TOKYO UNIVERSITY OF
AGRICULTURE AND TECHNOLOGY

Acknowledgements

The pursuit of the Ph.D. degree was one of the most challenging and also one of the most rewarding experiences in my life. The guidance and support of the following people was invaluable. Without their advice and encouragement, a successful outcome would be unimaginable. I sincerely appreciate their priceless help and support.

My deepest gratitude goes first and foremost to Professor Mingcong Deng, my supervisor, for his constant encouragement and guidance. His important guidance has been keeping me moving in the right direction, which highly improves my research ability. Without his consistent and illuminating instruction, this dissertation could not have reached its present form.

I would like to express my gratitude to my supervisors Professor Shinji Wakui, Professor Hitoshi Kitazawa, Professor Toshiaki Iwai and Professor Yasuhiro Takaki for their instructive advices and useful suggestions on my research, especially in writing this dissertation. I am deeply grateful of their help in the completion of this dissertation.

Thanks to my colleagues at Tokyo University of Agriculture and Technology who have supported me and offered help in various ways. Especially thanks go to Ph. D. students, Mr. Tomohito Hanawa, Mr. Yanfeng Wu, Miss Mengyang Li, Mr. Guang Jin and Mr. Xudong Gao, and other members of laboratory.

Last my thanks would go to my beloved family for loving considerations and great confidence all through these years. I am deeply indebted to my parents and sisters, who have provided much moral and material support on every aspect of my life, especially the long years of my education, as well as kept me away from family responsibilities and encouraged me to concentrate on my research.

Contents

1	Introduction	1
1.1	Background	1
1.2	Developments of coprime factorization	3
1.3	Motivations of the dissertation	7
1.4	Contributions of the dissertation	8
1.5	Organization of the dissertation	10
2	Mathematical preliminaries and problem statement	13
2.1	Introduction	13
2.2	Mathematical preliminaries	14
2.2.1	Definitions of spaces	14
2.2.2	Definitions of operators	16
2.2.3	Operator-based right coprime factorization	21
2.3	Problem setup	25
2.4	Conclusion	26
3	Extended right coprime factorization and robust control using L_α operator	27
3.1	Introduction	27
3.2	Extended right coprime factorization with L_α operator	29
3.2.1	L_α operator	29
3.2.2	An example to explain L_α operator	32

3.2.3	Extended right coprime factorization	33
3.3	Robust control using extended robust right coprime factorization	36
3.3.1	Problem statement	36
3.3.2	Mathematical preliminaries	38
3.3.3	Robust stability	41
3.3.4	Simulation example	44
3.4	Conclusion	51
4	Extended right coprime factorization and robust control using adjoint operator	53
4.1	Introduction	53
4.2	Right coprime factorization with adjoint operator	55
4.2.1	Problem statement	55
4.2.2	Adjoint operator	55
4.2.3	Factorization method based on adjoint operator	57
4.2.4	Existence of two controllers A and B	61
4.3	Robust control for perturbed nonlinear systems	62
4.3.1	An example showing the necessity of proposed method	62
4.3.2	Rational boundedness for robust control condition	63
4.3.3	Simulation example	66
4.4	Conclusion	69
5	Left coprime factorization realization based on right factorization and issues on robust stability of MIMO nonlinear systems	71
5.1	Introduction	71
5.2	Internal-output stability and left coprime factorization	74
5.2.1	Problem statement	74
5.2.2	Mathematical preliminaries	75

5.2.3	Internal-output stability and left coprime factorization	76
5.2.4	Simulation example	81
5.3	Robust stability of MIMO nonlinear systems	85
5.3.1	Problem statement	85
5.3.2	Robust stability of MIMO nonlinear systems	85
5.3.3	Simulation example	91
5.4	Conclusion	95
6	Conclusions	97
	Bibliography	101
A	Proof	117
A.1	Proof of Lemma 2.1	117
A.2	Proof of Lemma 2.2	118
A.3	Proof of Lemma 2.4	118
A.4	Proof of Lemma 2.5	118
B	Publications	121

List of Figures

2.1	An operator diagram	21
2.2	A nonlinear system with right coprime factorization	22
2.3	A nonlinear system with bounded perturbations	23
3.1	An example to explain the L_α operator	32
3.2	The proposed design scheme based on L_α operator	35
3.3	The equivalent of P	36
3.4	The perturbed nonlinear system	37
3.5	A nonlinear system satisfying L_α condition	45
3.6	Reference input r	46
3.7	The effectiveness of (3.11)	48
3.8	The effectiveness of (3.12)	49
3.9	Plant output y	50
4.1	The proposed system with the compensator S^{-1}	58
4.2	The nonlinear system with perturbations by isomorphism	59
4.3	New proposed nonlinear system with perturbations by isomorphism	63
4.4	Plant output y & reference input r	69
5.1	A nonlinear system with left factorization	75
5.2	Feedback system with the compensator S	77

5.3	The whole equivalent system with the compensator S	78
5.4	The designed scheme for the nonlinear system	79
5.5	Equivalent system of Figure 5.4	80
5.6	Reference input	82
5.7	Plant output	83
5.8	MIMO nonlinear systems with coupling effect.	87
5.9	Decoupling design scheme for MIMO nonlinear systems.	88
5.10	MIMO nonlinear systems with uncertainties.	88
5.11	Effectiveness (5.9) of P_1	92
5.12	Effectiveness (5.10) of P_1	92
5.13	Effectiveness (5.9) of P_2	93
5.14	Effectiveness (5.10) of P_2	93
5.15	Effectiveness (5.9) of P_3	94
5.16	Effectiveness (5.10) of P_3	94
5.17	Reference input r_1	95
5.18	Plant Output y_1	95
5.19	Reference input r_2	95
5.20	Plant Output y_2	95
5.21	Reference input r_3	96
5.22	Plant Output y_3	96

Chapter 1

Introduction

1.1 Background

With the development of modern control engineering and the requirement on the precise, reliable and safe control for systems, effective design scheme and accurate control aiming on guaranteeing perfect performance of systems and meeting the demand of real application have received much attentions from engineers and researchers [1], –, [18].

In terms of the development in control for systems, both linear systems and nonlinear systems have been developing greatly in the last three decades from many viewpoints. The traditional approaches of linear systems are still the most dominating due to its simple structure and mature theoretical support. Whereas, as a matter of fact, most of the real dynamic systems in application have nonlinear property, due to the intrinsic complexity in structures and the nonlinear dynamics in application. Therefore, considering the fact that linear controllers and design method cannot be satisfied with this demand, research and development of an effective approach for nonlinear systems control and design have always been top topic for improving performance and decreasing cost of systems to meet the needs in real application [4], [17], [19], –, [90] and [92], –, [105].

Among the nonlinear researches, uncertainties study for nonlinear systems has been attracting a lot of attention due to that robust phenomena always exist broadly in nonlinear systems leading to many unavoidable problems in real application [21], –, [29]. Generally, the uncertainties include parametric uncertainties, general uncertainties coming from modeling errors and external disturbances, which are always referred to uncertain nonlinearities or unknown nonlinear functions.

The existing uncertainties in the real systems usually have a great adverse effect on stability of the overall systems which could result in a serious damage to the systems. Thus, from the viewpoint of accuracy, reliability and safety for systems, it is necessary and important to eliminate or even remove the adverse effect resulted from uncertainties of the systems. One of the promising directions for solving this issue is to guarantee robust stability of the overall systems including uncertainties such that the overall systems can still work well in a normal way. Thus, robust researches have been obtaining much more attention from various areas, which have become one of the main concerns in the modern control and design.

Over the past decades, for dealing with the family of issues on robustness due to the existing uncertainties in systems, a great number of methods for nonlinear systems are proposed, such as Lyapunov-based method, gain scheduling method, feedback linearization method, backstepping technique, sliding mode control theory, right coprime factorization method, adaptive control approach and so on [41], –, [49]. Among these above mentioned methods, operator-based right coprime factorization has been proved to be a promising and effective method on robust control and system design since this method provides a convenient framework for nonlinear systems from the view of point of input-output relationship based on operator theory [50], –, [90]. In terms of the right coprime factorization method, there exist the comparative and main merits even though every appearing nonlinear control method

has its own merits and limitations on dealing with nonlinear systems. First, operator-based right coprime factorization is focused on the general case, only require the input-output relationship, which can be obtained relatively easy to take experimental data. Compared to other most of the techniques for control and design of nonlinear systems, there is no need to measure all the states of systems, which provides a relative convenient framework to study robustness of the nonlinear systems. Second, the usability of right coprime factorization is prominent, which just requires to build a Bezout identity for guaranteeing bounded input bounded output stability of nonlinear systems. Third, in terms of robustness study of the nonlinear systems, the right coprime factorization has a intrinsic advantage that it has a simple description for the uncertain nonlinear systems which reduce the difficulties in getting effect dependence on the uncertainties. This merit could lead to a direct analysis for control of the uncertain nonlinear systems.

In the following section, a detailed summary on the development of operator-based right coprime factorization is outlined [50], –, [90].

1.2 Developments of coprime factorization

The development of coprime factorization and its application in analysis, control and design for systems are fairly well understood for both linear and nonlinear systems by now. The origin of the idea leading to the coprime factorization method can be traced to the work of Rosenbrock [7], who considers polynomial matrix expression for linear time invariant (LTI) systems described by a family of ordinary differential equations. Based on polynomial matrix fraction description for multivariable transfer functions, parameterization of all stabilizing controllers for finite-dimensional linear time-invariant (FDLTI) systems is studied in the context of obtaining optimization for controllers [8]. Callier, F. M. and Desoer, C. A. extend the definition of coprime

factorization to distributed LTI systems. In the case of distributed LTI systems, attention is focused on the class of systems which form a Bezout domain and for which the existence of coprime fractional representations can be demonstrate. [9] proposes some sufficient conditions for existence of a doubly coprime factorization of a large class of infinite-dimensional systems well known as regular linear systems. Coprime factorization and well posed linear systems are discussed based on appropriate stabilizability and detectability, obtaining that every function with doubly coprime factorization in H_∞ is the transfer function of a jointly stabilizable and detectable well posed linear system. Besides these results on linear systems based on the notation of coprime factorization, the idea of coprime factorization also has made a great effect on nonlinear systems for provide a convenient framework to research the nonlinear systems from a view of point of the input-output stability.

In [82], the authors extend the case from FDLTI systems to nonlinear systems represented by the set of all stable input-output pairs based on right coprime factorization. Meanwhile, coprimeness property is proved to be equivalent to that of FDLTI systems. [83] considers the relationship between factorization representation and stability of time-invariant and discrete-time nonlinear systems, giving some mild necessary and sufficient conditions for existence of a factorization of recursive systems, where the factorization are stable recursive systems. Besides, the detailed construction of such a factorization is discussed. After that, the author of [84] considers a theory of coprimeness developed for a class of nonlinear systems, with the intention of constructing analytic tools for the solution of stabilizing a nonlinear system by using the additive nonlinear feedback technique. In addition, besides the viewpoint from the input-output of systems, many researchers consider coprime factorization from state-space equation view of point. In [69], the author shows that right coprime factorizations exist for the input-to-state mapping of a continuous time nonlinear system under the condition

of existing the smooth feedback stabilization solution for the system, and it shows that smooth stabilization implies smooth input-to-state stabilization. The authors of [70] first study the normalized right and left coprime factorizations of a nonlinear system from a state-space point of view and make use of the proposed normalized coprime factorization to give a method to balance the unstable nonlinear system based on a smooth solution of a Hamilton-Jacobi equation. Moreover, in [71], the Youla parameterization method is generalized from linear systems to nonlinear system, stabilizing a nonlinear system by using a unstable compensator and a stable compensator, and parameterizing a class of stabilizers in the context of a bounded-input bounded-output (BIBO) stable. Later, the state-space characterization concerned with Youla-Kucera parameterization is studied as well for a class of nonlinear systems via kernel representations, proposing a fair natural generalization of Youla-Kucera parameterization through observer based kernel representations. Besides right coprime factorization, left coprime factorization for nonlinear systems is also considered in [84], –, [86]. In detail, in [84], the authors construct a left coprime factorization for stabilization of nonlinear systems. That is, constructing a class of all controllers stabilize the given system, and the class of systems is stabilized by the given controller. Furthermore, necessary and sufficient conditions for the stabilization of the system are considered. The authors [85] study a class of nonlinear systems represented by both left and right coprime factorization. According to coprime factorization, the class of all stabilizing controllers of a particular structure for the considered systems is characterized. The results specialize to the Youla-Kucera parametrization in the linear cases.

Since 1980s, robust control and robust stabilization of the nonlinear uncertain system are researched by Chen and Han [74] in the context of operator-based right coprime factorization of a nonlinear system, which is considered in a general operator-based setting [57]. Roughly speaking, main idea of right

coprime factorization for nonlinear systems consists of two design steps: first, factorize a given plant P as a composition of two stable operators N and D such that $P = ND^{-1}$, where D is invertible, second, propose two stable controllers A, B such that Bezout identity $AN + BD = M$ is satisfied, where M is an unimodular operator. Then, the considered plant is said to have a right coprime factorization [4]. Based on the right coprime factorization approach, in [4], [59], [74] and [76], robustness and tracking performance for the nonlinear system with bounded perturbations are considered. In [74], the concept of robust right coprime factorization for nonlinear systems is introduced firstly. Moreover, based on the proposed concept, some sufficient conditions for robust stability of the nonlinear system with bounded perturbations are proposed. As for the conditions in [74], null set of perturbations is considered as main idea. That is, assume the perturbations in the null set of A in the form of $A(N + \Delta N) - AN = 0$, where ΔN is bounded perturbations. In [59], a generalized condition is considered for robustness of nonlinear system with bounded perturbations, whose merits compared to [74] is that the proposed condition in [59] can handle with a broader classes of nonlinear systems by using an inequality $\| [A(N + \Delta N) - AN]M^{-1} \| < 1$. In [76], robust stability and output tracking is discussed under some sufficient conditions even though the bounded perturbations exists. Moreover, the robust right coprime factorization method has been attracting an increasing attention and many important results for the real systems have been obtained [4], [17], [55], [61],[64] and [65], such as vibration control on an aircraft vertical tail with piezoelectric elements for guaranteeing robust stability [17] networked nonlinear control for an aluminum plate thermal process using robust coprime factorization [61], and an adaptive nonlinear sensorless control for an uncertain miniature pneumatic curling rubber actuator using passivity and the robust right coprime factorization approach [64]. Besides these, right coprime factorization for MIMO nonlinear systems is also studied in [106] and

[107]. In [107], some sufficient conditions for the MIMO nonlinear perturbed systems are derived. The proposed sufficient conditions are established by using Taylor expansion of a controller. In [106], for dealing with coupling effect and guaranteeing robust stability of MIMO nonlinear systems, internal operators of coupling effect are supposed to be satisfied with right factorization, which means the internal signal of the coupling effect could be observed for obtaining right factorization.

1.3 Motivations of the dissertation

Since it is effective and practical to apply the operator-based robust right coprime factorization method to the control design for nonlinear systems, there are a great number of results obtained in many fields, such as stability analysis, tracking performance, robust stability, passivity study and so on. However, there exist still some points worth to being researched for enriching and refining the operator-based robust right coprime factorization method.

Therefore, main concepts that motivate this dissertation are addressed from the following four aspects. First, as to the operator-based right coprime factorization approach, the fundamental tool is the Lipschitz operator, which provides a viewpoint to study the nonlinear system from operator theory to regard nonlinear systems as Lipschitz operators. However, there exist many cases where nonlinear systems cannot be satisfied with the Lipschitz requirement, leading to the operator-based right coprime factorization approach unavailable. This issue motivates the idea considering different operators to relax the restriction on Lipschitz operator. That is, to extend the operator-based right coprime factorization approach. Therefore, in this dissertation, L_α operators in Chapter 3 and adjoint operators in Chapter 4 are considered to address the above issue. Second, the study on how to factorize quantitatively nonlinear systems is importance for applying the operator-based

right coprime factorization approach, which motivates the following idea. In this dissertation, a systematic factorization for systems is considered using the proposed definition, inner product, adjoint operators and Hilbert space. Third, robust control of nonlinear systems plays an important role in the real application. Motivated by this, this dissertation is focused on the nonlinear control design for nonlinear systems by using the extended robust right coprime factorization approach to guarantee robust stability of the systems. Fourth, most of existing results for coprime factorization are on right coprime factorization for SISO nonlinear systems. There are few researches giving a study on left coprime factorization and MIMO nonlinear systems. Therefore, in Chapter 5, on the one hand, left coprime factorization is considered to guarantee a class of nonlinear system to be stable. Realization of left coprime factorization is obtained by using the proposed method. On the other hand, the issues on robust stability of MIMO nonlinear systems are discussed. By the proposed feasible design scheme, the designed system is overall stable.

1.4 Contributions of the dissertation

The proposed nonlinear control scheme and the research on extended robust right coprime factorization of this dissertation enriches the operator-based coprime factorization theory, which gives a promising direction to study the coprime factorization from a different view of point. Meanwhile, sufficient conditions on robust stability of nonlinear systems with perturbations are discussed, by which the robust control design for the perturbed nonlinear systems is convenient and feasible. Last, realization of left coprime factorization is discussed combining right factorization method, providing a method to study the internal-output stability of a class of nonlinear systems, and robust stability of MIMO nonlinear systems is guaranteed by the proposed

design scheme.

The proposed control design scheme is based on operator theory setting formulated in the context of extended norm linear space, which is suitable for nonlinear systems control theory and application in the context of stability, causality, robustness, uniqueness of internal control signals as well as coprime factorization. The reasons for considering extended norm linear spaces lie in that all control signals in engineering are time-limited, but in the study of a control processing we do not know when the processing will stop. The extended norm linear space can deal with the practical issue from theory, and most of the useful techniques and results can be carried over from the standard Banach space to the extended norm linear space, which is a basic requirement for a realizable physical control system.

Generally speaking, robust control is necessary and critical for nonlinear systems' control and design due to uncertainties usually existing in the real systems that always make an effect on stability and safety of the nonlinear systems. The main idea of robust control is to design controllers for the stable and excellent performance of nonlinear systems even in the cases of the uncertainties existing in the nominal nonlinear systems. Comparing to general methods for nonlinear systems such as, linear matrix inequality, sliding mode control, adaptive control, robust right coprime factorization method makes use of Bezout identity which is guaranteed by designing stable controllers while meeting a norm inequality condition in order to guarantee the robustness and to stabilize the unstable nonlinear system. In a word, the control and design process for robust stability of nonlinear systems with perturbations is more simple and precise. This is one of the merits of operator-based right coprime factorization. Of course, it is one of the contributions of the proposed design scheme in this dissertation as well.

This dissertation is mainly focusing on considering the extended right coprime factorization and nonlinear control of the considered nonlinear sys-

tems. Specially, by introducing the L_α operator, right coprime factorization is extended from the case where nonlinear systems are considered from the Lipschitz operator viewpoint to the case where nonlinear systems are considered from the L_α operator viewpoint. Then, based on the proposed operator and obtained extended right coprime factorization, feasible design schemes are proposed for the nonlinear system with perturbations to guarantee robust stability. Next, right factorization of a given nonlinear system is discussed based on a feasible framework by the proposed definition of Hilbert spaces, which extends the application of the isomorphism technique, meanwhile, sufficient conditions on robust stability are proposed, with which robust design can avoid drawbacks to use irrational boundedness of robust condition of former results of some cases and reduce difficulties of calculation in Lipschitz norm. Besides the above contributions, in this dissertation, left coprime factorization of a class of nonlinear systems and issues on robust control of MIMO nonlinear systems are addressed. In terms of left coprime factorization, its realization is obtained combining right factorization method from the input-output view of point. As to robust control of MIMO nonlinear system, decoupling effect and robust stability of the considered systems are studied, which enriches the coprime factorization methods.

In summary, this dissertation considers extended right coprime factorization and nonlinear control for nonlinear systems, which complements the theoretical analysis and control design of nonlinear systems.

1.5 Organization of the dissertation

This dissertation is organized as follows.

In Chapter 2, mathematical preliminaries for developing main results of this dissertation consisting of definitions of important spaces and operators are introduced. First, the definitions of extended linear space and generalized

Lipschitz operator are introduced, which serve as foundations for the research of this dissertation. Then, right factorization, right coprime factorization and robust right coprime factorization of a nonlinear system in a fairly general operator setting are introduced, which provide the theoretical basis for this dissertation.

In Chapter 3, the extended right coprime factorization and corresponding nonlinear robust control of nonlinear systems with perturbations are considered and designed by using the proposed operator and methods. Firstly, a kind of operators is introduced, by which operator-based right coprime factorization approach is extended for a class of nonlinear systems. By regulating the exponent of the proposed operator, a broader class of nonlinear systems can be handled using the extended right coprime factorization approach. Then, based on the obtained extended right coprime factorization, a feasible control design scheme is proposed to guarantee robust stability of the considered nonlinear systems with perturbations. The main idea of the practical design scheme is to prove a stabilizing operator to be an unimodular operator, so we can utilize the proposed unimodular operator to omit the complicated calculation in process of control and design for the systems with perturbations. Finally, a simulation example is involved to illustrate the proposed design scheme for confirming effectiveness of the proposed method.

In Chapter 4, adjoint-based right coprime factorization and robust stability of nonlinear systems with perturbations are considered based on Hilbert spaces. First, a framework is proposed to study right factorization using inner product, which provides fundamental for the following study. Second, a sufficient condition based on adjoint operators is given, based on which a compensator is designed to eliminate difficulties in obtaining internal signal of the perturbed systems. After that, a realizable design for robust stability is proposed based on unimodular property. Rational boundedness of robust condition is provided. Finally, a simulation is shown for confirming effective-

ness of proposed methods.

In Chapter 5, a realization approach to left coprime factorization for the nonlinear system is obtained, which provides an effective framework for constructing left coprime factorization based on right factorization method. Meanwhile, internal-output stability of the nonlinear system is guaranteed. The left coprime factorization from the operator theory view of point is studied and a simulation example is shown to confirm the validity of proposed methods in final.

In Chapter 6, the proposed method and nonlinear control for nonlinear systems are summarized. It is concluded that by using the proposed methods, extended robust right coprime factorization are studied for nonlinear system and robust control design schemes of the perturbed nonlinear systems are also given. Meanwhile, a special class of nonlinear systems are considered using left factorization combining with right factorization to guarantee input-output stability.

Chapter 2

Mathematical preliminaries and problem statement

2.1 Introduction

The mathematical preliminaries and problems statement are given in this chapter.

In Section 2.2, firstly, the definitions of spaces such as linear space, normed space, Banach space, Hilbert space, extended linear space associated with Banach space are recalled. Secondly, the definition of operator and some important operators are given including linear and nonlinear operator, invertible operator, stable operator, unimodular operator, Lipschitz operator and generalized Lipschitz operator. After that, the relationship between generalized Lipschitz operator and causality is discussed. Meanwhile, the definitions of bounded input bounded output (BIBO) stability and well-posedness are provided.

In Section 2.3, operator-based right coprime factorization is given for nonlinear systems at first. Then, a sufficient condition is given to show the relationship between the coprimeness and stability of nonlinear feed-back systems. According to this relationship, universal conditions are proposed

to guarantee the coprimeness of the factorization for the nonlinear systems. Meanwhile, a sufficient robust condition is provided to guarantee robust stability of the nonlinear systems with perturbations.

In Section 2.4, the main problems of this dissertation are given for developing the main results of this dissertation. That is, the extended right coprime factorization for a broader class of nonlinear systems and robust control for the nonlinear system with perturbation are described. Meanwhile, left coprime factorization is considered combining right factorization to guarantee a special class of nonlinear system to be internal-output stability.

2.2 Mathematical preliminaries

In this section, the related definitions and notations on kinds of spaces and operators are recalled. Moreover, some important results are listed.

2.2.1 Definitions of spaces

In mathematics analysis, the definition of space plays a fundamental role, whose definition can be described as a set with added structures. In this dissertation, the used spaces are linear space that is also named vector spaces.

Linear spaces

A space over F that is an arbitrary field, like the field R of real numbers or the field C of complex numbers is a set V endowed with structure by the prescription of

- (1) an addition operation $V: V \times V \longrightarrow V$,
- (2) an scalar multiplication in $V: F \times V \longrightarrow V$
- (3) an element $0 \in V$ called the zero of V

(4) a mapping opposition in V : $V \rightarrow V$ provided that the following axioms are satisfied for all $u, v, w \in V$ and all $\alpha, \beta \in \mathbb{F}$:

$$A(1) \quad u + (v + w) = (u + v) + w$$

$$A(2) \quad u + v = v + u$$

$$A(3) \quad u + 0 = u$$

$$A(4) \quad u - u = 0$$

$$A(5) \quad \alpha(\beta u) = (\alpha\beta)u$$

$$A(6) \quad (\alpha + \beta)u = \alpha u + \beta u$$

$$A(7) \quad \alpha(u + v) = \alpha u + \beta v$$

$$A(8) \quad 1u = u$$

Thus, note that a linear space is a commutative group endowed with additional structure by the prescription of a scalar multiplication subject to the conditions $A(5)$, $-$, $A(8)$.

Subspace of linear spaces

A nonempty subset U of a linear space V is called a subspace of V if it is stable under the addition and scalar multiplication in V .

Normed linear spaces

Considering a linear space V of time functions, the linear space V is said to be normed if each element v in V is endowed with norm $\| \cdot \|$, which can be defined in any way so long as the following three properties are fulfilled:

$$(1) \quad \| v \| \geq 0; \text{ with equality only when } x = 0;$$

$$(2) \quad \| av \| = |a| \| v \|;$$

$$(3) \quad \| v + w \| \leq \| v \| + \| w \| \text{ (the triangle inequality), whenever } v, w \in U, \text{ and } a \in \mathbb{R},$$

Banach space

Banach space is defined as a complete normed linear space. This means that a Banach space is a normed linear space V over the real or complex numbers with a norm $\| \cdot \|$ such that every Cauchy sequence (with respect to the metric) in V has a limit in V . Many spaces of sequences or functions are infinite dimensional Banach spaces, like L_p the set of all measurable complex-valued functions, for which $\int |f(t)|^p dt$ is finite.

Extended linear space

Let M be the class of real-valued measurable functions defined on $[0, \infty)$. As to every constant $T \in [0, \infty)$, suppose P_T be the projection operator mapping from M to other space, M_T , of measurable function such that

$$f_T(t) := P_T(f)(t) = \begin{cases} f(t), & t \leq T \\ 0, & t > T \end{cases}$$

$f_T(t) \in M_T$ is said to truncation of $f(t)$ with respect to T . Then, for a given Banach space X of measurable functions, set

$$X^e = \{f \in M : \|f_T\| < \infty \text{ for all } T < \infty\} \quad (2.1)$$

Obviously, X^e is a linear subspace of X .

2.2.2 Definitions of operators

Let U and Y be linear spaces and U_s and Y_s be two normed linear spaces, respectively. Define two suitable certain norm denoted $U_s = \{u \in U : \|u\| < \infty\}$ and $Y_s = \{y \in Y : \|y\| < \infty\}$. Moreover, U_s and Y_s are called the stable subspaces of U and Y .

Operator

An operator $Q : U \rightarrow Y$ is a mapping from U to Y . The operator Q can be described as shown in Figure 2.1 and its mathematical expression form can

be written as

$$y(t) = Q(u)(t)$$

where $u(t)$ and $y(t)$ are the element of U and Y , respectively.

Note that in the rest dissertation, denote $\mathcal{D}(Q)$ and $\mathcal{R}(Q)$ as the domain and range of the operator Q .

Linear and nonlinear operator

Let $Q : U \rightarrow Y$ be an operator defined from input space U to the output space Y . Provided that Q is satisfied with the following condition

$$Q : au_1 + bu_2 \rightarrow aQ(u_1) + bQ(u_2)$$

for all $u_1, u_2 \in U$ and all a, b are real numbers, then Q is called to be linear; otherwise, it is called to be nonlinear. According to the definition of linear operator, it can be founded a linear operator is satisfied with addition rule and multiplication rule for different elements. Note that linearity is a special case of nonlinearity. In what follows, nonlinear will always mean not necessarily linear unless otherwise indicated.

Bounded input bounded output (BIBO) stability

Let Q be a nonlinear operator with its domain $\mathcal{D}(Q) \subseteq U^e$ and range $\mathcal{R}(Q) \subseteq Y^e$. Provided that $Q(U) \subseteq Y$, Q is said to be input output stable. If Q maps all input functions from U_s into the output space Y_s , that is $Q(U_s) \subseteq Y_s$, then operator Q is said to be bounded input bounded output (BIBO) stable or simply, stable. Otherwise, namely, if Q maps some inputs from U_s to the set $Y^e \setminus Y_s$ (if not empty), then Q is said to be unstable. For any stable operators defined here and later in this dissertation, they always mean BIBO stable.

Invertible

An operator Q is said to be invertible if there exists an operator P such that

$$Q \circ P = P \circ Q = I$$

where, P is called inverse of Q and is denoted by Q^{-1} , where I is identity operator and \circ denotes the operation defined in the operator theory which can be simply presented as $Q \circ P$.

Unimodular operator

Let $\mathcal{S}(U, Y)$ be the set of stable operators from U to Y . Then $\mathcal{S}(U, Y)$ contains a subset defined by

$$\mathcal{U}(U, Y) = \{Q : Q \in \mathcal{S}(U, Y) \text{ is invertible with } Q^{-1} \in \mathcal{S}(U, Y)\}.$$

Each element of $\mathcal{U}(U, Y)$ is called unimodular operator.

Lipschitz operator

Let $Q : U \rightarrow Y$ be an operator mapping from U to Y and denote $\mathcal{N}(U, Y)$ be the family of operators from U to Y . A semi-norm on $\mathcal{N}(U, Y)$ is denoted by

$$\|Q\| := \sup_{\substack{u_1, u_2 \in U \\ u_1 \neq u_2}} \frac{\|Q(u_1) - Q(u_2)\|}{\|u_1 - u_2\|},$$

if $\|Q\|$ is finite. In general, it is a semi-norm in the sense that $\|Q\| = 0$ does not necessarily imply $Q = 0$.

Note that an element Q of $\mathcal{N}(U, Y)$ is in $Lip(U, Y)$ if and only if there is a number $L \geq 0$ such that

$$\|Q(u_1) - Q(u_2)\| \leq L \|u_1 - u_2\|$$

for all $u_1, u_2 \in U$. Moreover, $\|Q\|$ is the least such number L .

Generalized Lipschitz operator

Let U^e and Y^e be two extended linear spaces which are associated respectively with two given Banach spaces U and Y of measurable functions defined on the time domain $[0, \infty)$. Let D^e a subset of U^e . An operator $Q : D^e \rightarrow Y^e$ is called a generalized Lipschitz operator on D^e if there exists a constant L such that

$$\| [Q(u_1)]_T - [Q(u_2)]_T \| \leq L \| [u_1]_T - [u_2]_T \|$$

for all $u_1, u_2 \in D^e$ and for all $T \in [0, \infty)$.

Note that the least such constant L is given by

$$\| Q \| := \sup_{T \in [0, \infty)} \sup_{\substack{u_1, u_2 \in D^e \\ u_1 \neq u_2}} \frac{\| [Q(u_1)]_T - [Q(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|} \quad (2.2)$$

which is a semi-norm for general nonlinear operators and is the actual norm for linear Q . The actual norm for a nonlinear operator Q is given by

$$\begin{aligned} \| Q \|_{Lip} = & \| Q(u_0) \| \\ & + \sup_{T \in [0, \infty)} \sup_{\substack{u_1, u_2 \in D^e \\ u_1 \neq u_2}} \frac{\| [Q(u_1)]_T - [Q(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|} \end{aligned}$$

for any fixed $u_0 \in D^e$. This can be easily verified by the definition of norm.

Note that according to different domains and ranges of standard Lipschitz operator and generalized Lipschitz operator, they are not comparable. Generalized Lipschitz operator has been proved more useful than standard Lipschitz operator for nonlinear system control design and engineering under stability, robustness, uniqueness of internal control signals. In addition, in this dissertation, denote $Lip(D^e) = Lip(D^e, D^e)$. In the following, causality will be introduced, which is a basic requirement for realizing a physical system.

Causality for generalized Lipschitz operator

Let U^e be the extended linear space associated with a given Banach space U , and let $Q : U^e \rightarrow U^e$ be a nonlinear operator describing a nonlinear control system. Then, Q is said to be causal if and only if

$$P_T Q P_T = P_T Q$$

for all $T \in [0, \infty)$, where P_T is a projection operator.

The physical meaning behind the definition of causality is addressed as follows. If the system outputs depend only on the present and past values of the corresponding system inputs, then we have $Q P_T(u) = Q(u)$ for all input signals u in the domain of Q , so that $P_T Q P_T = P_T Q$. Conversely, if $P_T Q P_T = P_T Q$ for all $T \in [0, \infty)$, then we have $P_T Q (I - P_T)(u) = 0$ for all input u in the domain of Q , which implies that any future value of a system input, $(I - P_T)(u)$, does not affect the present and past values of the corresponding system output given by $P_T Q(\cdot)$, or in other words, system outputs depend only on the present and past values of the corresponding system inputs.

Lemma 2.1 *A nonlinear operator $Q : U^e \rightarrow U^e$ is causal if and only if for any $x, y \in U^e$ and $T \in [0, \infty)$, $x_T = y_T$ implies $[Q(x)]_T = [Q(y)]_T$.*

Proof. The proof is given in Appendix A.1 [57].

Lemma 2.2 *If $Q : U^e \rightarrow U^e$ is a generalized Lipschitz operator, then Q is causal.*

Proof. The proof is given in Appendix A.2 [57].

Lemma 2.2 and the following **Lemma 2.3** imply that the uniqueness requirement can be guaranteed by introducing the generalized Lipschitz operator, which means that in real systems, the internal signals of the systems are required to be unique.

Lemma 2.3 *A nonlinear generalized Lipschitz operator produces a unique output from an input in the sense that if the input x and output y are related*

by a generalized Lipschitz operator Q such that $y = Q(x)$, then $x_T = \tilde{x}_T$ implies that $y_T = \tilde{y}_T$ for all $T \in [0, \infty)$.

2.2.3 Operator-based right coprime factorization

A nominal operator based nonlinear control system is shown in Figure 2.1, where the given system $P : U \rightarrow Y$ is from the input space U to the output space Y . Let control input and system output be u and y , respectively.

Right factorization

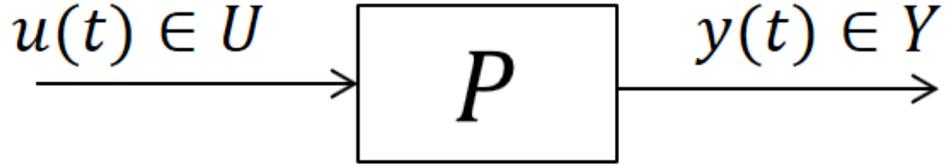


Figure 2.1: An operator diagram

Then the given system operator P as shown in Figure 2.1 is said to have a right factorization if there are a linear space W and two stable operators D and N such that $P = ND^{-1}$ on U , where D is invertible. The linear space W is called a quasi-state space of P .

Right coprime factorization

Provided that P exists a right factorization (N, D) , there are two stable operators $A : Y \rightarrow U$ and $B : U \rightarrow U$ and B is invertible, satisfying Bezout identity

$$AN + BD = M, \text{ for } M \in \mathcal{U}(W, U),$$

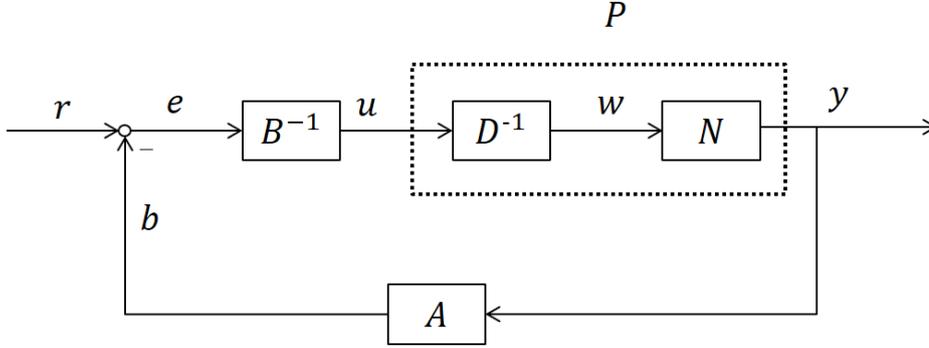


Figure 2.2: A nonlinear system with right coprime factorization

then the right factorization of P is said to be coprime.

It is worth mentioning that the initial state is supposed to be considered, that is, $AN(w_0, t_0) + BD(w_0, t_0) = M(w_0, t_0)$ should be satisfied. In this dissertation, the initial condition is chosen as $t_0 = 0$ and $w_0 = 0$.

Well-posedness

The nonlinear system shown in Figure 2.2 is well-posed, if as to each input signal $r \in U$, all signals in the system are uniquely determined.

Overall stable

The nonlinear system shown in Figure 2.2 is said to be overall stable, if $r \in U_s$, implies that $u \in U_s$, $y \in Y_s$, $w \in W_s$, $e \in U_s$ and $b \in U_s$.

Lemma 2.4 *Suppose the system shown in Figure 2.2 is well-posed and the system has a right factorization $P = ND^{-1}$, then the system is overall stable if and only if the operator M is unimodula.*

Proof. The proof is given in Appendix A.3 [57].

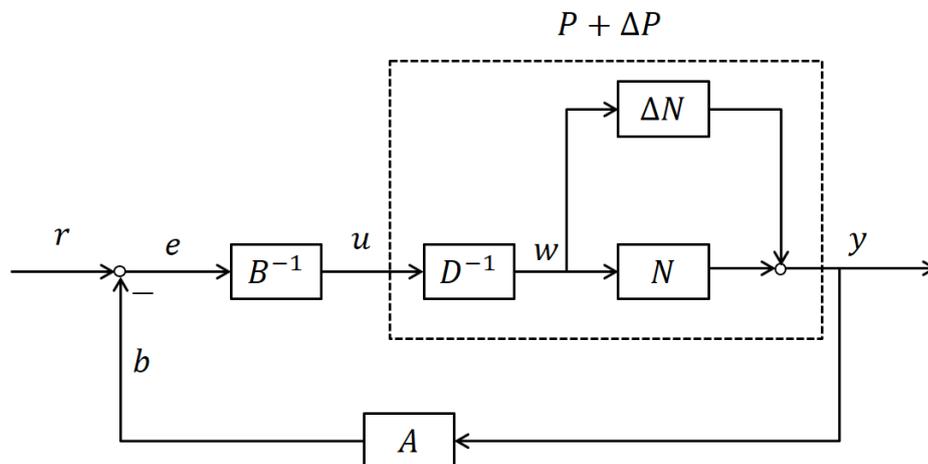


Figure 2.3: A nonlinear system with bounded perturbations

Robust right coprime factorization

Consider the nonlinear perturbed system shown in Figure 2.3, where the nominal system and the perturbed system are given as P and \bar{P} , where $\bar{P} = P + \Delta P$, ΔP is denoted as the bounded perturbations. The right factorization of the nominal system P and the overall system \bar{P} are

$$P = ND^{-1}$$

and

$$P + \Delta P = (N + \Delta N)D^{-1}$$

respectively, where N and D are stable operators, D is invertible and ΔN is denoted as the bounded perturbations.

According to the definition of null set, in [74], the following condition is proposed to guarantee the nonlinear system with unknown bounded perturbations to be robustly stable,

$$A(N + \Delta N) - AN = 0. \quad (2.3)$$

under the condition of satisfaction of $\mathcal{R}(\Delta N) \subseteq \mathbf{N}(A)$, where $\mathbf{N}(A)$ is the null set defined by

$$\mathbf{N}(A) = \{x : x \in \mathcal{D}(A) \text{ and } A(x + y) = A(y) \text{ for all } y \in \mathcal{D}(A)\}$$

Based on the proposed sufficient condition, the fact that

$$A(N + \Delta N) + BD = AN + BD = M$$

is obtained, which guarantee the robust stability of the nonlinear systems with unknown bounded perturbations.

However, the proposed design scheme for the nonlinear systems with unknown bounded perturbations in [74] is restrictive for some case due to the condition is difficult to satisfy. Therefore, a generalized sufficient condition compared to [74] is proposed in [59] in order to improve and extend the condition.

Lemma 2.5 *Let D^e be a linear subspace of the extended linear space U^e associated with a given Banach space U , and let $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$. Let the Bezout identity of the nominal system and the exact system be $AN + BD = M$, $A(N + \Delta N) + BD = \tilde{M}$, respectively. If*

$$\| (A(N + \Delta N) - AN)M^{-1} \| < 1$$

then the system shown in Figure 2.3 is robust stable.

Proof. The proof is given in Appendix A.4 [57].

Lemma 2.6 *Assume right factorization of the system shown in Figure 2.3 is given as $P + \Delta P = (N + \Delta N)D^{-1}$, where $N + \Delta N$ is an unimodular operator. If two designed operators A and B is satisfied with the Bezout identity $A(N + \Delta N) + BD = M + \Delta M$, moreover, $(N + \Delta N)(M + \Delta M)^{-1} = I$, then output can track to reference input while the nonlinear system is overall stable.*

2.3 Problem setup

Operator-based right coprime factorization method has attached much attention on analysis, stabilization, design and control for the nonlinear systems. Most of the existing results are obtained based on the assumption that the given nonlinear system is satisfied with Lipschitz condition. The fundamental tool is the Lipschitz operator, which provides a viewpoint to study the nonlinear system from operator theory to regard nonlinear systems as Lipschitz operators. However, there exist many cases where nonlinear systems cannot be satisfied with the Lipschitz requirement, leading to the operator-based right coprime factorization approach unavailable. In order to solve this fundamental and critical problem, the extended right coprime factorization method using L_α operator for dealing with a broader class of given nonlinear systems is proposed and robust control for this kind of nonlinear systems is considered as well.

In terms of operator-based right coprime factorization, in most cases, right factorization cannot be directly obtained, whose role is of importance for studying the nonlinear systems. That is, how to quantitatively factorize a given nonlinear systems. Therefore, in this dissertation, a systematic factorization for systems is considered using the proposed definition, inner product, adjoint operators, Hilbert space to extend right coprime factorization. Meanwhile, rational boundedness of robust condition is also proposed to avoid the practical difficulties in real application. Considering that most of existing results for coprime factorization are on right coprime factorization. There are few researches giving a study on left coprime factorization. Therefore, for extending the application of operator-based coprime factorization, left coprime factorization is considered combining right factorization to guarantee a class of nonlinear system to be internal-output stable.

2.4 Conclusion

In this chapter, the mathematical preliminaries including the basic definitions and notations are introduced. In detail, the definitions like extended linear spaces and generalized Lipschitz operators are introduced, which serve as foundation for this dissertation. For nonlinear systems, the concept of right coprime factorization and robust right coprime factorization condition are described. Moreover, two sufficient conditions of guaranteeing robust stability of the nonlinear perturbed system are given in a fairly general operator setting, which provide the theoretical basis for developing the main results of this dissertation. Finally, the concerned problems are also summarized in this chapter.

Chapter 3

Extended right coprime factorization and robust control using L_α operator

3.1 Introduction

Nonlinear control and design for nonlinear systems are always important and challenging considering the fact that most of real systems exists more or less nonlinearity dynamics property and complex structures. Among the studied aspects, robust control design for nonlinear systems with perturbations plays a significant role in control engineering owing to robust phenomenon always happening in nonlinear systems resulting from unnoticeable and unavoidable factors in real systems. Therefore, many researchers have devoted themselves to studying some methods to deal with the robust issue for the nonlinear uncertain systems. Among the proposed methods, operator-based right coprime factorization has been regarded as an effective and practical method in dealing with nonlinear systems, including stability analysis, control design scheme, output tracking and so on.

Although there are a vase amount of general results available in the literatures on the existence, uniqueness, characterization and construction of the

right coprime factorization approach for the nonlinear systems [50], –, [82], however, there is yet a promising direction which seems to be relatively ignored by researchers: *the application range of the right coprime factorization approach*. In terms of the right coprime factorization approach, the fundamental tool is the Lipschitz operator, which provides a viewpoint to study the nonlinear system from operator theory. However, there are some nonlinear systems that cannot be satisfied with the Lipschitz condition, resulting in the right coprime factorization approach unavailable.

Thus, in this chapter, main concepts that motivate the present research are the observation that the Lipschitz operator has a restriction on dealing with the nonlinear systems and the former methods for guaranteeing robust stability of the nonlinear system with bounded perturbations. Specially, by introducing the L_α operator, right coprime factorization is extended from the case where nonlinear systems are considered from the Lipschitz operator viewpoint to the case where nonlinear systems are considered from the L_α operator viewpoint. Then, based on the proposed operator, feasible design schemes are proposed for the nonlinear system with bounded perturbations to guarantee robust stability.

In Section 3.2, extended right coprime factorization of a nonlinear system is considered by using the proposed L_α operator. At first, the definition of L_α operator is introduced for extending right coprime factorization for nonlinear systems. Meanwhile, some properties of L_α operators are proposed. Then, in order to explain the proposed operator, an example is given to show the merit of L_α operators, which serves as proof that L_α operator can deal with a broader class of nonlinear systems. Further, according to the obtained results, a fundamental theorem is obtained and the effectiveness on applying the proposed operator to the robust right coprime factorization approach is discussed. Then, extended right coprime factorization of a nonlinear system is given to guarantee stability of the considered system.

In Section 3.3, nonlinear robust control design for the nonlinear system with bounded perturbations is considered. In detail, first, an inequality on generalized L_α norm is proposed for developing the sufficient condition for robust stability of nonlinear systems with bounded perturbations. Second, a new unimodular operator based on the proposed extended right coprime factorization is given. Third, the practical robust control design scheme for the nonlinear system with bounded perturbations is proposed to guaranteeing robust stability of the considered systems. The given numerical example illustrates the effectiveness of the proposed methods in final.

In Section 3.4, the extended right coprime factorization and robust control using L_α operator for the considered nonlinear systems are summarized.

3.2 Extended right coprime factorization with L_α operator

3.2.1 L_α operator

In this section, firstly, L_α operators are proposed. According to the proposed operator, fundamental theorems are discussed for applying the proposed operator to extend right coprime factorization approach.

Definition 3.1 Let U and Y be two Banach spaces, denoting as input space and output space of the considered systems, respectively. Let $T : U \rightarrow Y$ be an operator mapping from U to Y . If the operator T satisfies that

$$\| T \|_\alpha := \sup_{\substack{u_1, u_2 \in U \\ u_1 \neq u_2}} \frac{\| T(u_1) - T(u_2) \|}{\| u_1 - u_2 \|^\alpha} \quad (3.1)$$

is finite, for all $u_1, u_2 \in U$ and where the parameter $\alpha > 0$, then T is said to be a L_α operator.

The $\| \cdot \|_\alpha$ is a semi-norm for the L_α operator in the sense that $\| T \|_\alpha = 0$ does not necessarily imply $T = 0$. We can find a case where $T \neq 0$, but

$\|T\|_\alpha$ is 0, for example a constant operator which maps any input to a fixed output.

Note that, the proposed L_α has finite incremental gain to guarantee stability of nonlinear feedback systems. A real plant is regarded as an L_α operator, whose domain and range are input space and output spaces of the real plant, respectively. The considered plant should be satisfied with the L_α condition such that the finite gain exists. In terms of the proposed operators, the parameter α can be considered as one freedom for guaranteeing existence of finite gain issue by designing the suitable value in order to adjust input of the real plant.

It is noted that when α equals to 1, the L_α operator is reduced to the Lipschitz operator. The main merit of the L_α operator lies in that the L_α operator can include a broader class of nonlinear systems than Lipschitz operators. This fact provides a framework to deal with these nonlinear system that cannot be coped with the Lipschitz operator based on the right coprime factorization approach.

Corollary 3.1 *The class $Lip_\alpha(U, Y)$ of the all L_α operators form U to Y defined as **Definition 3.1** is a linear space over the real number field \mathbf{R} . Moreover, if $T_1, T_2 \in Lip_\alpha(U, Y)$ and $a \in \mathbf{R}$, then the following results are obtained:*

- (1) $\|T_1\|_\alpha = 0$ only if T_1 is a constant operator;
- (2) $\|T_1 + T_2\|_\alpha \leq \|T_1\|_\alpha + \|T_2\|_\alpha$;
- (3) $\|aT_1\|_\alpha \leq |a| \|T_1\|_\alpha$.

Note that $Lip_\alpha(U, Y)$ be the subset of $\mathcal{N}(U, Y)$ (the family of all nonlinear operators mapping from U to Y) with each element T with $\|T\|_\alpha < \infty$. It is clear that an element T of $\mathcal{N}(U, Y)$ is in $Lip_\alpha(U, Y)$ if and only if there is a number $L \geq 0$ such that $\|T(u_1) - T(u_2)\| \leq L \|u_1 - u_2\|^\alpha$ for all $u_1, u_2 \in U$.

After that, the norm of the L_α operator is considered, which is the fundamental premises using the definition of the L_α operator for the robust right

coprime factorization approach. For any fixed $u_0 \in U$, define a norm for all $T \in Lip_\alpha(U, Y)$ as follows,

$$\| T \|_{Lip}^\alpha := \| T(u_0) \| + \| T \|_\alpha$$

where $\| T \|_\alpha$ is defined as (3.1). $\| T \|_{Lip}^\alpha$ is called the L_α norm of the L_α operator T .

A convenient choice for u_0 is $u_0 = 0$, where note that $T(0)$ is not zero in general if T is nonlinear. To prove $\| T \|_{Lip}^\alpha$ to be a norm of the L_α operator, T , it amounts to showing that $\| T \|_{Lip}^\alpha = 0$ implies $T = 0$, where 0 is the zero operator. This, however, is an immediate consequence of result (1) of **Corollary 3.1**.

Definition 3.2 Let U^e and Y^e be two extended linear spaces, which are associated respectively with two given Banach spaces U and Y , where a Banach space is a complete vector space with a norm. An operator $D : U^e \rightarrow Y^e$ is called a generalized L_α operator on U^e if there exists a constant $L \geq 0$ such that

$$\| [D(u_1)]_T - [D(u_2)]_T \| \leq L \| [u_1]_T - [u_2]_T \|^\alpha$$

for all $u_1, u_2 \in U^e$ and for all $T \in [0, \infty)$.

Note that the least such constant L is given by

$$L := \sup_{T \in [0, \infty)} \sup_{\substack{u_1, u_2 \in U^e \\ u_1 \neq u_2}} \frac{\| [D(u_1)]_T - [D(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|^\alpha} \quad (3.2)$$

which is a semi-norm for the generalized L_α operator. The actual norm for a generalized L_α operator D is given by

$$\begin{aligned} \| D \|_{Gli}^\alpha &= \| D(u_0) \| \\ &+ \sup_{T \in [0, \infty)} \sup_{\substack{u_1, u_2 \in U^e \\ u_1 \neq u_2}} \frac{\| [D(u_1)]_T - [D(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|^\alpha} \end{aligned}$$

for any fixed $u_0 \in U^e$.

For simplicity, throughout the following chapter, by a L_α operator we always mean one defined in this generalized sense. The reason for considering the extended linear space is that all control signals in real application are time-limited, but in the control processing, we sometimes do not know when the processing will stop. Hence, because of the finite time duration of practice, the function $f(t) = e^t + t^2, t \geq 0$ and the like should be considered under the underlying spaces.

3.2.2 An example to explain L_α operator

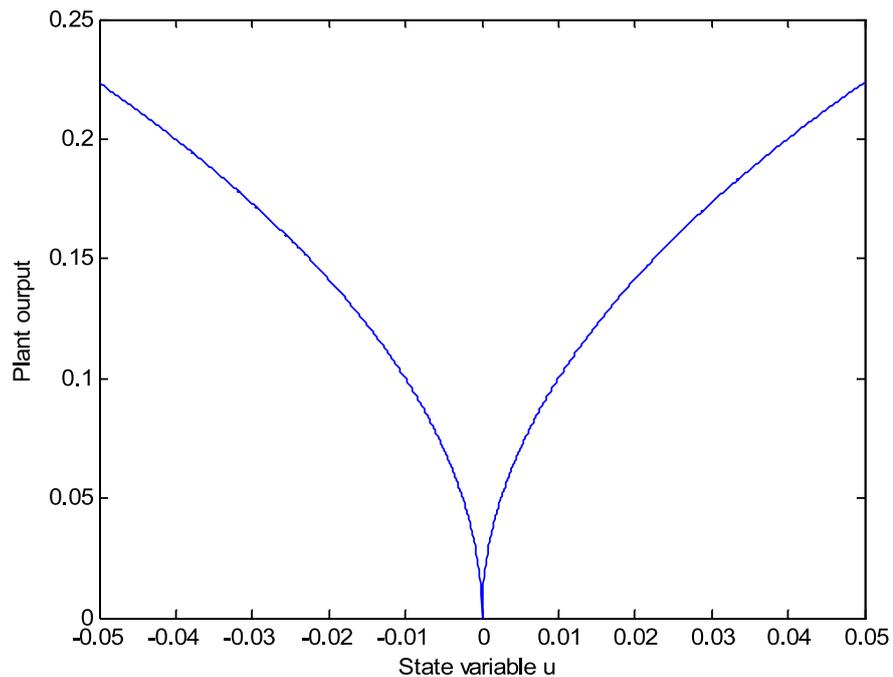


Figure 3.1: An example to explain the L_α operator

The nonlinear system $P(u)(t) = |u(t)|^{\frac{1}{2}}$ defined on $[-1, 1]$. It is not a

Lipschitz operator, because this nonlinear system becomes infinitely steep as u approaches 0 as shown in Figure 3.1 based on fact that derivation of the nonlinear system trends to infinite as u approaches 0. That is, there is no constant finite number to satisfy the Lipschitz condition. However, it is a L_α operator by choosing the parameter such that $0 < \alpha < \frac{1}{2}$. Thus, the proposed L_α operator can deal with a broader class of nonlinear systems based on the proposed example than the Lipschitz operator.

Note that as to the design of the proposed operator exponent, that while considering the nonlinear system, there are some points in some case like the example shown, which leads to the norm tending to infinitely. For dealing with these systems, the exponent should be designed to be satisfied with the norm requirement according to the whole infinitely points.

3.2.3 Extended right coprime factorization

In this subsection, after introducing the definition of the generalized L_α operator of the extended linear space, the extended right coprime factorization approach will be discussed under the framework of the proposed operator for stabilizing the nonlinear system. Firstly, as the Lipschitz operator, the following lemma are proposed for guaranteeing the fundamental of factorized operators.

Lemma 3.1 it The set $Lip_\alpha(U, Y)$ of all L_α operators from the normed space U to Y is a Banach space under the L_α norm.

Proof. Based on **Corollary 3.1**, $Lip_\alpha(U, Y)$ is a linear space since Y is a linear space. Hence, it suffices to verify its completeness under the L_α norm.

Let T_n be a Cauchy sequence in $Lip_\alpha(U, Y)$ such that $\|T_m - T_n\|_{Lip}^\alpha \rightarrow 0$

as $m, n \rightarrow \infty$. Then, for any $u \in U$,

$$\begin{aligned} \|T_m(u) - T_n(u)\| &\leq \| (T_m - T_n)(u) - (T_m - T_n)(u_0) \| \\ &\quad + \|T_m(u_0) - T_n(u_0)\| \\ &\leq \| (T_m - T_n) \|_{Lip}^\alpha \|u - u_0\| \\ &\quad + \|T_m(u_0) - T_n(u_0)\| \end{aligned}$$

which shows that the sequence T_n is in fact uniformly Cauchy on each bounded subset of U . Since Y is complete, $T(u)$ exists and so is unique. Moreover, since T_n is a Cauchy sequence, $\lim_{n \rightarrow \infty} \|T_n\|_{Lip}^\alpha = c$, where c is a constant number, so that

$$\begin{aligned} \|T(u_1) - T(u_2)\| &= \lim_{n \rightarrow \infty} \|T_n(u_1) - T_n(u_2)\| \\ &\leq \lim_{n \rightarrow \infty} \|T_n\|_{Lip}^\alpha \|u_1 - u_2\| \\ &= c \|u_1 - u_2\| \end{aligned}$$

for all $u_1, u_2 \in U$. This shows that $T \in Lip_\alpha(U, Y)$ with

$$\|T\|_{Lip}^\alpha \leq c + \|T(u_0)\|$$

We finally verify that $\|T_n - T\|_{Lip}^\alpha \rightarrow 0$ as $n \rightarrow \infty$. Since the above also proves $\|T_n(u_0) - T(u_0)\| \rightarrow 0$ as $n \rightarrow \infty$, for $\varepsilon > 0$, we can let N be such that $\|T_m - T_n\|_{Lip}^\alpha \leq \frac{\varepsilon}{2}$ and $\|(T_m - T)(u_0)\| \leq \frac{\varepsilon}{2}$ for $m, n \geq N$. Then, for any $u_1, u_2 \in U$, it follows that

$$\begin{aligned} \|(T - T_n)(u_1) - (T - T_n)(u_2)\| &= \lim_{m \rightarrow \infty} \|(T_m - T_n)(u_1) - (T_m - T_n)(u_2)\| \\ &\leq \lim_{m \rightarrow \infty} \|T_m - T_n\|_{Lip}^\alpha \|u_1 - u_2\| \\ &\leq \frac{\varepsilon}{2} \|u_1 - u_2\| \end{aligned}$$

So that $\|T_n - T\|_{Lip}^\alpha \leq \varepsilon$ for $n \geq N$. This implies that $\|T_n - T\|_{Lip}^\alpha \leq \varepsilon$ as $n \rightarrow \infty$, completing the proof of the lemma.

Next the extended right coprime factorization will be discussed based on the proposed L_α operators. The extended right coprime factorization is mainly built by the framework of L_α operators, whose parameter is supposed to be determined using off-line method. In order to formulate a nonlinear analogue of the extended right coprime factorization, a commonly used input-output stability principle is employed.

Theorem 3.1 *Suppose that $P : U \rightarrow Y$ be a causal, well-posedness and stabilizable nonlinear systems satisfied with the L_α condition defined as **Corollary 3.1**, and P has a right factorization $P = ND^{-1}$ on U , where N and D are stable, causal and satisfied with L_α condition. Provided that there exist two controllers A , B , such that both of them are stable, causal and satisfied with the following Bezout identity:*

$$AN(w)(t) + BD(w)(t) = M(w)(t)$$

where M is unimodular from U to U , then P is bounded input bounded output stable.

Proof. First, based on the condition, the following diagram can be obtained in the context of L_α operator.

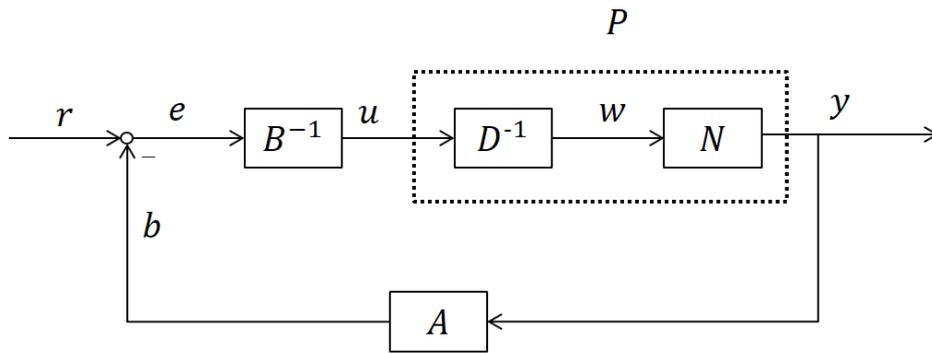


Figure 3.2: The proposed design scheme based on L_α operator

From Figure 3.2, we can obtain

$$\begin{aligned} AN(w)(t) + BD(w)(t) &= A(y)(t) + B(u)(t) \\ &= b(t) + e(t) \\ &= r(t) \end{aligned} \tag{3.3}$$

According to $AN(w)(t) + BD(w)(t) = M(w)(t)$,

$$w(t) = M^{-1}r(t).$$

Thus, Figure 3.2 can be rewritten as Figure 3.3 and combining with the stable property of N , the nonlinear system P are bounded input bounded output stable.

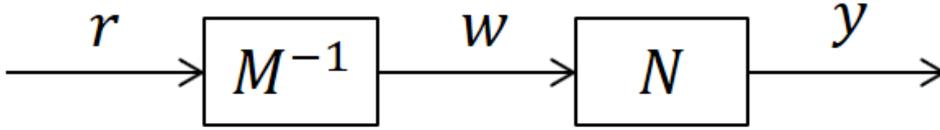


Figure 3.3: The equivalent of P

The proof is completed.

Based on **Theorem 3.1**, the P is said to have an extended right coprime factorization under the L_α condition. In the following section, we will apply it to consider robust control for the nonlinear systems with bounded perturbations.

3.3 Robust control using extended robust right coprime factorization

3.3.1 Problem statement

As to the nonlinear system with bounded perturbations shown in Figure 3.4, the overall system ΔP is denoted as $\Delta P = P + \Delta P$. Assume that the

right factorization of P and ΔP are denoted as $P = ND^{-1}$ and $P + \Delta P = (N + \Delta N)D^{-1}$, respectively, where N is stable; D is stable and invertible, and ΔN is bounded perturbations of the nonlinear system. Note that, P , $P + \Delta P$ are in the context of L_α operators.

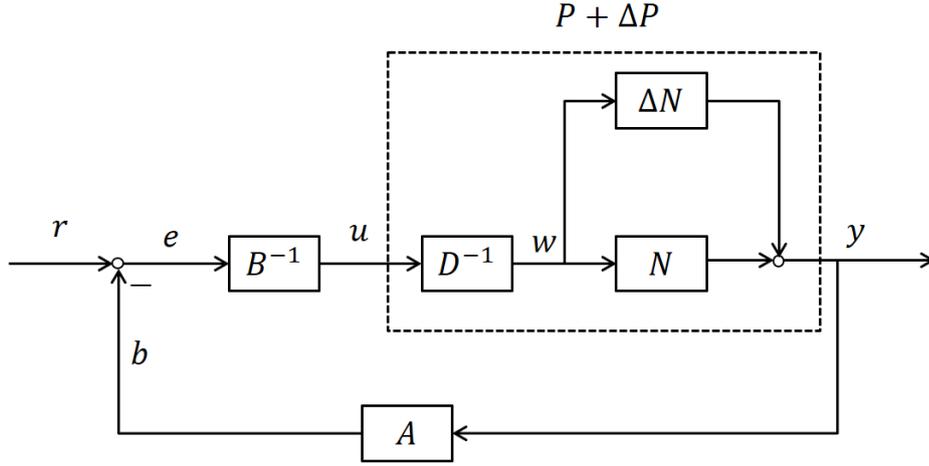


Figure 3.4: The perturbed nonlinear system

For the right coprime factorization, there are some results on robust stability for nonlinear system satisfied with Lipschitz operator. In [74], robust stability of the nonlinear system with bounded perturbations shown in Figure 3.4 can be guaranteed provided that

$$A(N + \Delta N) - AN = 0. \quad (3.4)$$

In [59], authors propose the following condition to guarantee robust stability of the perturbed nonlinear system

$$\| [A(N + \Delta N) - AN]M^{-1} \| < 1. \quad (3.5)$$

Under condition (3.5), the operator $A(N + \Delta N) + BD$ can be guaranteed to be unimodular.

Note that when the right coprime factorization approach is applied to consider a nonlinear system, the property of the nonlinear system would be explicit, verifying whether the nonlinear system is satisfied with the Lipschitz condition. If not, the right coprime factorization approach cannot be used to establish the Bezout identity for guaranteeing stability of the nonlinear system.

In this dissertation, in order to consider the issues mentioned above, a practical design scheme is proposed for stabilizing the perturbed nonlinear systems using a new unimodular operator instead of M in [59], which relaxes the restriction of the previous methods, at least for the mathematical aspects of the perturbed nonlinear systems. That is, in the following subsection, the robustness of the nonlinear system with bounded perturbations is considered based on the L_α operator. Practical design schemes for dealing with the bounded perturbations is proposed to guarantee robust stability of the perturbed nonlinear system.

3.3.2 Mathematical preliminaries

The following preparatory results which are used throughout this subsection for developing the sufficient conditions for guaranteeing robust stability for the nonlinear system with bounded perturbations. Throughout this section, the whole operators are considered in the context of the definition of the L_α operator.

Lemma 3.2 *Provided that a L_α operator $H \in Lip_\alpha(U_s)$, where U_s is the stable space of U which is a Banach space, is satisfied with $\|H\|_{Lip}^\alpha < 1$, then $I - H$ is invertible, in $Lip_\alpha(U_s, U_s)$ and*

$$\|(I - H)^{-1}\|_{Lip}^\alpha \leq (1 - \|H\|_{Lip}^\alpha)^{-1} \quad (3.6)$$

Proof. In fact, for each $u_1, u_2 \in U_s$,

$$\begin{aligned} \|(I - H)u_1 - (I - H)u_2\| &\geq \|u_1 - u_2\| - \|Hu_1 - Hu_2\| \\ &\geq (1 - \|H\|_{Lip}^\alpha) \|u_1 - u_2\| \end{aligned}$$

Thus, the operator $I - H$ is injective.

After that, the fact that $I - H$ is surjective is verified as follows.

Define that $K_0 := I$ and $K_n := I + HK_{n-1}, \forall n = 1, 2, \dots$, for each $u \in U$

$$\|K_{n+1}(u) - K_n(u)\| \leq [\|H\|_{Lip}^\alpha]^n \|H(u)\| \quad n = 1, 2, \dots$$

Then for any positive integer m , obtain

$$\begin{aligned} \|K_{n+m}(u) - K_n(u)\| &= \left\| \sum_{k=0}^{m-1} (K_{n+k+1}(u) - K_{n+k}(u)) \right\| \\ &\leq \sum_{k=0}^{m-1} [\|H\|_{Lip}^\alpha]^{n+k} \|H(u)\| \\ &\leq \frac{[\|H\|_{Lip}^\alpha]^n \|H(u)\|}{1 - \|H\|_{Lip}^\alpha} \end{aligned} \quad (3.7)$$

Since $\|H\|_{Lip}^\alpha < 1$ and U is a Banach space, then

$$S(u) = \lim_{n \rightarrow \infty} K_n(u)$$

exists and

$$\begin{aligned} \|S(u) - K_n(u)\| &= \lim_{n \rightarrow \infty} \|K_{n+m}(u) - K_n(u)\| \\ &\leq \frac{[\|H\|_{Lip}^\alpha]^n \|H(u)\|}{1 - \|H\|_{Lip}^\alpha} \end{aligned} \quad (3.8)$$

Since H is a L_α operator and thus is continuous, we have

$$S(u) = \lim_{n \rightarrow \infty} K_n(u) = \lim_{n \rightarrow \infty} (I + HK_{n-1})u = I(u) + HS(u)$$

that is,

$$S = I + HS$$

namely, $(I - H)S = I$, which implies that $I - H$ is surjective in $Lip_\alpha(U_s)$.

Then for $u_1, u_2 \in \mathcal{R}(I - H)$,

$$\| (I - H)^{-1}u_1 - (I - H)^{-1}u_2 \| \leq (1 - \| H \|_{Lip}^\alpha)^{-1} \| u_1 - u_2 \|^\alpha$$

Thus, we can get the conclusion, $I - H$ is invertible and

$$\| (I - H)^{-1} \|_{Lip}^\alpha \leq (1 - \| H \|_{Lip}^\alpha)^{-1}$$

The proof of this lemma is completed.

Lemma 3.3 *The nonlinear system satisfied with L_α condition has a right coprime factorization if and only if the composite operator $I + APB^{-1}$ is injective and its inverse is causal, and all the operators A, B, D, N, B^{-1} , and $(I + APB^{-1})$ are causally stable.*

Proof. Observe that provided that a nonlinear system has a right coprime factorization, there are two causal operators $N : U \rightarrow U^e$ and $D : U \rightarrow U^e$ with a causal inverse D^{-1} such that $ND^{-1} = P$ and $AN + BD = M$. Since $B : U \rightarrow U$ is one-to-one and is onto $\mathcal{R}(B)$, $\mathcal{R}(B^{-1}) = X \subset \mathcal{D}(D^{-1})$. Moreover, $\mathcal{R}(D) = U^e$.

$$\begin{aligned} I + APB^{-1} &= I + AND^{-1}B^{-1} \\ &= [BD + AN]D^{-1}B^{-1} \\ &= MD^{-1}B^{-1} \end{aligned}$$

which implies that the operator $(I + APB^{-1}) : \mathcal{R}(B) \rightarrow U$ is causal, one-to-one and onto, and hence is causally invertible, with the inverse equal to BD .

Conversely, provided that the operator $(I + APB^{-1}) : \mathcal{R}(B) \rightarrow U$ is causally invertible, then we can define

$$D = B^{-1}(I + APB^{-1})^{-1}M$$

Due to $(I + APB^{-1})^{-1} : U \rightarrow \mathcal{R}(B)$ is onto, $\mathcal{R}((I + APB^{-1})^{-1}) = \mathcal{R}(B) = \mathcal{D}(B^{-1})$. Combining with $\mathcal{R}(B^{-1}) = U$, the operator D defined is causal, one-to-one and onto. Therefore,

$$D = [M^{-1}(I + APB^{-1})B]^{-1} = [M^{-1}(B + AP)]^{-1}$$

Consequently, $D^{-1} = M^{-1}(AP + B) : U^e \rightarrow U$ exists and is causal. If define $N = PD$, then N is causal, and obtain both $ND^{-1} = PDD^{-1} = P$ and

$$AN + BD = (AND^{-1} + B)D = (AP + B)D = MD^{-1}D = M$$

Completes the proof of the lemma.

3.3.3 Robust stability

Theorem 3.2 *Suppose that U and Y be Banach spaces. Let $G, R \in Lip_\alpha(U_s, Y_s)$, where U_s, Y_s are the stable spaces of U, Y , respectively, such that G is invertible in $Lip(U_s, Y_s)$ and satisfied with*

$$\|G - R\|_{Lip}^\alpha \|G^{-1}\|_{Lip}^\alpha < 1$$

Then, R is invertible in $Lip_\alpha(U_s, Y_s)$ with

$$\begin{aligned} \|R^{-1}\|_{Lip}^\alpha &\leq \|G^{-1}\|_{Lip}^\alpha \|R^{-1}(u_0)\| \\ &+ \frac{\|G^{-1}\|_{Lip}^\alpha}{1 - \|G - R\|_{Lip}^\alpha \|G^{-1}\|_{Lip}^\alpha} \end{aligned} \quad (3.9)$$

for any $u_0 \in U$.

Proof. By the **Lemma 3.2**, show that $\|G^{-1}R\|_{Lip}^\alpha$ is invertible in $Lip_\alpha(U_s, Y_s)$, considering

$$\|I - RG^{-1}\|_{Lip}^\alpha = \|G - R\|_{Lip}^\alpha \|G^{-1}\|_{Lip}^\alpha < 1$$

Also from (3.6),

$$\begin{aligned} \|(RG^{-1})^{-1}\|_{Lip}^\alpha &\leq \frac{1}{1 - \|I - RG^{-1}\|_{Lip}^\alpha} \\ &\leq \frac{1}{1 - \|G - R\|_{Lip}^\alpha \|G^{-1}\|_{Lip}^\alpha} \end{aligned} \quad (3.10)$$

Since $R = (RG^{-1})G$, we see that R has an inverse in $Lip_\alpha(U_s, Y_s)$, namely,

$$R^{-1} = G^{-1}(RG^{-1})^{-1}$$

Hence,

$$\|R^{-1}\|_{Lip}^\alpha \leq \|G^{-1}\|_{Lip}^\alpha \|(RG^{-1})^{-1}\|_{Lip}^\alpha$$

The estimate (3.9) follows from the above inequality, (3.10) and the definition of norm for the generalized L_α operator.

Note that **Theorem 3.2** provides a condition on how to guarantee an operator be invertible. This condition in **Theorem 3.2** is sufficient, not necessary. The reason for this result lies in that **Theorem 3.2** is obtained based on **Lemma 3.2**, but in the proof of **Lemma 3.2**, for proving the surjective property of the operator $I - H$, the condition $\|H\|_{Lip}^\alpha < 1$ is sufficient, not necessary.

After the preparatory work, we give the design scheme for the nonlinear system with bounded perturbations.

Theorem 3.3 *In terms of the nonlinear feedback system with bounded perturbations in Figure 3.4, provided that the following condition is guaranteed,*

$$\|(A(N + \Delta N) - B)\|_{Lip}^\alpha \|B^{-1}\|_{Lip}^\alpha < 1 \quad (3.11)$$

then $A(N + \Delta N)$ is an unimodular operator.

Proof. The fact that B is unimodular implies that B is invertible and B^{-1} is also stable.

Therefore, according to **Theorem 3.2**, we can obtain $A(N + \Delta N)$ is invertible.

$$\begin{aligned} & \| A(N + \Delta N)^{-1} \|_{Lip}^\alpha \leq \| B^{-1} \|_{Lip}^\alpha \| A(N + \Delta N)^{-1}(x_0) \| \\ & + \frac{\| B^{-1} \|_{Lip}^\alpha}{1 - \| B^{-1} - A(N + \Delta N) \|_{Lip}^\alpha \| B^{-1} \|_{Lip}^\alpha} \end{aligned}$$

Since

$$\begin{aligned} A(N + \Delta N) &= B - (B - A(N + \Delta N)) \\ &= [I - (B - A(N + \Delta N))B^{-1}]B \end{aligned}$$

Thus, $I - [B - A(N + \Delta N)]B^{-1}$ is proved to be invertible. Considering the inverse of $I - (B - A(N + \Delta N))B^{-1}$ is stable, the obtained inverse of $A(N + \Delta N)$ is stable. The proof of the theorem is completed.

Based on the **Theorem 3.3**, we will prove the perturbed Bezout identity is unimodular as the following theorem. Therefore, the proposed design scheme can guarantee the nonlinear system with bounded perturbations to be stable.

Theorem 3.4 *For the perturbed nonlinear system as shown in Figure 3.4, if the following condition*

$$\| BD \|_{Lip}^\alpha \| [A(N + \Delta N)]^{-1} \|_{Lip}^\alpha < 1 \quad (3.12)$$

is satisfied, then the nonlinear system with bounded perturbations is robust stable, i.e. $A(N + \Delta N) + BD$ is unimodular.

Proof. Since

$$\| BD \|_{Lip}^\alpha \| [A(N + \Delta N)]^{-1} \|_{Lip}^\alpha < 1$$

Thus,

$$\| [BD + A(N + \Delta N)] - A(N + \Delta N) \|_{Lip}^\alpha \| [A(N + \Delta N)]^{-1} \|_{Lip}^\alpha < 1$$

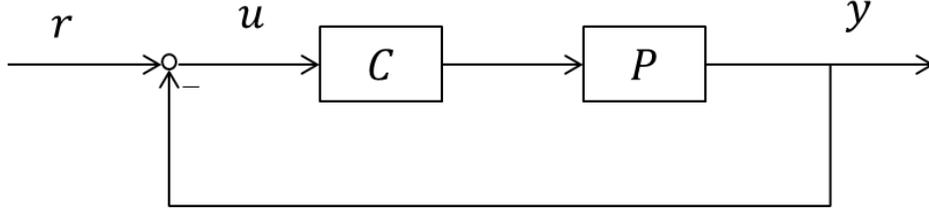
It follows that $A(N + \Delta N) + BD$ is unimodular by **Theorem 3.3**. Thus, the nonlinear system with bounded perturbations is satisfied with robust right coprime factorization, which results in that the overall system is stable.

Note that **Theorem 3.3** and **Theorem 3.4** are both sufficient conditions on guaranteeing robust stability of the nonlinear system with bounded perturbations. From the view of point of guaranteeing the unimodular property, the proposed conditions can guarantee it, but the proposed conditions are not necessary because of the conservativeness of **Theorem 3.2** and the nonlinearity property of the considered operator.

As for the perturbation ΔN existing in the considered nonlinear feedback system, we consider the perturbation ΔN is unknown, but ΔN has upper bound and lower bound. That is, there are exist two constants α and β such that $\alpha \leq \Delta N \leq \beta$. For guaranteeing the two conditions (3. 11) and (3.12) in general, we should do our best to use the knowledge of the above bounds, meanwhile, to consider the relationship of $\| (A(N + \Delta N) - B) \|_{Lip}^\alpha$ and $\| [A(N + \Delta N)]^{-1} \|_{Lip}^\alpha$ to design controllers A and B , where the two controllers can be selected by trial and error so far.

3.3.4 Simulation example

In this section, a numerical example is given to show the effectiveness of the proposed method. Consider a nonlinear feedback system shown as in Figure 3.5, where reference input, control input and plant output are r , u and y , respectively. We assume, in this nonlinear feedback system, that $X = L_\infty$ is the standard Banach space of real-valued measurable functions defined on $[0, \infty)$, with the associated extended linear space $X^e = L_\infty^e$. Suppose that the plant operator P is given by the following unstable, L_α and time-varying

Figure 3.5: A nonlinear system satisfying L_α condition

nonlinear operator, where $\alpha = \frac{1}{4}$:

$$P(u)(t) = \int_0^t [u(\tau)]^{\frac{1}{4}} d\tau + e^{\frac{2}{15}t} [u(t)]^{\frac{2}{5}}$$

Based on the proposed plant, the operators D , N are given as follows:

$$N(w)(t) = \int_0^t e^{-\frac{1}{12}\tau} [w(\tau)]^{\frac{1}{10}} d\tau + [w(t)]^{\frac{4}{25}}$$

$$D(w)(t) = e^{-\frac{1}{3}t} [w(t)]^{\frac{2}{5}}$$

The stability in terms of D , N is verified easily. And we can get the inverse operator of D is unstable from L_∞ to L_∞ .

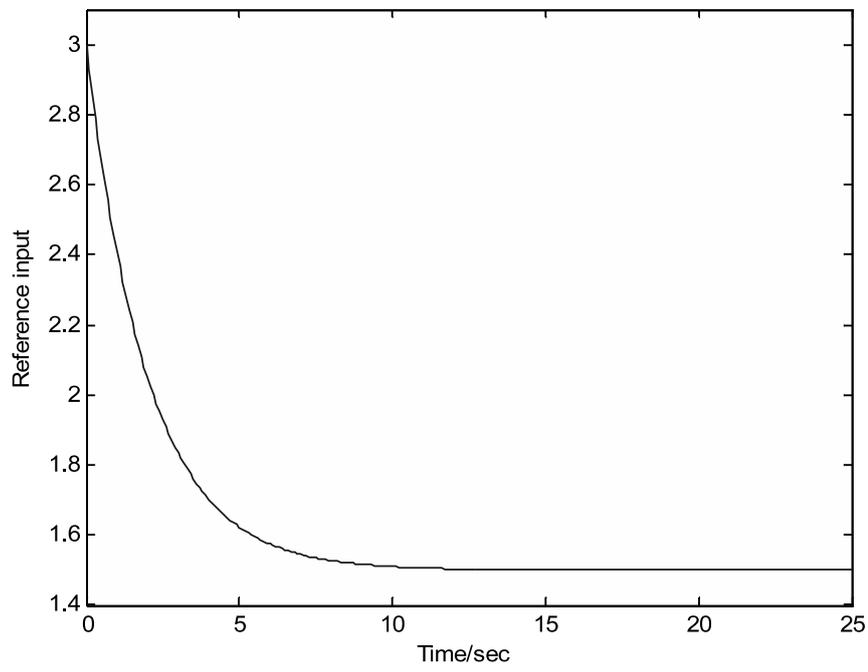
Next step for establishing a Bezout identity, we pick a stable controller A such that the $I - AN$ is invertible as follows,

$$A(y)(t) = (e^{-\frac{5}{6}t} - 1) \left[\int_0^t [u(\tau)]^{\frac{1}{4}} d\tau - y(t) \right]^{\frac{25}{4}}$$

Then, we have

$$AN(w)(t) = (e^{-\frac{5}{6}t} - 1) \int_0^t [u(\tau)]^{\frac{1}{4}} d\tau$$

$$- (e^{-\frac{5}{6}t} - 1) \left(\int_0^t e^{-\frac{1}{12}\tau} [w(\tau)]^{\frac{1}{10}} d\tau - [w(t)]^{\frac{4}{25}} \right)$$

Figure 3.6: Reference input r

Therefore, the controller B is provided based on the proposed controllers A ,

$$B(u)(t) = (I - AN)D^{-1}(u(t)) = [u(t)]^{\frac{5}{2}}$$

Finally, it can be verified that A and B satisfy the Bezout identity. Indeed, we have

$$(AN + BD)(w)(t) = I(w)(t) \quad (3.13)$$

According to the above analysis, the proposed unstable nonlinear system, P is stable by the proposed design scheme. Note that in order to realize the Bezout identity, the controllers are chosen from the the simplicity viewpoint. The controller A is chosen from the following set: $\mathcal{A} = \{A \in Lip_\alpha(X) : (I - AN)D^{-1} \in Lip_\alpha(X)\}$. In this case the controller A will stabilize the unstable operator D^{-1} . There is a cancellation of unstable factor between D^{-1} and $I - AN$, hence, the stability of the controller B is guaranteed. Aside from that, there exist a great number of chooses only if the designed controllers are satisfied with the Bezout identity condition. After that, the case of the nonlinear system with bounded perturbations is proposed to confirm the effectiveness of the robust design scheme.

After that, the perturbations and right factorization of the overall plant are given as follows, where the perturbations $\delta(t)$ is chosen as $\delta(t) = 0.5e^{-\frac{1}{15}t}$ for confirming the effectiveness of the proposed design scheme.

$$\begin{aligned} (P + \Delta P)(u)(t) &= \delta(t) \left(\int_0^t [u(\tau)]^{\frac{1}{4}} d\tau + e^{\frac{2}{15}t} [u(t)]^{\frac{2}{5}} \right) \\ (N + \Delta N)(w)(t) &= \delta(t) \int_0^t e^{-\frac{1}{12}\tau} [w(\tau)]^{\frac{1}{10}} d\tau + \delta(t) [w(t)]^{\frac{4}{25}} \\ D(w)(t) &= e^{-\frac{1}{3}t} [w(t)]^{\frac{2}{5}} \end{aligned}$$

Based on the above perturbed system, the conditions (3.11) and (3.12) are verified as shown in Figure 3.7 and Figure 3.8 to make the design scheme

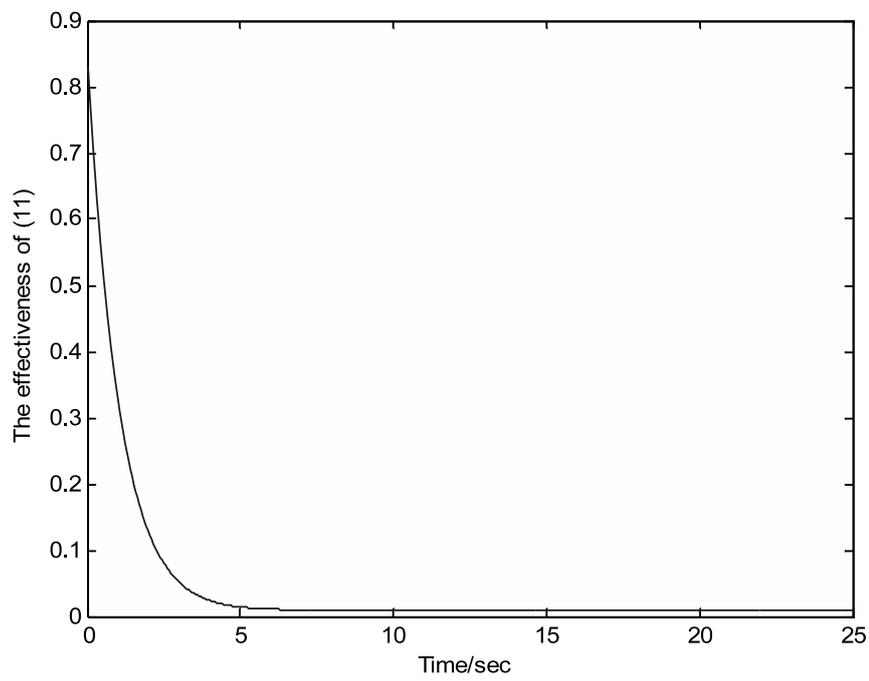


Figure 3.7: The effectiveness of (3.11)

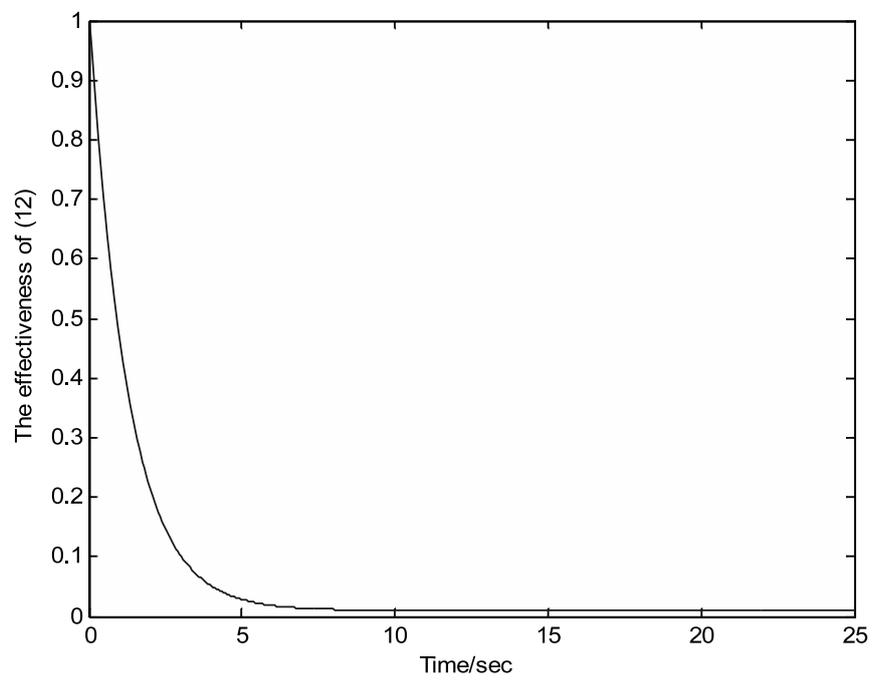
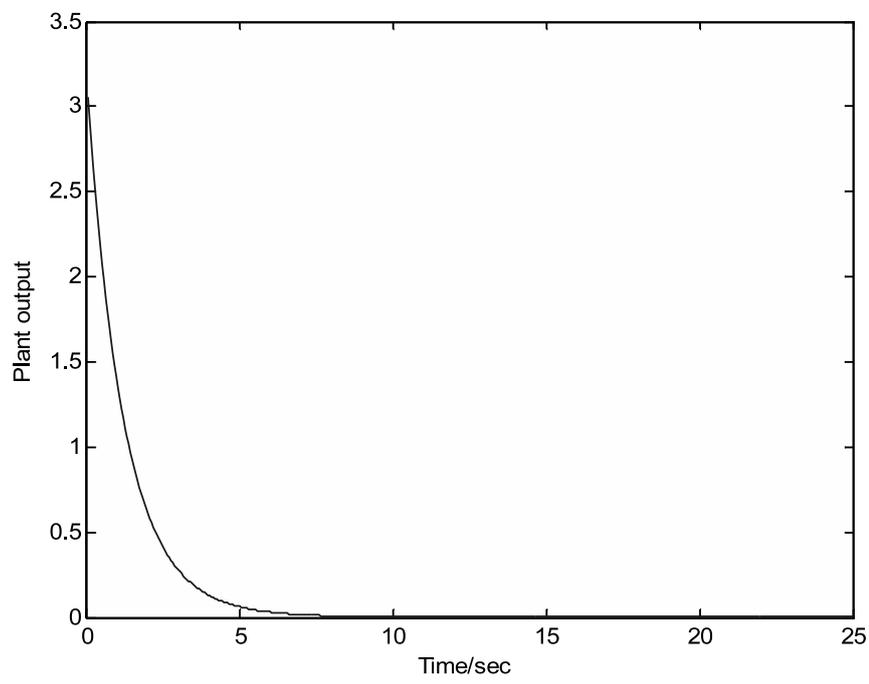


Figure 3.8: The effectiveness of (3.12)

Figure 3.9: Plant output y

available, where the reference input is chosen as $r(t) = 1.5(1 + e^{-\frac{1}{2}t})$ shown in Figure 3.6. As Figure 3.7 and Figure 3.8 are shown, the bounded perturbations is satisfied with the conditions for guaranteeing robust stability.

Next, the simulation result of the plant output of the nonlinear system with bounded perturbations is given in Figure 3.9. Thus, based on the simulation results, robust stability of the nonlinear system with bounded perturbations is obtained by the proposed design scheme.

3.4 Conclusion

In this chapter, extended right coprime factorization is discussed based on the proposed L_α operator and robust control for the nonlinear system with bounded perturbations is considered using a new unimodular operator for guaranteeing robust stability. The application range of the robust right coprime factorization approach was extended by the proposed operator. Feasible design schemes were proposed for avoiding the difficulties in calculation based on the proposed operator, which meant that robust stability of the perturbed nonlinear system can be guaranteed based on the proposed unimodular operator B . Finally, the effectiveness of the proposed design scheme was confirmed by a simulation example.

Chapter 4

Extended right coprime factorization and robust control using adjoint operator

4.1 Introduction

In Chapter 3, extended right coprime factorization and robust control for nonlinear systems satisfying L_α operator condition are discussed by using the proposed L_α operators. According to the proposed method, right coprime factorization is extended, which means that the application of operator-based right coprime factorization becomes broader. However, besides the above considered issues, there is few attention on how to factorize nonlinear systems with perturbations for a robust right coprime factorization and the rational boundedness for robust condition issue. Therefore, in this chapter for dealing with these issues, inner product and adjoint operators of Hilbert spaces are introduced. Adjoint operator-based extended robust right coprime factorization is proposed, which not only guarantees existence of the compensator for the isomorphism, but also combines with inner product providing a quantitative right factorization for the nonlinear system with perturbations. As to robust stability of the nonlinear system, a practical design scheme is

proposed by two steps, which is realizable due to avoiding the irrational number of robust stability and complex calculation for the considered nonlinear systems.

In Section 4.2, how to factorize a given nonlinear system is considered based on the adjoint operator of Hilbert spaces. The isomorphism relationship is reconstructed based on Hilbert spaces, which extends the application range of the isomorphism technique. That is, adjoint operator combining inner product of Hilbert spaces is employed to factorize a unstable system using isomorphism idea. Then, in terms of the obtained factors, coprime-ness property of the nonlinear system is discussed and the design schemes of stable controllers for Bezout identity are given, which guarantee that the obtained right factorization is coprime. In this section, stability of nonlinear systems is guaranteed using the extended right coprime factorization based on the proposed adjoint operator combining inner product of Hilbert spaces.

In Section 4.3, robust control design for the nonlinear system with perturbations is considered based on the obtained right coprime factorization in Section 4.2. Specially, first, the considered problem is discussed on irrational boundedness of the robust conditions. Second, mathematical preliminaries are given for developing the sufficient condition for robust stability of nonlinear systems with perturbations. Third, the robust control design scheme for the nonlinear system with perturbations is proposed to guaranteeing robust stability of the considered systems, while the proposed design scheme can avoid the issue that irrational boundedness holds the norm inequalities. That is, in the real application, the proposed design scheme is more practical. Finally, The given numerical example illustrates the effectiveness of the proposed methods.

In Section 4.4, extended right coprime factorization using adjoint operator of Hilbert spaces and rational boundedness of robust condition for the considered nonlinear systems are summarized.

4.2 Right coprime factorization with adjoint operator

4.2.1 Problem statement

There have been many significant results on robust control using robust right coprime factorization for nonlinear systems, however, among the results, there exists few attention on how to factorize a given nonlinear system with perturbations such that the perturbed system has a robust right coprime factorization. Therefore, in this section, the problem will be discussed for a class of nonlinear systems.

The aim of this section lies in obtaining robust right coprime factorization of the nonlinear system with perturbations quantitatively. Based on the proposed adjoint operator, a generalized form of the isomorphism relationship is defined extending the application range of isomorphism technique.

4.2.2 Adjoint operator

Definition 4.1. An isomorphism is defined from U to Y with a single binary operation, if an operator ψ from U to Y is bijective and satisfied with the following conditions,

$$\psi(u_1 \circ u_2) = \psi(u_1) \star \psi(u_2), \text{ for each } u_1, u_2 \in U$$

where \circ and \star are the suitable operations defined on U and Y , respectively.

In order to apply the isomorphism technique, a compensator is designed based on adjoint operators of Hilbert spaces. Firstly, the definition of inner product is given as follows.

Definition 4.2. A mapping $(u_1, u_2) \rightarrow \langle u_1, u_2 \rangle$, from $U \times U$ to the field of complex numbers \mathbb{C} , such that

$$1. \langle au_1 + bu_2, u_3 \rangle = a \langle u_1, u_3 \rangle + b \langle u_2, u_3 \rangle;$$

$$2. \langle u_2, u_1 \rangle = \overline{\langle u_1, u_2 \rangle};$$

$$3. \langle u_1, u_1 \rangle \geq 0. \langle u_1, u_1 \rangle = 0 \text{ only when } u_1 = 0.$$

whenever $u_1, u_2, u_3 \in U$ and $a, b \in \mathbb{C}$. Then, the mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on U .

Based on proposed inner product, a norm of an element of the space U can be obtained.

Lemma 4.1. *If $\langle \cdot, \cdot \rangle$ is an inner product on a complex vector space U , the equation*

$$\|x\| = \sqrt{\langle x, x \rangle} \quad (4.1)$$

defines a norm $\|\cdot\|$ on U , where $x \in U$.

Proof. Based on (4.1) defined, it is apparent that $\|x\| \geq 0$ and $\|x\| = a\|x\|$ whenever $x \in U$ and $a \in \mathbb{C}$. Moreover, owing to the definition of the inner product, $\|x\| = 0$ only when $x = 0$. The following inequality can be written in the form

$$|\langle x, y \rangle| \leq \sqrt{\langle x, x \rangle} \sqrt{\langle y, y \rangle} = \|x\| \|y\|$$

Also we have,

$$\begin{aligned} \|x + y\|^2 &\leq \|x\|^2 + 2|\langle x, y \rangle| + \|y\|^2 \\ &\leq \|x\|^2 + 2\|x\|\|y\| + \|y\|^2 \\ &= (\|x\| + \|y\|)^2 \end{aligned}$$

Thus, $\|x + y\| \leq \|x\| + \|y\|$ for each $x, y \in U$. Based on the definition of norm, proof of the lemma is completed.

Definition 4.3. A norm space U is called to be a Hilbert space if its norm can be defined and is complete, as in (4.1).

In this chapter, the input space U and output space Y are considered in the context of the Hilbert space. Next, the definition of adjoint operator is introduced.

Definition 4.4. If $T \in Lip(U, Y)$, there is an element T^* of $Lip(Y, U)$ such that

$$\langle T^*u_1, y_1 \rangle = \langle u_1, Ty_1 \rangle$$

then T^* is called to be an adjoint operator of T .

Moreover, the following properties can be obtained:

1. $(aS + bT)^* = \bar{a}S^* + \bar{b}T^*$
2. $(RS)^* = S^*T^*$
3. $(T^*)^* = T$

Based on the proposed adjoint operator, a sufficient condition is proposed in following subsection to guarantee existence of isomorphism for the nonlinear system with perturbations.

4.2.3 Factorization method based on adjoint operator

Theorem 4.1. *In terms of the input space and output space, U and Y , respectively, let $T \in Lip(U, Y)$, then T is an isomorphism from U onto Y if $T^{-1} = T^*$.*

Proof. The operator T is an isomorphism from U to Y if it is both invertible and norm preserving. Accordingly, suppose T has an inverse, and it suffices to show that T preserves norms if $T^*T = I$. Since

$$\langle T^*Tx, x \rangle - \langle x, x \rangle = \langle Tx, Tx \rangle - \langle x, x \rangle \quad (4.2)$$

for each $x \in U$. Thus, if $T^* = T^{-1}$, the operator T is an isomorphism.

The proof of the theorem is completed.

As to the existence of isomorphism for the nonlinear system with perturbations, a compensator S is designed to make $\tilde{D} = DS$ satisfied with $\tilde{D}^{-1} = (DS)^{-1} = S^{-1}D^{-1} = \tilde{D}^*$ as shown in Figure 4.1 based on **Theorem 4.1**. The nonlinear system with perturbations can be rewritten as shown in Figure 4.2. After that, robust right factorization of the nonlinear system with

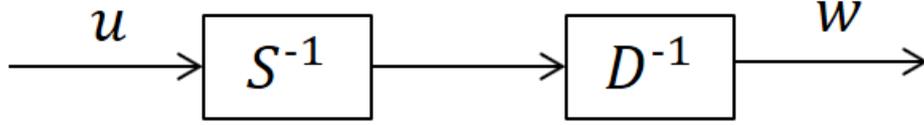


Figure 4.1: The proposed system with the compensator S^{-1}

perturbations will be considered quantitatively based on the inner product of Hilbert space.

In this dissertation, \circ and \star are defined as follows, if each $u_1, u_2 \in U$,

$$\circ : u_1 \circ u_2 = \langle u_1, u_2 \rangle \quad (4.3)$$

$$\star : \phi(u_1) \star \phi(u_2) = \langle \phi(u_1), \phi(u_2) \rangle = \phi(u_1 + \phi(u_2)). \quad (4.4)$$

After that, we can propose different form of inner product to satisfy the different requirements of nonlinear systems depending on analysis, which leads to the extension of application range of the isomorphism technique.

Theorem 4.2. *Suppose that the system shown in Figure 4.2, robust right factorization of the nonlinear system with perturbations can be realized using isomorphism defined as (4.3) and (4.4) based on the inner product of Hilbert space.*

Proof. Considering the linear space W is the isomorphic space of U , assume the isomorphism to be ϕ , thus, $w = \phi(u)$. Obtain

$$\phi(u_1 \circ u_2) = \phi(u_1) \star \phi(u_2).$$

where \circ and \star are defined as (4.3) and (4.4), respectively.

Form the bijective property of isomorphism, the following equation can be established:

$$\langle \phi(u), u \rangle = 2\phi(u). \quad (4.5)$$

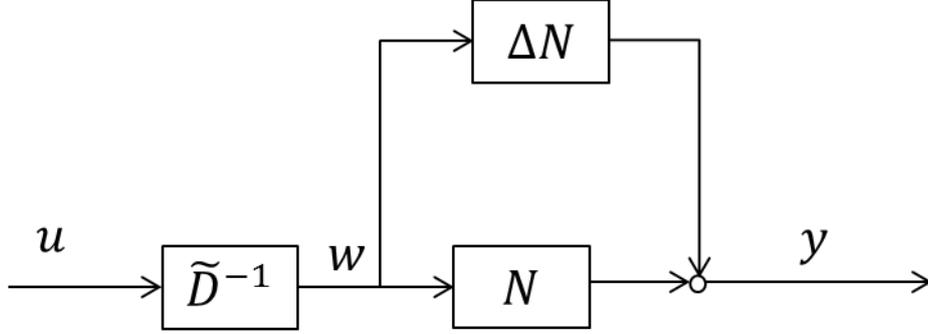


Figure 4.2: The nonlinear system with perturbations by isomorphism

Then according to the form of (8), its solution can be obtained as follows,

$$\phi(u)(t) = \psi(t)\Psi(u)(t),$$

where $0 < \min(\psi(t)) < \psi(t) < \max(\psi(t)) < \infty$ for $t \in [0, \infty)$, both $\psi(t)$ and $\Psi(u)$ are known operators.

Thus, robust right factorization of the nonlinear system with perturbations can be obtained as shown in (4.6) and (4.7), respectively:

$$\tilde{D}^{-1}(u)(t) = \psi(t)\Psi(u)(t) \quad (4.6)$$

$$(N + \Delta N)(w)(t) = \psi(t)\Psi(w)(t) + \Delta(t)\Psi(w)(t) \quad (4.7)$$

The proof of the theorem is completed.

From **Theorem 4.2**, using the isomorphism, robust right factorization of nonlinear system with perturbations is obtained. Next, two stable controllers A and B of Bezout identity will be considered. How to design A and B directly determines whether the nonlinear system with perturbations can be robustly stable. Therefore, in the following subsection, existences of two stable controllers A and B will be discussed.

4.2.4 Existence of two controllers A and B

Theorem 4.3. *As to the nonlinear system shown in Figure 4.2, there exist two stable controllers A and B such that the output can track to the reference input.*

Proof. Suppose that the two controllers A and B exist, according to the **Lemma 2.6**, the following condition of the obtained nonlinear system with perturbations can be satisfied :

$$A(N + \Delta N)(w)(t) + BD(w)(t) = N + \Delta N(w)(t)$$

Thus, the controller A is proposed as follows:

$$A(y)(t) = \frac{m}{n}y(t) + b \quad (4.8)$$

where n , m are real numbers, b is a parameter, $0 < m < \infty$, $0 < n < \infty$. According to the designed controller A, we can find the fact that A is stable and the operator $I - A$ is also invertible.

Since,

$$(I - A)^{-1}(e)(t) = \frac{n}{n - m}e(t) + b.$$

Therefore, based on **Lemma 2.6** B is obtained as follows:

$$B(u)(t) = (I - A)(N + \Delta N)\tilde{D}^{-1}(u)(t) \quad (4.9)$$

According to the controller B, B^{-1} can be driven:

$$B^{-1}(e)(t) = \tilde{D}(N + \Delta N)^{-1}(I - A)^{-1}(e)(t)$$

The proof of the theorem is completed.

From **Theorem 4.3**, the existence of the two controllers A and B is confirmed for the obtained robust right factorization. In the following section, the rational boundedness of robust condition will be discussed for guaranteeing robust stability of the nonlinear perturbed system.

4.3 Robust control for perturbed nonlinear systems

4.3.1 An example showing necessity of proposed method

In order to show necessity of redesigning robust stability condition, an example is given.

An Example Showing Necessity of Redesigning

Considering the real plant $P + \Delta P$, it is defined as follows [80], where $|\delta(t)| < 1$

$$(P + \Delta P)(u)(t) = \frac{1}{15}(e^{2t} + (1 + \delta(t))e^t + \delta(t) + (\delta(t) + \delta^2(t))e^{-t})u^2(t)$$

Form the isomorphism technique in [17], the robust right factorization are obtained as follows:

$$(N + \Delta N)(w)(t) = \frac{1 + (1 + \delta(t))e^{-t}}{15}w^2(t)$$

$$(D + \Delta D)^{-1}(u)(t) = \sqrt{e^{2t} + \delta(t)}u(t)$$

Then, the controllers A and B can be designed as follows:

$$A(y)(t) = \frac{b}{a}y(t),$$

$$B(u)(t) = \frac{b}{a} \left(\frac{1 + (1 + \delta(t))e^{-t}}{15} \right)^3 u^4(t)$$

where $0 < b < a < \infty$.

In terms of the robust stability condition [80], $(8 - 4\sqrt{3})b < a < (8 + 4\sqrt{3})b$, the controller A is supposed to satisfy the above condition, then the overall system can be robustly stable.

When it is applied to the practical systems, the irrational number boundary cannot be precisely determined because the irrational number boundary cannot measure the practical system in many cases like the affection of uncertainties. Thus, a precise design scheme is necessary to deal with the bounded perturbation of the nonlinear system.

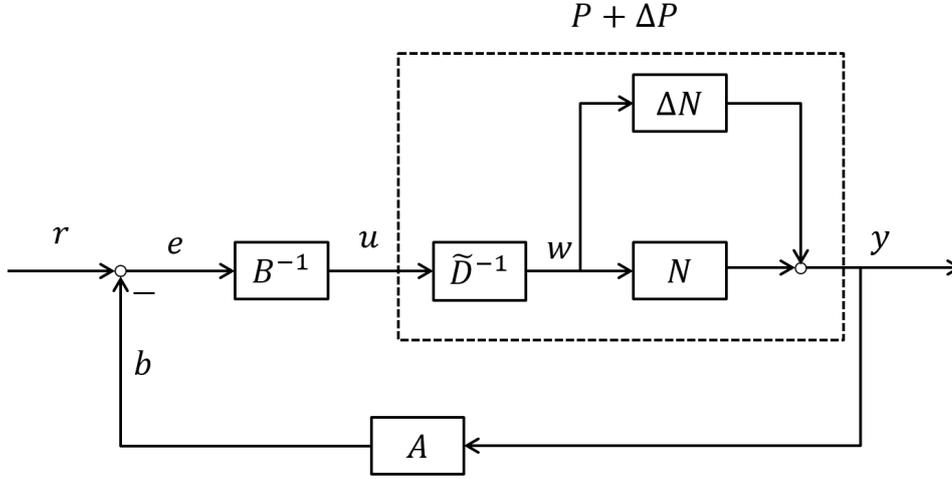


Figure 4.3: New proposed nonlinear system with perturbations by isomorphism

4.3.2 Rational boundedness for robust control condition

In the following subsection, robust control of the considered system as shown in Figure 4.3 will be studied.

Based on **Theorem 4.3**, two stable controllers A and B of robust right coprime factorization exist for the nonlinear system with perturbations. Thus, in this subsection, robust stability of the nonlinear system with perturbations will be discussed.

Theorem 4.4. *As to the nonlinear system shown in Figure 4.3, for the two stable controllers A and B are in forms of (4.8) and (4.9), respectively, if the following conditions are satisfied:*

$$0 < m < 2n \quad (4.10)$$

$$m < n < 2m \quad (4.11)$$

then, the following equations

$$\| [A(N + \Delta N) - (N + \Delta N)] \| \| (N + \Delta N)^{-1} \| < 1$$

$$\| B\tilde{D} \| \| [A(N + \Delta N)]^{-1} \| < 1$$

can be satisfied. That is, $A(N + \Delta N)$ and $A(N + \Delta N) + B\tilde{D}$ are unimodular.

Proof. First, $\| [A(N + \Delta N) - (N + \Delta N)] \| \| (N + \Delta N)^{-1} \| < 1$ will be proved.

In order to prove the inequality of the Lipschitz norm, an operator $H(y)$ is proposed as shown in (4.12).

$$\begin{aligned} H(y)(t) &= [A(N + \Delta N) - (N + \Delta N)](N + \Delta N)^{-1}(y)(t) \\ &= A(y)(t) - I(y)(t) \end{aligned} \quad (4.12)$$

where $I(\cdot)$ is the identity operator.

From (4.12) and the Lipschitz norm, the coefficient of (4.12) can be obtained:

$$\begin{aligned} \alpha(H) &= \alpha(A) - \alpha(I) \\ &= \frac{m}{n}y + b - y \\ &= \left(\frac{m}{n} - 1\right)y + b \end{aligned} \quad (4.13)$$

Thus, based on condition (4.10), $0 < m < 2n$, the following inequality is obtained,

$$\begin{aligned} &\| [A(N + \Delta N) - (N + \Delta N)] \| \| (N + \Delta N)^{-1} \| \\ &= \sup_{T \in [0, \infty)} \sup_{\substack{y_1, y_2 \in Y \\ y_1 \neq y_2}} \frac{\| [H(y_1)] - H(y_2) \|}{\| [y_1] - [y_2] \|} \\ &< \sup_{T \in [0, \infty)} \left\| \frac{m}{n} - 1 \right\| \\ &< 1 \end{aligned} \quad (4.14)$$

Therefore, based on **Theorem 3.2**, the operator $A(N + \Delta N)$ is unimodular.

Next, $\| B\tilde{D} \| \| [A(N + \Delta N)]^{-1} \| < 1$ will be proved. The operator $K(r)$ is established as follows,

$$\begin{aligned} K(r)(t) &= B\tilde{D}[A(N + \Delta N)]^{-1}(r)(t) \\ &= (I - A)A^{-1}(r)(t) \\ &= A^{-1}(r)(t) - I(r)(t) \end{aligned} \quad (4.15)$$

where $I(\cdot)$ is the identity operator.

According to (4.15), its coefficient can be shown as follow:

$$\begin{aligned} \alpha(K) &= \alpha(A^{-1}) - \alpha(I) \\ &= \frac{n}{m}r - b - r \\ &= \left(\frac{n}{m} - 1\right)r - b \end{aligned} \quad (4.16)$$

Based on the condition (4.11),

$$\begin{aligned} &\| B\tilde{D} \| \| [A(N + \Delta N)]^{-1} \| \\ &= \sup_{T \in [0, \infty)} \sup_{\substack{r_1, r_2 \in U \\ r_1 \neq r_2}} \frac{\| [K(r_1)] - K[(r_2)] \|}{\| [r_1] - [r_2] \|} \\ &< \sup_{T \in [0, \infty)} \left\| \frac{n}{m} - 1 \right\| \\ &< 1 \end{aligned} \quad (4.17)$$

From the first result and **Theorem 3.2**, $A(N + \Delta N) + B\tilde{D}$ is unimodular. The proof of the theorem is completed.

Therefore, from **Theorem 4.4**, the design scheme for robust stability of the nonlinear system with perturbations is obtained.

Therefore, in this dissertation, robust stability and extended robust right coprime factorization for the given nonlinear systems with perturbations are addressed. In the next section, a simulation example is given to confirm the effectiveness of the proposed methods.

4.3.3 Simulation example

First, the input space U and the output space Y are given: $U = Y = \mathbb{C}_{[0,\infty)}^1$. The nonlinear system with perturbations is shown,

$$(P + \Delta P)(u)(t) = \frac{1 + te^{-t}(1 + \delta(t))(1 + te^{2t})}{8}u(t)$$

Next, based on the given plant operator, right factorization is deduced from the design scheme in this dissertation. Thus, the following inner product is employed to consider robust right factorization of the given plant,

$$\langle \phi(u)(t), u(t) \rangle = 2u(t) \int_0^t \frac{(1-t)e^{-t}}{(te^{-t} + 1)u^2(\tau)} \phi(u)(\tau)u(\tau) d\tau$$

As to the proposed isomorphism, the following equation is obtained,

$$2u(t) \int_0^t \frac{(1-t)e^{-t}}{(te^{-t} + 1)u^2(\tau)} \phi(u)(\tau)u(\tau) d\tau = 2\phi(u)(t) \quad (4.18)$$

By calculating, solution of the equation (4.18) of $\phi(u)(t)$ is shown,

$$\phi(u)(t) = \frac{1}{8}(te^{-t} + 1)u(t)$$

Based on the solution, the operators $(N + \Delta N)(w)(t)$ and $\tilde{D}^{-1}(u)(t)$ can be obtained,

$$(N + \Delta N)(w)(t) = \frac{1 + te^{-t}(1 + \delta(t))}{8}w(t)$$

$$\tilde{D}^{-1}(u)(t) = \frac{1 + te^{-t}}{8}u(t)$$

From the right factorization principle, D^{-1} is obtained ,

$$D^{-1}(u)(t) = (te^{2t} + 1)u(t).$$

Meanwhile, the compensator S is founded

$$\begin{aligned} S(w)(t) &= D^{-1}\tilde{D}(w)(t) \\ &= (te^{2t} + 1)\frac{8}{1 + te^{-t}}w(t) \end{aligned}$$

The two stable controllers A and B for the nonlinear system with perturbations are designed.

$$\begin{aligned} A(y)(t) &= \frac{2}{3}y(t) + 15 \\ B(u)(t) &= \frac{(1 + te^{-t}(1 + \delta(t)))(1 + te^{-t})}{24}u(t) \end{aligned}$$

Moreover, based on the designed controller A , the following two conditions are also satisfied with requirement on inequality of the Lipschitz norm.

Since,

$$\begin{aligned} & \| [A(N + \Delta N) - (N + \Delta N)] \| \| (N + \Delta N)^{-1} \| \\ &= \sup_{T \in [0, \infty)} \sup_{\substack{y_1, y_2 \in Y \\ y_1 \neq y_2}} \frac{\| [H(y_1)] - H(y_2)] \|}{\| [y_1] - [y_2] \|} \\ &< \sup_{T \in [0, \infty)} \left\| \frac{2}{3} - 1 \right\| \\ &< 1 \end{aligned} \tag{4.19}$$

and

$$\begin{aligned} & \| B\tilde{D} \| \| [A(N + \Delta N)]^{-1} \| \\ &= \sup_{T \in [0, \infty)} \sup_{\substack{r_1, r_2 \in U \\ r_1 \neq r_2}} \frac{\| [K(r_1)] - K(r_2)] \|}{\| [r_1] - [r_2] \|} \\ &< \sup_{T \in [0, \infty)} \left\| \frac{3}{2} - 1 \right\| \\ &< 1 \end{aligned} \tag{4.20}$$

Then, the nonlinear system with perturbations can be guaranteed robust stable.

$$\begin{aligned} [A(N + \Delta N) + B\tilde{D}](w)(t) &= \frac{1 + te^{-t}(1 + \delta(t))}{8}w(t) \\ &= \tilde{M}(w)(t) \\ &= (N + \Delta N)(w)(t) \end{aligned} \tag{4.21}$$

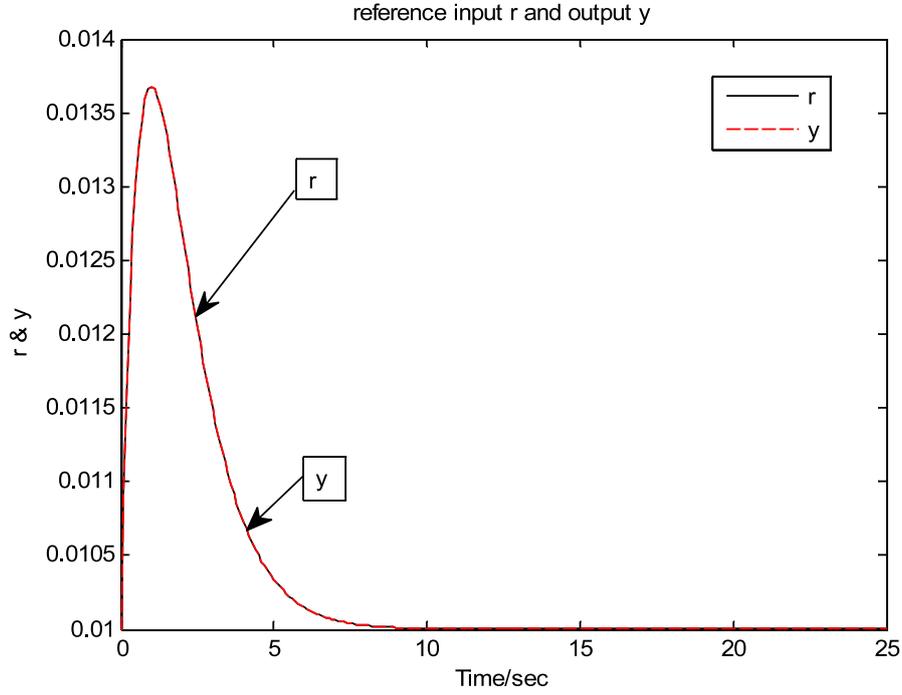


Figure 4.4: Plant output y & reference input r

Thus, the output of the nonlinear system tracking to the reference input can be guaranteed owing to

$$y(t) = (N + \Delta N)\tilde{M}^{-1}(r)(t) = r(t).$$

The simulation results of the output and reference input are shown in Figure 4.4, where the reference input is $r = 0.01(1 + te^{-t})$, $\delta(t) = e^{-t}$. From Figure 4.4, it is easy to find that robust stability of the considered system is guaranteed, meanwhile, the plant output can track to the reference input while the reference input. Thus, the effectiveness of the proposed scheme is confirmed with simulation example.

4.4 Conclusion

In this chapter, extended robust right coprime factorization and robust stability for the nonlinear systems with perturbations are considered based on adjoint operator and inner product of Hilbert. The proposed method based on the inner product extended the isomorphism for the nonlinear systems with perturbations. Simultaneously, by the adjoint operator of Hilbert spaces, the existence of a compensator for the systems was obtained. That is, robust right coprime factorization is extended by the proposed design scheme. Further, a practical and realizable design scheme for robust stability was proposed using the proposed controller, which avoided the irrational boundedness of robust condition of former results. Finally, the effectiveness of the proposed design scheme was confirmed by the simulation example.

Chapter 5

Left coprime factorization realization based on right factorization and issues on robust stability of MIMO nonlinear systems

5.1 Introduction

In Chapter 3 and Chapter 4, extended robust right coprime factorization and the nonlinear control problem are considered for nonlinear systems based on L_α operator and adjoint operator, respectively.

As we addressed in the former chapter, operator-based coprime factorization has been proved to be a promising and effective method, which provides a rather convenient framework for researching nonlinear systems. A great number of results on operator-based right coprime factorization has been consistently pursued with tremendous effort by many researchers in the fields [65], –, [90]. As we addressed, almost of the researchers on nonlinear systems are employed right coprime factorization. As to the technique of left coprime factorization, very little insights and researchers arise for nonlinear system.

As for the right coprime factorization method, the main idea is to factorize a given system operator P as a composition of two different operators N and D such that $P = ND^{-1}$, where N is a stable operator and D is a stable and invertible operator, then to design other two stable operators A, B satisfying the Bezout identity $AN + BD = M$, based on N and D , where M is an unimodular operator. Therefore, in this chapter, we will extend the right factorization idea to consider left coprime factorization for nonlinear systems from the input-output viewpoint.

On the other hand, however, comparing with a great number of results on single-input-single-output (SISO) nonlinear uncertain systems [54], –, [68], there are relative fewer results available for MIMO nonlinear systems with uncertainties due to complications and difficulties in dealing with uncertainties [106], –, [118]. In this chapter, we will discuss an issue on robust control of MIMO nonlinear systems with uncertainties from the input-output point of view using the operator-based right coprime factorization approach.

There exists some results based on right coprime factorization for MIMO nonlinear systems [106] and [107]. In [107], some sufficient conditions for the MIMO nonlinear perturbed systems are derived. The proposed sufficient conditions are established by using Taylor expansion of a controller. In terms of Taylor expansion, in some cases, it is a hard condition to satisfy with derivative condition, owing to calculating the higher derivation of the controller. Therefore, there are some restrictive conditions for applying the proposed design scheme [107]. In [106], for dealing with coupling effect and guaranteeing robust stability of MIMO nonlinear systems, internal operators of coupling effect are supposed to be satisfied with right factorization, which means the internal signal of the coupling effect could be observed for obtaining right factorization. Generally speaking, most coupling effect is unknown for nonlinear systems. Therefore, some cases where internal signal of coupling effect is not available could be difficult to employ the proposed design

scheme. Therefore, considering the above issues, a feasible design scheme for MIMO nonlinear systems with uncertainties is motivated. First, the definition of quotient operators is proposed to deal with coupling effect of the MIMO nonlinear systems. Based on the proposed quotient operator, a sufficient condition is proposed for decoupling the MIMO nonlinear systems, which relaxes the former restrictive conditions on Taylor expansion and right factorization of coupling effect. From the proposed design scheme, a unified framework is established to deal with the existed coupling effect directly. Then, a feasible control is proposed to deal with the MIMO nonlinear systems with uncertainties based on two sufficient conditions using a new unimodular operator, whose merits lie in avoiding obtaining right factorization of a control operator and difficult calculation of the normal Bezout identity.

In Section 5.2, the main results are developed by the proposed methods. Firstly, the problem statement for this chapter is given, which includes the concerned issues. Then, the definition of left coprime factorization is recalled. Based on the given definition, internal-output stability and left coprime factorization for a class of nonlinear system is considered based on extending right factorization method. Finally, a simulation example is given to confirm the effectiveness of the proposed design scheme.

In Section 5.3, main results on robust control of MIMO nonlinear systems are proposed. Coupling effect of the MIMO nonlinear systems is considered. Then, sufficient conditions for guaranteeing robust stability of the MIMO nonlinear systems with uncertainties are given. A simulation example is given to show effectiveness of the proposed design scheme.

In Section 5.4, the summary of left coprime factorization by extending right factorization for a class of nonlinear systems and robust control for MIMO nonlinear systems are given.

5.2 Internal-output stability and left coprime factorization

5.2.1 Problem statement

In the linear case, the equivalent relationship between the Bezout identity and coprime factorization exists. However, when a nonlinear system using the left coprime factorization approach is considered, it is difficult to find the equivalent relationship between the Bezout identity and the coprimeness of the left factorization. On the other hand, as for right factorization on the nonlinear systems, unstable factors lie in the process from the input space to the internal signal space. To guarantee stability of the nonlinear systems, the internal signal is employed to establish a Bezout identity based on the obtained right factorization. However, considering left factorization definition, unstable factors lie in the process from internal signal space to the output space, which cannot be handled as the way of right factorization.

That is, internal-output stability of nonlinear systems is considered, which cannot be handled by right coprime factorization, and how to realize left coprime factorization of nonlinear systems is discussed. Specifically, for dealing with the issue on internal-output stability, the part of left factorization is designed to transform the issue to a relative convenient framework to study the nonlinear system based on a right factorization. After that, a systematic scheme for a family of nonlinear systems is proposed to realize left coprime factorization combining the right factorization technique. The proposed scheme provides a realization approach to left coprime factorization, without using the left coprime factorization definition to clarify whether left factorization of a nonlinear system is coprime or not.

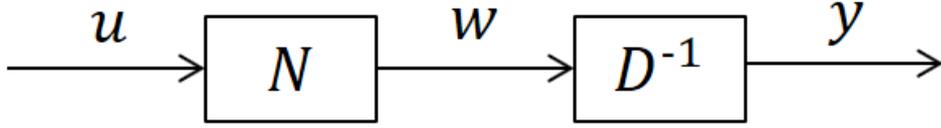


Figure 5.1: A nonlinear system with left factorization

5.2.2 Mathematical preliminaries

Definition 5.1. A nonlinear system $P : U \rightarrow Y$ is shown in Figure 5.1, where U and Y are employed to denote the input space and the output space, respectively. u , w and y represent the control input, the internal signal and the output, respectively. Then, the given system P is said to have left factorization, provided that if there exist a linear space W and two stable operators $N : U \rightarrow W$ and $D : Y \rightarrow W$ such that $P = D^{-1}N$ where D^{-1} exists (not stable) and the space W is called to be a factorization space.

Note that the unstable factors of left factorization exist in the part D , so from the whole system angle, the nonlinear system P is not internal-output stability in the sense that the bounded internal signal maps bounded output signal, which is different with right factorization.

Definition 5.2. As for the nonlinear system with left factorization shown in Figure 1, when the set of all unbounded u such that Pu is bounded and Nu is bounded is the empty set, then left factorization D and N of P is said to be coprimeness. This is equivalent to require that for all bounded w , $D^{-1}w$ is bounded or $\{u : Nu = w\}$ is bounded.

5.2.3 Internal-output stability and left coprime factorization

In this section, a relative convenient framework is proposed to study internal-stability of the nonlinear system with left factorization shown in Figure 5.1. For dealing with this issue, an idea is proposed to transform the left factorization to an equivalent framework for studying internal-output stability. That is, the unstable part of the nonlinear system with left factorization D^{-1} is designed to have right factorization, denoted as $D = QR^{-1}$, where Q, R are stable and R^{-1} is not necessary stable. Let w^* be the internal signal of D . Based on the proposed scheme, the fact that Q is invertible can be obtained, since $Q = DR$ and R, D being invertible imply the composite operator RD is invertible. Therefore, the internal-output stability is transformed an internal part issue to the nonlinear system. Through this transformation, the unstable factor of the nonlinear system is described by the internal part Q^{-1} . The internal stability of the nonlinear system will be discussed by combining right factorization with left factorization.

In this chapter, in order to guarantee the internal-output stability of the nonlinear system and realize left coprime factorization of the nonlinear systems, an feasible approach is proposed by designing Bzout identity combining the right factorization technique. However, because of the universal cases of the composite operator $Q^{-1}N$, there exist some difficulties in the process of designing Bezout identity for the nonlinear system. Thus, before designing the Bezout identity, a preparatory result on the composite operator is shown as follows.

Lemma 5.1. *As for the nonlinear system shown in Figure 5.1, if a feedback compensator is designed as shown in Figure 5.2 such that*

$$\| S - Q^{-1}N \| \| S^{-1} \| < 1 \quad (5.1)$$

where S is the designed controller, then the inverse of $Q^{-1}N$ exists and is

stable. Its equivalent system is denoted as $Q^{-1}\tilde{N}$ in the rest chapter and the whole system is shown in Figure 5.3.

Proof. According to **Lemma 3.2**, show that $Q^{-1}NS^{-1}$ is invertible. Since

$$\| I - Q^{-1}NS^{-1} \| \leq \| S - Q^{-1}N \| \| S^{-1} \| < 1$$

Also,

$$\begin{aligned} \| (Q^{-1}NS^{-1})^{-1} \| &\leq \frac{1}{1 - \| I - Q^{-1}NS^{-1} \|} \\ &\leq \frac{1}{1 - \| S - Q^{-1}N \| \| S^{-1} \|} \end{aligned} \quad (5.2)$$

Since $Q^{-1}N = (Q^{-1}NS^{-1})S$, we see that $Q^{-1}N$ has an inverse, namely, $(Q^{-1}N)^{-1} = S^{-1}(Q^{-1}NS^{-1})^{-1}$. Also,

$$\| (Q^{-1}N)^{-1} \| \leq \| S^{-1} \| \| (Q^{-1}NS^{-1})^{-1} \|$$

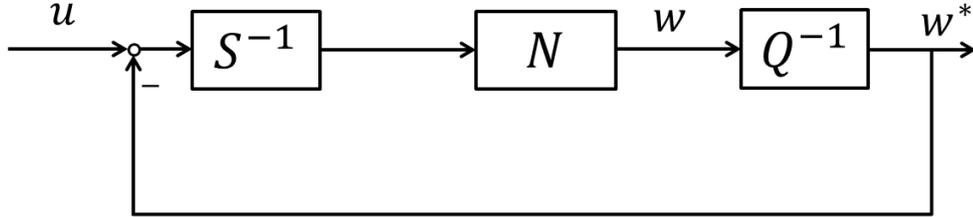


Figure 5.2: Feedback system with the compensator S

Therefore, according to the designed compensator S , the proposed equivalent operator $Q^{-1}\tilde{N}$ is proved to be invertible and its inverse is stable. After that, relationship from w^* to u can be obtained. Based on the proposed design compensator, the design scheme of the realization to left coprime factorization for a specific class of nonlinear systems will be shown in the following theorem.

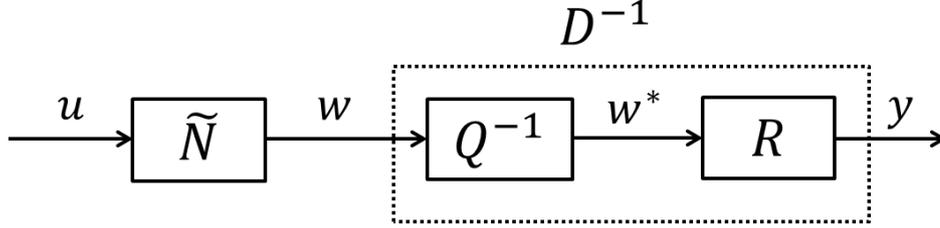


Figure 5.3: The whole equivalent system with the compensator S

Theorem 5.1. *As for the nonlinear system shown in Figure 5.3 if there exist two stable operators A and B , where $\|AR\| < 1$ and B^{-1} exists, such that $AR(w^*)(t) + B\tilde{L}(w^*)(t) = I(w^*)(t)$, where I is identity operator from w^* to r and $\tilde{L}(w^*)(t) = (Q^{-1}\tilde{N})^{-1}(w^*)(t)$, then the nonlinear system P is internal-output stable and has left coprime factorization.*

Proof. Firstly, in terms of the existence of a controller A , there are a great number of choices in finding an operator A such that the Lipschitz norm of AR is less one. For simplicity, in this chapter, we choose

$$A(y)(t) = \frac{m(t)}{n(t)}y(t)$$

where $m(t)$, $n(t)$ are two designed operators.

$A(y)$ can be found to be stable. And according to the chosen operator A , the controller B can be designed as follows,

$$B(u)(t) = (I - AR)\tilde{L}^{-1}(u)(t)$$

From **Lemma 3.2**, the composite operator $I - AR$ is invertible. Combining with the fact that \tilde{L} is also invertible, the proposed operator B is invertible.

After obtaining the two controllers A and B , the Bezout identity

$$AR(w^*)(t) + B\tilde{L}(w^*)(t) = I(w^*)(t)$$

can be obtained.

Moreover, according to the proposed approach and the connection between signals of the nonlinear system, Figure 5.4 is shown to describe the designed scheme, where r , u and y are the reference input, control input and output of the nonlinear system, respectively.

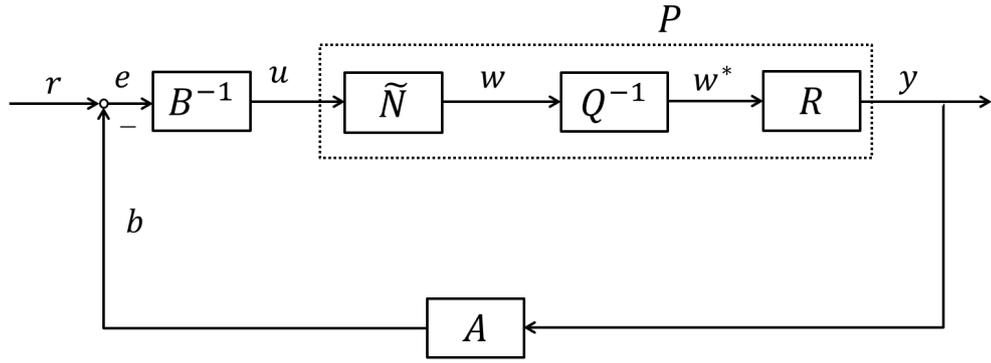


Figure 5.4: The designed scheme for the nonlinear system

Form $AR(w^*)(t) + B\tilde{L}(w^*)(t) = I(w^*)(t)$, we can get

$$\begin{aligned} AR(w^*)(t) + B\tilde{L}(w^*)(t) &= A(y)(t) + B(u)(t) \\ &= b(t) + e(t) \end{aligned} \quad (5.3)$$

Combining with the signals relationship as shown in Figure 5.4, the following relationship between w^* and r is obtained.

$$w^*(t) = I^{-1}(r)(t) \quad (5.4)$$

Thus, the design of the nonlinear system as shown in Figure 5.4 can be simplified to Figure 5.5 as follow.

Owing to that I is unimodular and R is stable, so internal-output stability of the nonlinear system P is guaranteed in the sense that a bounded input maps a bounded output.

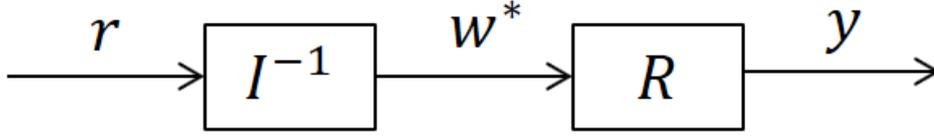


Figure 5.5: Equivalent system of Figure 5.4

Based on the definition of left coprime factorization proposed in [84], all bounded w , $D^{-1}(w)$ is bounded or $\{u : N(u) = w\}$ is bounded. For the nonlinear system shown in Figure 5.5, all given bounded w , the output $y = D^{-1}(w)$ is bounded. Indeed, based on the design scheme, the nonlinear system P is proved to be stable, that is, the output of P is bounded. Thus, the obtained left factorization of P is coprime through the proposed design scheme.

The proof of the theorem is completed.

Note that from **Theorem 5.1**, the realization approach on left coprime factorization to a class of nonlinear systems is obtained using the proposed method. Meanwhile, the internal-output stability is guaranteed by combining left factorization and right factorization. The merits of the proposed methods lie in two following points that the proposed method can be employed to study the internal-output stability of the nonlinear systems, transforming the internal-output issue to a feasible issue, and that left coprime factorization makes more sense than the right coprime factorization for the specific class of nonlinear system, at least for the commonly employed mathematical description of operator-based nonlinear equations describing systems.

In this section, the proposed method provides some new insights and thoughts in studying the internal-output stability and establishing left coprime factorization for nonlinear systems. After that, the cases where the

nonlinear systems with the left coprime factorization have bounded perturbations will be discussed.

5.2.4 Simulation example

In this section, a numerical simulation is given for confirming the effectiveness of the proposed method. Let $C_{[0,\infty]}$ be the space of continuous functions, and $C_{[0,\infty]}^1$ consists of all the functions having a continuous first derivative, which both are defined on $[0, \infty)$. Suppose that the input space U and the output space Y are included in $C_{[0,\infty]}^1$. After that, the nonlinear system P is given as shown in the following unstable and time-varying nonlinear operator from U to Y :

$$P(u)(t) = (1+t)^2 \left(\frac{1}{2t+1} u(t) - \frac{1}{t+1} \right) - t - 1$$

where $I(u)(t)$ is the identity operator.

Based on the proposed nonlinear system P , left factorization can be obtained as follows,

$$\begin{aligned} N(u)(t) &= \frac{1}{2t+1} u(t) - \frac{1}{t+1} \\ D(w^*)(t) &= \frac{1}{(1+t)^2} w^*(t) + \frac{1}{1+t} \end{aligned}$$

It can be found that N and D are both stable operators, D^{-1} is unstable. Since, according to the obtained D , its inverse can be shown as follow,

$$D^{-1}(w^*)(t) = (1+t)^2 w^*(t) - t - 1$$

Then, based on the proposed method for internal-output stability of the nonlinear system, left factorization on D is designed as follows,

$$\begin{aligned} R(w^*)(t) &= \frac{1}{t+1} w^*(t) - \frac{1}{(1+t)^2} \\ Q(w^*)(t) &= \frac{1}{(t+1)^3} w^*(t) \end{aligned}$$

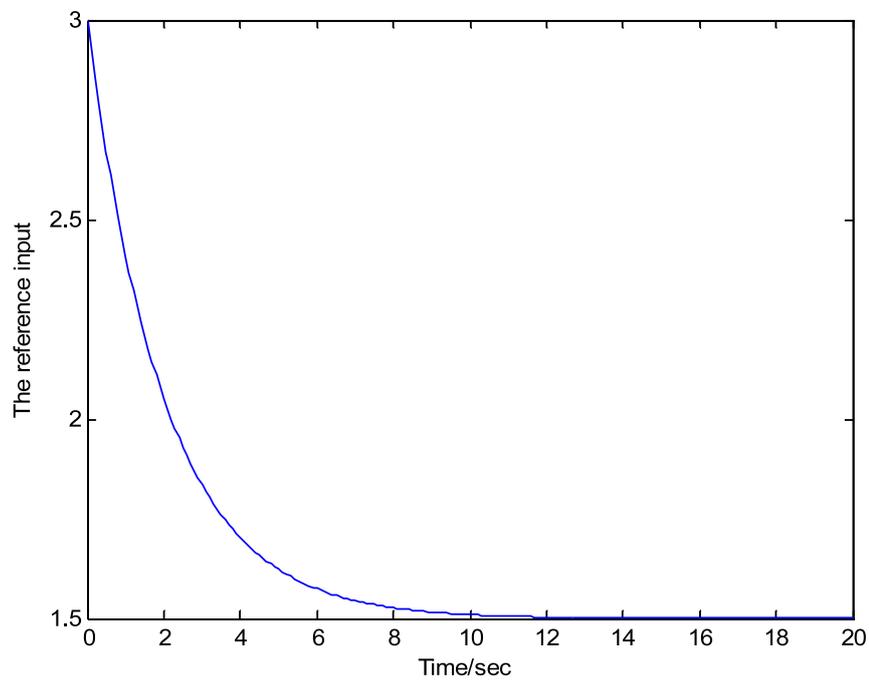


Figure 5.6: Reference input

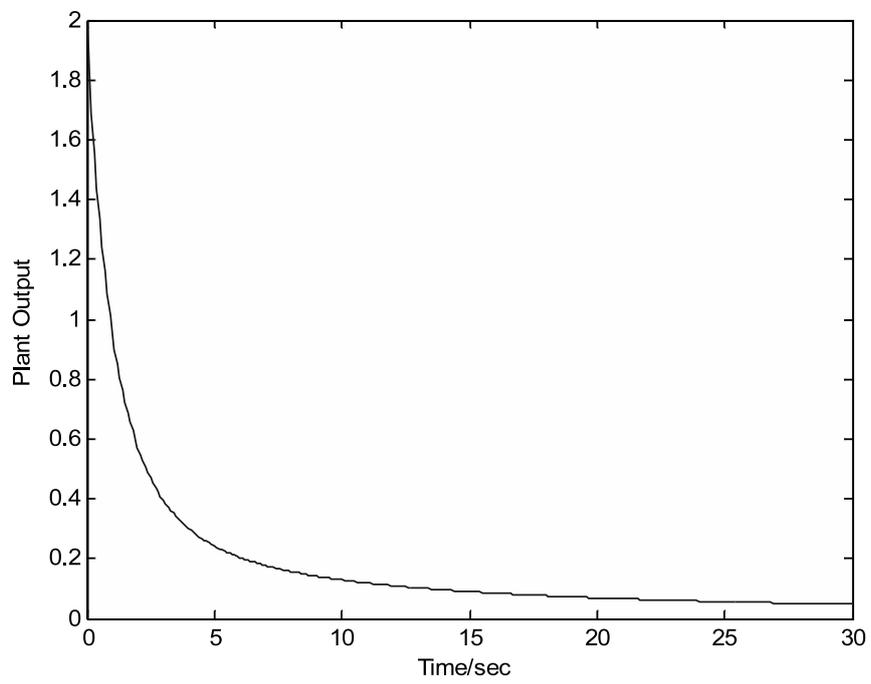


Figure 5.7: Plant output

where Q and R are stable, and the inverse of R^{-1} is not stable.

After that, according to **Lemma 5.1**, a compensator will be designed to make the composite operator $Q^{-1}N(u)(t)$ be unimodular, which is employed to establish a Beout identity to guarantee the internal-output stability and the coprime property of the nonlinear system.

From the above factorization, we can get

$$Q^{-1}N(u)(t) = \frac{(t+1)^3}{2t+1}u(t) - (t+1)^3$$

Therefore, the feedback compensator S is designed as follow,

$$S(u)(t) = \frac{1}{(t+1)^2}u(t) + \frac{2t+1}{t+1}$$

Next, the controllers A and B are designed according to **Theorem 5.1** as follows:

$$A(y)(t) = \frac{(1+t)(2t+1)}{2t^2}y(t)$$

$$B(u)(t) = \frac{(2t^2 - 2t - 1)(t+1)}{2t^2(2t+1)}u(t) + \frac{2t+1}{2t^2(t+1)}$$

Based on the designed controllers, it can be verified that A and B satisfy the following Bezout identity. Indeed, we have, $AR(w^*)(t) + B\tilde{L}(w^*)(t) = I(w^*)(t)$.

In order to show the effectiveness the proposed design scheme for the nonlinear system without perturbation, simulation results are given in Figure 5.6 and Figure 5.7, which are the reference input $r(t)$ and the output $y(t)$, respectively. The reference input is chosen as $r(t) = 1.5(1 + e^{-0.5t})$ in this chapter. Thus, based on the simulation results, the internal-output stability of the nonlinear system is obtained using the proposed design scheme.

5.3 Robust stability of MIMO nonlinear systems

5.3.1 Problem statement

Generally speaking, uncertainties for the MIMO nonlinear system are considered in many researches. However, the previous methods for dealing with coupling effect and uncertainties restrict their application to some extent because of complicated calculation of invertible operators and difficulties in practice. Therefore, in this chapter, robust control design is discussed for the MIMO nonlinear system with uncertainties based on the proposed quotient operator. That is, the proposed quotient operator controller is employed to deal with coupling effect existing in systems effectively, then the proposed sufficient conditions relax restriction of the previous methods for stabilizing the overall systems.

5.3.2 Robust stability of MIMO nonlinear systems

In this chapter, the MIMO nonlinear systems with uncertainties are considered, however, there exist some particular characteristics compared with the SISO nonlinear systems due to coupling effect. Therefore, first, we will discuss coupling effect of the MIMO nonlinear systems.

Assume that a MIMO nonlinear system with right factorization, whose input space, quasi-state space and output space are denoted as U , W and Y , respectively, shown in Figure 5.8 exists coupling effect, where $\mathbf{u} = (u_1, u_2, \dots, u_n)$ and $\mathbf{y} = (y_1, y_2, \dots, y_n)$ are input and output, respectively. $\mathbf{P} = (P_1, P_2, \dots, P_n) : U \rightarrow V$ is a nominal system and has right factorization $\mathbf{P} = \mathbf{N}\mathbf{D}^{-1}$, where $\mathbf{N} = (N_1, N_2, \dots, N_n) : W \rightarrow V$ and $\mathbf{D} = (D_1, D_2, \dots, D_n) : W \rightarrow U$ are stable and \mathbf{D} is invertible, $\Delta\mathbf{D}$ is denoted as immanent coupling effect. As for coupling effect of the MIMO nonlinear system, assume that the existing

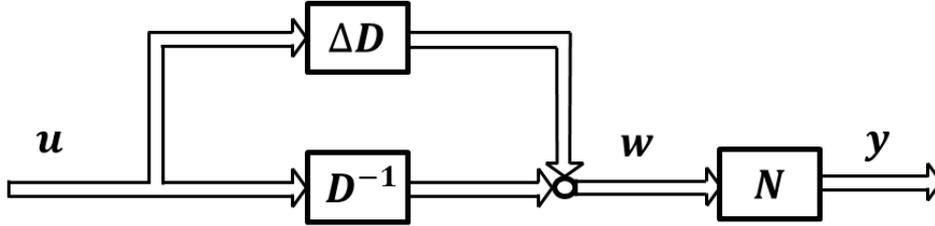


Figure 5.8: MIMO nonlinear systems with coupling effect.

coupling effect is related to the input signals of the MIMO nonlinear system, which leads to that internal signals of the system exists coupling effect. For these cases, there are many real applications where coupling effect exists between input signal and internal signal in the MIMO nonlinear systems. In this chapter, the coupling effect of the MIMO nonlinear systems is assumed to belong to one certain subspace W_0 of W .

Next, decoupling for the MIMO nonlinear system will be discussed by quotient operators.

Definition 5.3. *Provided that X be a linear space X_0 is a linear subspace of X , denote by X/X_0 the set of all cosets $x + X_0$, for each $x \in X$ with addition defined by $(x + X_0) + (y + X_0) = x + y + X_0$ and multiplication by scalars defined by $a(x + X_0) = ax + X_0$, then X/X_0 is called to be a quotient space of X by X_0 . Based on the quotient space of X by X_0 , define an operator Q from X to X/X_0 as follows, $Q(x) = I(x) + X_0$ then Q is said to be a quotient operator, where I is the identity operator.*

According to the proposed operator, a nonlinear control design for decoupling the MIMO nonlinear system is provided in Figure 5.9. Then, the decoupling problem can be solved by the following lemma.

Lemma 5.2. *As for the MIMO nonlinear system as shown in Figure 5.8, if the controller Q is designed to be a quotient operator from W to W/W_0 ,*

mapping \mathbf{d} to W/W_0 , \mathbf{F} is designed to be an operator from W/W_0 to W , and \mathbf{R} is designed such that $\mathbf{R}\mathbf{D} = \mathbf{I}$, where \mathbf{Q} , \mathbf{F} and \mathbf{R} are stable, then the MIMO nonlinear system is decoupled.

Proof. The control design of Figure 5.9 indicates that \mathbf{Q} guarantees output of \mathbf{Q} is not involved in output of \mathbf{D}^{-1} , which means that the quotient space W/W_0 merely contains the element of W_1 .

According to Figure 5.9 and \mathbf{F} , obtain

$$\begin{aligned}\mathbf{w} &= (\mathbf{D}^{-1} + \mathbf{\Delta D})(\mathbf{u}) - \mathbf{FQR}(\mathbf{u}) = \mathbf{b} - \mathbf{FQ}(\mathbf{d}) \\ &= \mathbf{b} - \mathbf{g}\end{aligned}\tag{5.5}$$

Since, the proposed controller \mathbf{Q} is quotient, which guarantees \mathbf{w} is not involved in the coupling effect resulting from $\mathbf{\Delta D}$. The proof is completed.

Note that based on the proposed design scheme for decoupling, \mathbf{D} is reconstructed as $\tilde{\mathbf{D}}$, and the internal signal w_i is not involved the coupling effect $\mathbf{\Delta D}$. Compared to the previous method [106] and [107], the merits of the proposed design scheme are that Taylor expansion of B_i is not needed to obtain, which can lead to the higher approximation part.

In this chapter, if the decoupled MIMO nonlinear systems, $\mathbf{P} = \tilde{\mathbf{D}}^{-1}\mathbf{N}$ has a right coprime factorization, which indicates \mathbf{P} is satisfied with the Bezout identity

$$\mathbf{A}\mathbf{N} + \mathbf{B}\tilde{\mathbf{D}} = \mathbf{M}\tag{5.6}$$

where $\mathbf{A} = (A_1, A_2, \dots, A_n) : V \rightarrow U$ is stable, $\mathbf{B} = (B_1, B_2, \dots, B_n) : U \rightarrow U$ is stable and invertible, and $\mathbf{M} = (M_1, M_2, \dots, M_n)$ is one unimodular operator from W to U . It is worth mentioning that the initial state is supposed to be considered, that is, $\mathbf{A}\mathbf{N}(\mathbf{w}_0, t_0) + \mathbf{B}\tilde{\mathbf{D}}(\mathbf{w}_0, t_0) = \mathbf{M}(\mathbf{w}_0, t_0)$ should be satisfied.

Note that generally the given system \mathbf{P} is unstable in the sense of bounded-input-bounded-output stability. As for the MIMO nonlinear system, we

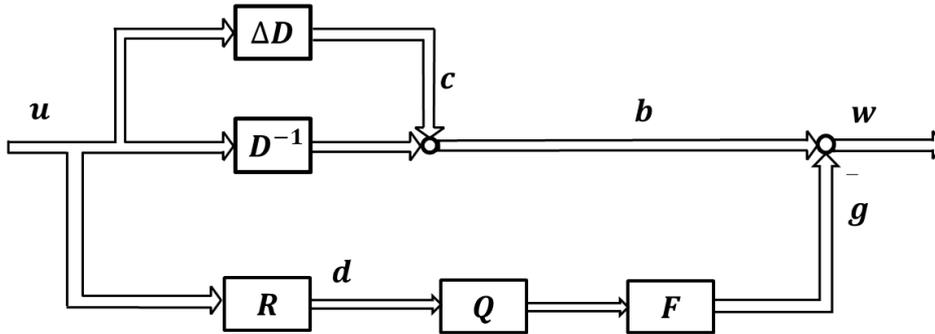


Figure 5.9: Decoupling design scheme for MIMO nonlinear systems.

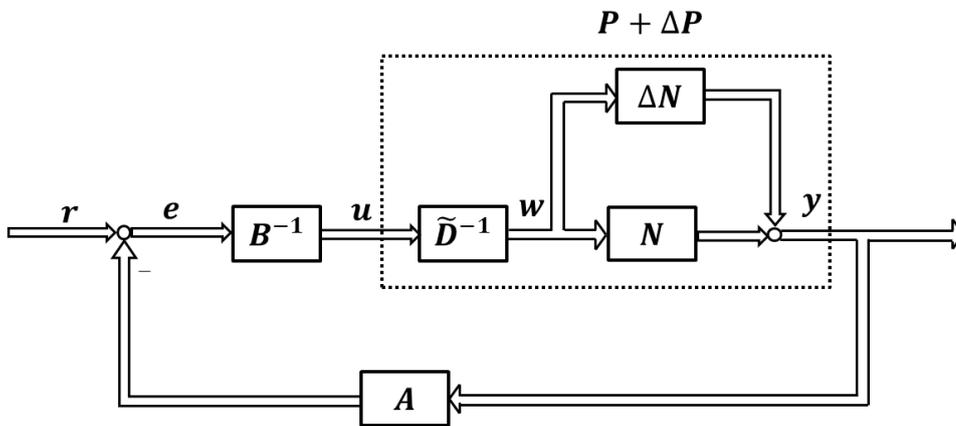


Figure 5.10: MIMO nonlinear systems with uncertainties.

mainly consider the each subsystem P_i , ($i = 1, 2, \dots, n$) to satisfy the Bezout identity responding to the controllers.

The MIMO nonlinear system without uncertainties is stable based on the right coprime factorization. However, for some cases, there are inevitable factors existing in systems, leading to uncertainties. Therefore, in this chapter, we are in a position to consider robust stability of the MIMO nonlinear system with uncertainties shown in Figure 5.10.

Issue: Under what conditions does that the MIMO nonlinear system with uncertainties $\mathbf{N} \rightarrow \mathbf{N} + \Delta\mathbf{N}$ not influence stability for the same control operators \mathbf{A} and \mathbf{B} .

Main objectives of this chapter is to provide some sufficient conditions to guarantee the MIMO nonlinear system with uncertainties to be robust stability. Before we develop sufficient conditions for robust stability of the MIMO nonlinear systems with uncertainties, the following preparatory results will be provided firstly.

Proposition 5.1. *Provided that $Q, R \in Lip(U_s, V_s)$, where U_s and V_s are the stable spaces of U and V , respectively, suppose that Q is invertible in $Lip(U_s, V_s)$ with $\|Q^{-1}\| \|Q - R\| < 1$, then R is invertible in $Lip(U_s, V_s)$ with*

$$\|R^{-1}\| \leq \|Q^{-1}\| \|R^{-1}(u_0)\| + \frac{\|Q^{-1}\|}{1 - \|Q - R\| \|Q^{-1}\|} \quad (5.7)$$

for any $u_0 \in U_s$.

Proof. Show that $\|Q^{-1}R\|$ is invertible in $Lip(U_s, V_s)$, because $\|I - RQ^{-1}\| = \|Q - R\| \|Q^{-1}\| < 1$. Also, from (2),

$$\|(RQ^{-1})^{-1}\| \leq \frac{1}{1 - \|I - RQ^{-1}\|} \leq \frac{1}{1 - \|Q - R\| \|Q^{-1}\|} \quad (5.8)$$

Since $R = (RQ^{-1})Q$, we see that R has an inverse in $Lip(U_s, V_s)$, namely, $R^{-1} = Q^{-1}(RQ^{-1})^{-1}$. $\|R^{-1}\| \leq \|Q^{-1}\| \|(RQ^{-1})^{-1}\|$. The estimation (5) follows from the above equation and (6).

Theorem 5.2. *As for the MIMO nonlinear system with uncertainties in Figure 5.10, if the following condition is satisfied,*

$$\| (A_i(N_i + \Delta N_i) - B_i) \| \| B_i^{-1} \| < 1 \quad (5.9)$$

then $A_i(N_i + \Delta N_i)$ is an unimodular operator.

Proof. The fact that B_i is unimodular implies that B_i is invertible and B_i^{-1} is stable. Hence, according to **Proposition 5.1**, we can obtain $A_i(N_i + \Delta N_i)$ is invertible.

$$\begin{aligned} \| A_i(N_i + \Delta N_i)^{-1} \| \leq & \| B_i^{-1} \| \| A_i(N_i + \Delta N_i)^{-1}(x_0) \| \\ & + \frac{\| B_i^{-1} \|}{1 - \| B_i^{-1} - A_i(N_i + \Delta N_i) \| \| B_i^{-1} \|} \end{aligned}$$

Since $A_i(N_i + \Delta N_i) = B_i - (B_i - A_i(N_i + \Delta N_i)) = [I - (B_i - A_i(N_i + \Delta N_i))B_i^{-1}]B_i$. Hence, $I - [B_i - A_i(N_i + \Delta N_i)]B_i^{-1}$ can be proved is invertible. And since the inverse of $I - (B_i - A_i(N_i + \Delta N_i))B_i^{-1}$ is stable. Therefore, we can obtain the inverse of $A_i(N_i + \Delta N_i)$ is stable. Hence, we can get the conclusion that $A_i(N_i + \Delta N_i)$ is unimodular. The proof is completed.

In the following theorem, we give main result of a new condition which can guarantee the MIMO nonlinear systems with uncertainties to be robust stability.

Theorem 5.3. *In Figure 5.10, if*

$$\| B_i \tilde{D}_i \| \| [A_i(N_i + \Delta N_i)]^{-1} \| < 1 \quad (5.10)$$

is satisfied, then the MIMO nonlinear system with uncertainties is robust stable, i.e. $A_i(N_i + \Delta N_i) + B_i \tilde{D}_i$ is an unimodular operator. *Proof.* Since we have $\| B_i \tilde{D}_i \| \| [A_i(N_i + \Delta N_i)]^{-1} \| < 1$, hence $\| [B_i \tilde{D}_i + A_i(N_i + \Delta N_i)] - A_i(N_i + \Delta N_i) \| \| [A_i(N_i + \Delta N_i)]^{-1} \| < 1$. It follows that $A_i(N_i + \Delta N_i) + B_i \tilde{D}_i$ is unimodular by **Proposition 5.1**. Thus, the MIMO nonlinear system with uncertainties is satisfied with operator-based right coprime factorization, which results in that the overall system is stable.

Compared with previous methods, on the one hand, one merit of this chapter lie in that based on the proposed quotient operator, the coupling effect of the MIMO nonlinear system is decoupled, which reduce the respective design for the subsystem, providing a relative uniform framework to consider, at least available for the kind of MIMO nonlinear systems. The realization of internal signal w is challenging. It is considered to be as the future work. On the other hand, as for the MIMO nonlinear systems with uncertainties, feasible design scheme for guaranteeing robust stability is discussed by using the shown unimodular operator. From **Theorem 5.2** and **Theorem 5.3**, the proposed design scheme dose not employ the unimodular operator M_i^{-1} of the previous work, by which the complicated work of calculating the Bezout identity and the inverse of the unimodular operator M_i is reduced.

5.3.3 Simulation example

In this chapter, a numerical example is given to show the effectiveness of the proposed method for dealing with coupling effect and robust stability. A three-input/three-output nonlinear system is considered to demonstrate the proposed control method.

Let $C_{[0,\infty]}$ be the space of continuous functions and $C_{[0,\infty]}^1$ be the subspace of $C_{[0,\infty]}$ that is comprised of all the functions having a continuous first derivative, both defined on $[0, \infty)$. Considering two linear spaces, U and V : $U = C_{[0,\infty]}$, $V = C_{[0,\infty]}^1 \subset U$ as the input space and output space, respectively, suppose that the three-input/three-output nonlinear system \mathbf{P} is given by the following unstable and time-varying nonlinear operator,

$$P_1 : \begin{cases} P_1(u_1)(t) = (t^2 + e^t)u_1(t) + 1 \\ D_1(w_1)(t) = \frac{1}{t^2 + e^t}w_1(t) \\ N_1(w_1)(t) = w_1(t) + 1 \end{cases} \quad P_2 : \begin{cases} P_2(u_2)(t) = \frac{1 + te^{-t} + t^2e^t}{8}u_2(t) \\ D_2(w_2)(t) = (te^{2t} + 1)^{-1}w_2(t) \\ N_2(w_2)(t) = \frac{1 + te^{-t}}{8}w_2(t) \end{cases}$$

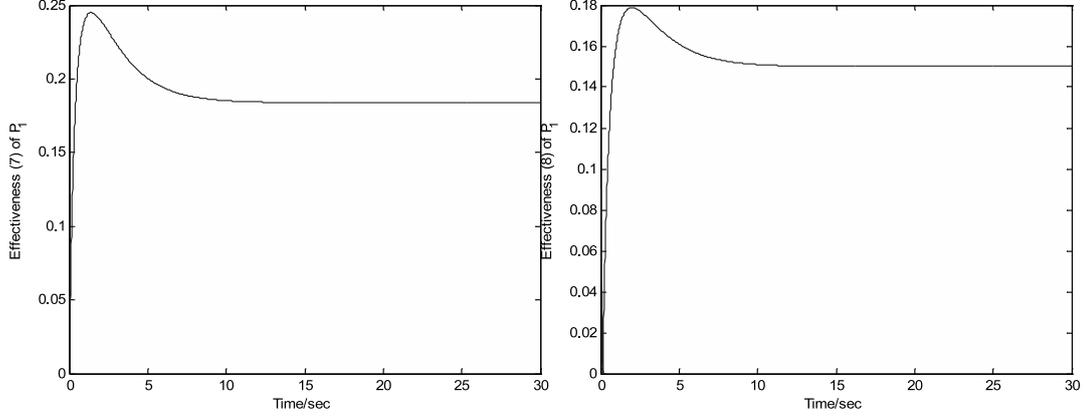


Figure 5.11: Effectiveness (5.9) of P_1 . Figure 5.12: Effectiveness (5.10) of P_1 .

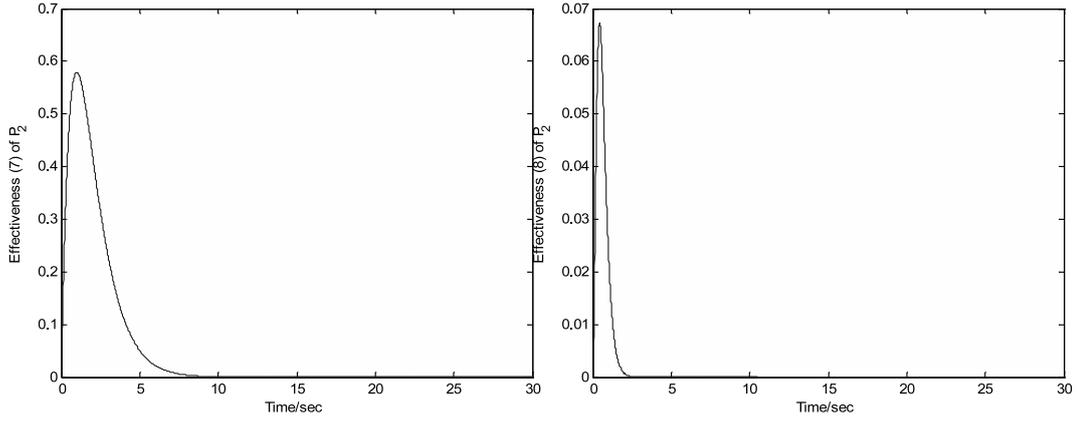


Figure 5.13: Effectiveness (5.9) of P_2 . Figure 5.14: Effectiveness (5.10) of P_2 .

$$P_3 : \begin{cases} P_3(u_3)(t) = (1+t)^2 u_3(t) + 2t^2 + 3t + 2 \\ D(w_3)(t) = \frac{1}{(1+t)^2} w_3(t) - \frac{2t+1}{1+t} \\ N(w_3)(t) = w_3(t) + 1 \end{cases}$$

Without loss of generality, coupling effects of the three-input/three-output nonlinear system are considered as $\Delta D_{12}(u_2(t)) = K_{12}u_2(t)$, $\Delta D_{13}(u_3(t)) = K_{13}u_3(t)$, $\Delta D_{21}(u_1(t)) = K_{21}u_1(t)$, $\Delta D_{23}(u_3(t)) = K_{23}u_3(t)$, $\Delta D_{31}(u_1(t)) = K_{31}u_1(t)$, $\Delta D_{32}(u_2(t)) = K_{32}u_2(t)$. The ranges of coupling effects $\Delta D_{12}(u_2(t))$

and $\Delta D_{13}(u_3(t))$, $\Delta D_{21}(u_1(t))$ and $\Delta D_{23}(u_3(t))$, $\Delta D_{31}(u_1(t))$ and $\Delta D_{32}(u_2(t))$ belong to W_1 , W_2 , W_3 , respectively. For simplicity, in this chapter assume that $\Delta D_{13}(u_3(t)) = 0$, $\Delta D_{21}(u_1(t)) = 0$, $\Delta D_{32}(u_2(t)) = 0$. Therefore, for dealing with the existed coupling effects, the controllers $R_i(u_i(t))$, $Q_i(d_i(t))$ ($i = 1, 2, 3$), respectively are designed to be $R_i(u_i(t)) = D_i^{-1}(u_i(t))$, $Q_i(d_i(t))$ is the quotient operator from W to W/W_i ($i = 1, 2, 3$). In the simulation results, $K_{12} = 1.7$, $K_{23} = 1.3$, $K_{31} = 1.9$.

Next, uncertainties of the three-input/three-output nonlinear system are considered. Assumed that $\Delta N_1(w_1(t)) = \delta_1(t)w_1(t)$, $\Delta N_2(w_2(t)) = \frac{1 + \delta_2(t)te^{-t}}{8}$, $\Delta N_3(w_3(t)) = \frac{\delta_3(t)}{2t + 1}$, where δ_i ($i = 1, 2, 3$) should be bounded. In simulation results, $\delta_1(t) = 0.5$, $\delta_2(t) = 0.5e^{-t}$, $\delta_3(t) = 5 + 2te^{-0.35t}$. Note that, as to the operator-based right coprime factorization method, the boundedness or stability of ΔN_i , ($i = 1, 2, 3$) usually does not imply the overall stability, not even input-output stability, of the perturbed system due to the feedback configuration. Hence, the robustness issue under consideration is not trivial, particularly for MIMO nonlinear systems. After that, based on the proposed control design, the control operators are designed as follows;

$$A_1(y_1)(t) = \left(1 - \frac{1}{2t^2 + e^t}\right)y_1(t), \quad B_1(u_1)(t) = u_1(t) + \frac{1}{2t^2 + e^t} - 1$$

$$A_2(y_2)(t) = \frac{2}{3}y_2(t) + 15, \quad B_2(u_2)(t) = \frac{(1 + te^{-t} + t^2e^{-2t})}{24}u_2(t)$$

$$A_3(y_3)(t) = \left(1 - \frac{1}{(2t + 1)^2}\right)y_3(t)$$

$$B_3(u_3)(t) = \frac{t^2 + 2t + 1}{4t^2 + 4t + 1}u_3(t) + \frac{1 + t}{2t + 1} + \frac{t^2 + 2t}{t^2 + 2t + 1}$$

Based on the proposed method, the conditions (5.9) and (5.10) should be guaranteed to make the design scheme available. In other words, calculation of the left item of the conditions (5.9) and (5.10) should be less than 1. The

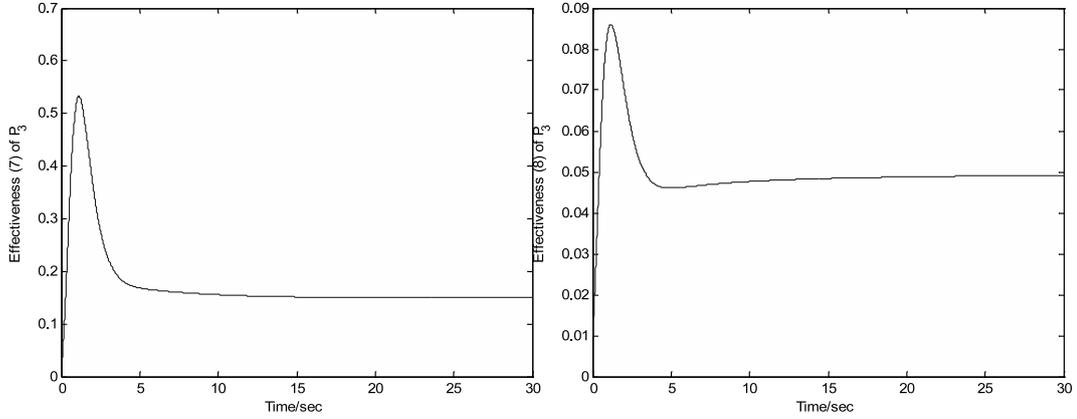
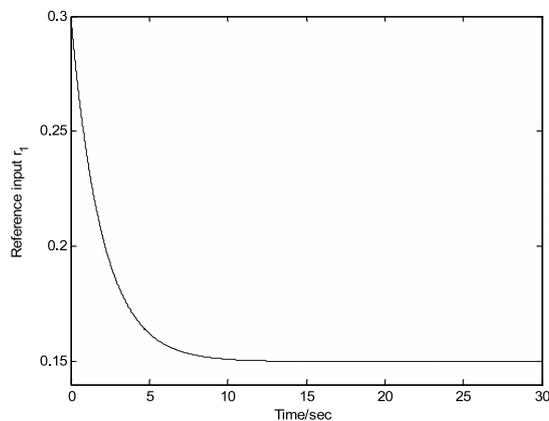
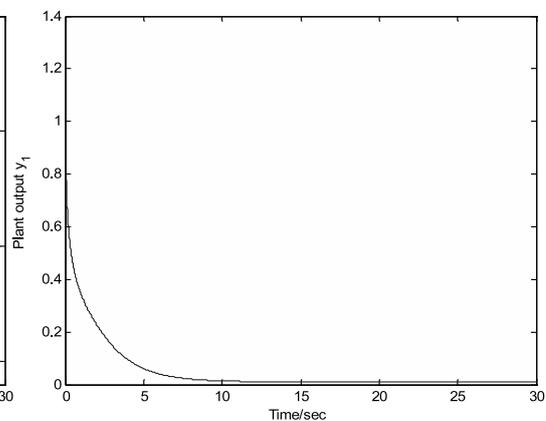
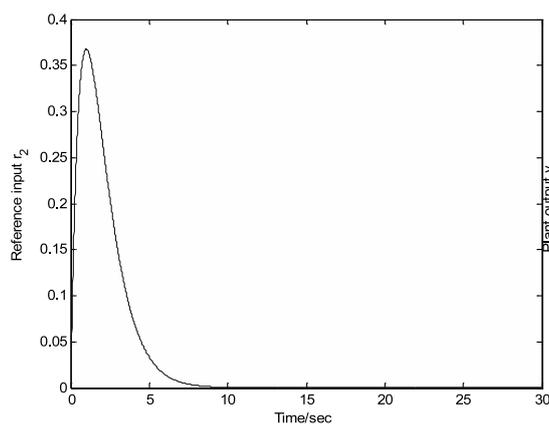
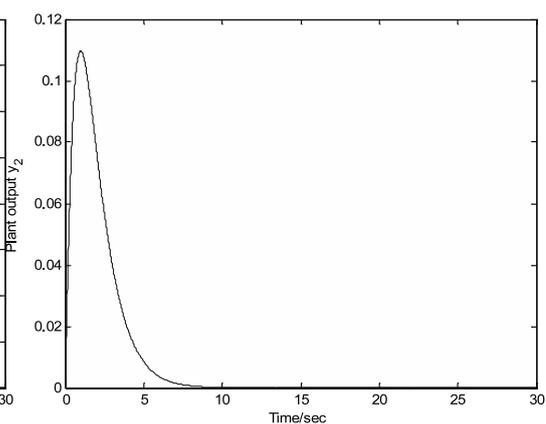


Figure 5.15: Effectiveness (5.9) of P_3 . Figure 5.16: Effectiveness (5.10) of P_3 .

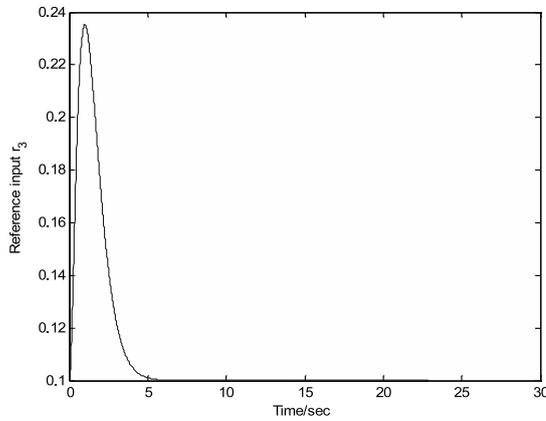
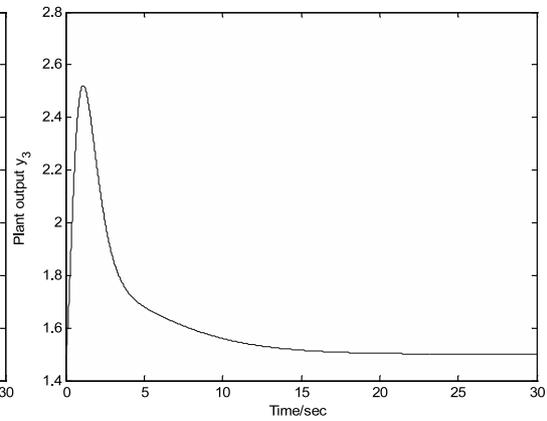
simulation results of these conditions are shown in Figures 5.11-5.16, respectively. For showing robust stability of the considered systems, the reference inputs of P_1 , P_2 and P_3 are chosen as $r_1(t) = 0.15(1 + e^{-\frac{t}{2}})$, $r_2(t) = te^{-t}$, $r_3(t) = 0.1 + t^2e^{-2t}$, respectively. For demonstrating the process, simulation results on reference inputs and plant outputs of the three-input/three-output nonlinear system are shown in Figures 5.17-5.22. According to Figure 5.18, Figure 5.20 and Figure 5.22, robust stability of the proposed nonlinear system is guaranteed by using the proposed design scheme.

5.4 Conclusion

In this chapter, a nonlinear control method using operator-based coprime factorization for a class of nonlinear systems is considered, and issues on robust control of MIMO nonlinear systems are discussed. On one hand, firstly, a part of the nonlinear system was factorized to provide a relative convenient framework to investigate left coprime factorization. Secondly, the invertible property of the composite operator for left factorization was guaranteed by the designed compensator for combining left factorization and right factor-

Figure 5.17: Reference input r_1 .Figure 5.18: Plant Output y_1 .Figure 5.19: Reference input r_2 .Figure 5.20: Plant Output y_2 .

ization. Thirdly, two stable controllers were proposed to establish the Bezout identity. Based on the proposed designed scheme, left factorization for the nonlinear systems was proved to be coprime and internal-output stability was obtained. On the other hand, based on operator-based right coprime factorization, a class of MIMO nonlinear systems with uncertainties is considered for guaranteeing robust stability of the MIMO nonlinear systems. That is, based on right coprime factorization of the MIMO nonlinear systems, a feasible design scheme was proposed by using a new unimodular operator. Based

Figure 5.21: Reference input r_3 .Figure 5.22: Plant Output y_3 .

on the obtained conditions, the designed system was overall stable. Finally, the effectiveness of the proposed design schemes for left coprime factorization and MIMO nonlinear systems with uncertainties was also shown by the proposed simulation examples.

Chapter 6

Conclusions

In this dissertation, operator-based nonlinear control using extended robust right coprime factorization for the nonlinear systems is discussed. Meanwhile, a special class of nonlinear system are considered by using left factorization and right factorization. Firstly, using the L_α operator, right coprime factorization is extended to deal with a broader class of nonlinear systems, and robust control for the considered system is designed. Secondly, adjoint-based right coprime factorization are considered in the context of factorization quantitatively-factorization method of the given nonlinear system. Moreover, rational boundedness robust conditions are given for guaranteeing robust stability of nonlinear systems. Further, a special class of nonlinear system is considered by using left factorization and right factorization to guarantee internal-output stability. Meanwhile, realization of left coprime factorization is obtained.

In Chapter 2, firstly, mathematical preliminaries for developing main results of this dissertation consisting of definitions of important spaces and operators are recalled. In detail, the definitions of extended linear space and generalized Lipschitz operator are introduced, which serve as foundations for the research of this dissertation. Therein, right factorization, right coprime factorization and robust right coprime factorization of a nonlinear system

in a fairly general operator setting are recalled, which provide the theoretical basis for this dissertation. The concerned and researched problems are addressed in final.

In Chapter 3, right coprime factorization is extended using the proposed L_α operators, and nonlinear robust control design of nonlinear systems with perturbations is considered by using the proposed operator. Firstly, L_α operators is introduced, by which extended right coprime factorization approach is discussed for dealing with a broader class of nonlinear systems compared to right coprime factorization. Then, based on the obtained extended right coprime factorization, a feasible control design scheme is proposed to guarantee robust stability of the considered nonlinear systems with perturbations. Feasible design schemes were proposed for omitting the complicated calculation in process of control and design for the systems with perturbations, which means that robust stability of the perturbed nonlinear system can be guaranteed based on the proposed unimodular operator B . Finally, the effectiveness of the proposed design scheme was confirmed by a simulation example.

In Chapter 4, adjoint-based right coprime factorization and robust stability of nonlinear systems with perturbations are investigated. Firstly, a framework for considering nonlinear systems based on inner product is proposed to study right factorization, which proves fundamental knowledge for factorizing the systems. Secondly, a sufficient condition based on Hilbert spaces is given for the considered nonlinear systems to guarantee the isomorphism relationship. Thirdly, a robust control design for the considered nonlinear system is given based on the proposed controller and the unimodular property of controller. According to the proposed robust design scheme, the nonlinear systems with perturbations can be handled precisely and effectively. Finally, the numerical example is given to illustrate the validity of the proposed design methods.

In Chapter 5, in terms of left factorization and right coprime, a special class of nonlinear systems are considered to guarantee stability with the Bezout identity. Based on the designed method, the considered nonlinear systems are proved to have coprimeness. First, a design scheme is proposed to provide a convenient framework to deal with the internal-output stability of the nonlinear systems. The proposed scheme is motivated by the right factorization definition and the left factorization definition. By the proposed framework, a Bezout identity is designed, which can guarantee the internal-output stability and meanwhile realize left coprime factorization for nonlinear systems. Moreover, based on operator-based right coprime factorization, a class of MIMO nonlinear systems with uncertainties is considered for guaranteeing robust stability of the MIMO nonlinear systems. That is, based on right coprime factorization of the MIMO nonlinear systems, a feasible design scheme was proposed by using an unimodular operator. Based on the obtained conditions, the designed system was overall stable. Finally, a simulation examples are involved to illustrate the proposed design scheme for confirming effectiveness of the proposed methods.

Bibliography

- [1] A. Isidori, *Nonlinear control systems*, 3rd ed., Springer Verlag, Berlin, 1995.
- [2] V. I. Utkin, Sliding mode control design principles and applications to electric drives, *IEEE Transactions on Industrial Electronics*, vol. 40, no. 1, pp. 23-36, 1993.
- [3] W. Zhang and S. S. Ge, A global implicit function theorem without initial point and its applications to control of non-affine systems of high dimensions, *Journal of Mathematical Analysis and Applications*, vol. 313, no. 1, pp. 251-261, 2006.
- [4] M. Deng, *Operator-based nonlinear control systems design and applications*, 1st ed., Wiley-IEEE Press, 2014.
- [5] G. Zames, On the input-output stability of time-varying nonlinear feedback systems Part I: Conditions derived using concepts of loop gain, conicity, and positivity, *IEEE Transactions on Automatic Control*, vol. 11, no. 2, pp. 228-238, 1966.
- [6] M. M. Polycarpou and P. Ioannou, A robust adaptive nonlinear control design, *Automatica*, vol. 32, no. 3, pp. 423-427, 1996.
- [7] H. H. Rosenbrock, *State-space and multi-variable theory*, 1st ed., Wiley, New York, 1970.

- [8] D. C. Youla, H. A. Jabr and J. J. Bongiorno, Modern wiener-hopf design of optimal controllers, *IEEE Transactions on Automatic Control*, vol. 21, no. 6, pp. 319-338, 1976.
- [9] R. Curtain, G. Weiss and M. Weiss, Coprime factorization for regular linear systems, *Automatica*. vol. 32. no. 11, pp. 1519-1531, 1996.
- [10] P. Krishnamurthy, F. Khorrami and J. Jiang, Global output feedback tracking for nonlinear systems in generalized output-feedback canonical form, *IEEE Transactions on Automatic Control*, vol. 47, no. 5, pp. 814-819, 2002.
- [11] B. Labibi, H. Marquez and T. Chen, Decentralized robust output feedback control for control affine nonlinear interconnected systems, *Journal of Process Control*, vol. 19, no. 5, pp. 865-878, 2009.
- [12] G. Bartolini, A. Pisano, E. Punta and E. Usai, A survey of applications of second-order sliding mode control to mechanical systems, *International Journal of Control*, vol. 76, no. 9-10, pp. 875-892, 2003.
- [13] D. J. Leith and W. E. Leithead, Survey of gain-scheduling analysis and design, *International Journal of Control*, vol. 73, no. 11, pp. 1001-1025, 2000.
- [14] C. Bonivento, L. Marconi and R. Zanasi, Output regulation of nonlinear systems by sliding mode, *Automatica*, vol. 37, no. 4, pp. 535-542, 2001.
- [15] T. Zhang, S. Ge and C. Hang, Stable adaptive control for a class of nonlinear systems using a modified Lyapunov functions, *IEEE Transactions on Automatic Control*, vol. 45, no. 1, pp.129-132, 2000.

- [16] T. R. Johansen, Fuzzy model based control: Stability, robustness and performance issues, *IEEE Transactions on Fuzzy Systems*, vol. 2, no. 3, pp. 221-232, 1994.
- [17] Y. Katsurayama, M. Deng and C. Jiang, Operator based experimental studies on nonlinear vibration control for aircraft vertical tail with considering low order modes, *Transactions of the Institute of Measurement and Control*, vol. 38, no. 12, 2016.
- [18] Z. L. Lin, X. Y. Bao and B. M. Chen, Further results on almost disturbance decoupling with global asymptotic stability for nonlinear systems, *Automatica*, vol. 35, no. 4, pp. 709-717, 1999.
- [19] X. Chen, G. Zhai and T. Fukuda, An approximate inverse system for nonminimum-phase systems and its application to disturbance observer, *Systems & Control Letters*, vol. 52, no. 3-4, pp. 193-207, 2004.
- [20] J. Yao, Z. Jiao, D. Ma and L. Yan, High-Accuracy tracking control of Hydraulic rotary actuators with modeling uncertainties, *IEEE/ASME Transactions on Mechatronics*, vol. 19, no. 2, pp. 633-641, 2014.
- [21] Z. Li, T. Chai, C. Wen and C. Soh, Robust output tracking for nonlinear uncertain systems, *Systems & Control Letters*, vol. 25, no. 1, pp.53-61, 1995.
- [22] S. S. Ge and J. Wang, Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients, *IEEE Transaction on Automatic Control*, vol. 48, no. 8, pp. 1463-1469, 2003.
- [23] W. M. Haddad, V. Chellaboina and J. L. Fausz, Robust nonlinear feedback control for uncertain linear systems with nonquadratic performance criteria, *Systems & Control Letters*, vol. 33, no. 5, pp. 327-338, 1998.

- [24] M. Chiu and Y. Arkun, A methodology for sequential design of robust decentralized control systems, *Automatica*, vol. 28, no. 5, pp. 997-1001, 1992.
- [25] P. D. Christofides, Robust output feedback control of nonlinear singularly perturbed systems, *Automatica*, vol. 36, no. 1, pp. 45-52, 2000.
- [26] Q. Wang and C. Su, Robust adaptive control of a class of nonlinear systems including actuator hysteresis with Prandtl-Ishlinskii presentations, *Automatica*, vol. 42, no. 5, pp. 859-867, 2006.
- [27] Z. Jiang and I. Mareels, Robust nonlinear integral control, *IEEE Transaction on Automatic Control*, vol. 46, no. 8, pp. 1336-1342, 2001.
- [28] C. Su, Y. Stepanenko, J. Svoboda and T. P. Leung, Robust adaptive control of a class of nonlinear systems with unknown Backlash-Like Hysteresis, *IEEE Transaction on Automatic Control*, vol. 45, no. 12, pp. 2427-2432, 2000.
- [29] R. Rebarber and G. Weiss, Internal model based tracking and disturbance rejection for stable well-posed systems, *Automatica*, vol. 39, no. 9, pp. 1555-1569, 2003.
- [30] Z. Pan and T. Basar, Adaptive controller design for tracking and disturbance attenuation in parametric strict-feedback nonlinear systems, *IEEE Transaction on Automatic Control*, vol. 43, no. 8, pp. 1066-1083, 1998.
- [31] Z. Cai, M. S. de Queiroz and D. M. Dawson, Robust adaptive asymptotic tracking of nonlinear systems with additive disturbance, *IEEE Transaction on Automatic Control*, vol. 51, no. 3, pp. 524-529, 2001.

- [32] D. I. Landau, *Adaptive control: the model reference approach*, 1st ed., CRC Press, 1979.
- [33] J. Yao, Z. Jiao and D. Ma, Extended-state-observer-based output feedback nonlinear robust control of Hydraulic systems with backstepping, *IEEE Transactions on Industrial Electronics*, vol. 61, no. 11, pp. 6285-6293, 2014.
- [34] Y. Yang and J. Ren, Adaptive fuzzy robust tracking controller design via small gain approach and its application, *IEEE Transaction on Fuzzy Systems*, vol. 11, no. 6, pp. 783-795, 2003.
- [35] W. Lin and R. Pongvuthithum, Adaptive output tracking of inherently nonlinear systems with nonlinear parameterization, *IEEE Transaction on Automatic Control*, vol. 48, no. 10, pp. 1737-1749, 2003.
- [36] D. Verscheure, B. Demeulenaere, J. Swevers, J. Schutter and M. Diehl, Time-optimal path tracking for robots: a convex optimization approach, *IEEE Transaction on Automatic Control*, vol. 54, no. 10, pp. 2318-2327, 2009.
- [37] X. Liu and W. Wang, High order sliding mode and its application on the tracking control of piezoelectric systems, *International Journal of Innovative Computing, Information and Control*, vol. 4, no. 3, pp.697-704, 2008.
- [38] W. Baumann and W. J. Rugh, Feedback control of nonlinear systems by extended linearization, *IEEE Transaction on Automatic Control*, vol. 31, no. 1, pp. 40-46, 1986.
- [39] J. Choi, H. Lee and J. Zhu, Decoupling and tracking control using eigenstructure assignment for linear time-varying systems, *International Journal of Control*, vol. 74, no. 5, pp. 453-464, 2001.

- [40] S. D. Devasia, D. Chen and B. Paden, Nonlinear inversion based output tracking, *IEEE Transaction on Automatic Control*, vol. 41, no. 7, pp. 930-941, 1996.
- [41] R. Curtain and G. Weiss, Exponential stabilization of well-posed systems by colocated feedback, *SIAM Journal on Control and Optimization*, vol. 45, no. 1, pp. 273-297, 2006.
- [42] Y. Bo and W. Lin, Homogeneous observers, iterative design, and global stabilization of high-order nonlinear systems by smooth output feedback, *IEEE Transactions on Automatic Control*, vol. 49, no. 7, pp. 1069-1080, 2004.
- [43] D. Voytsekhovskiy and R. M. Hirschorn, Stabilization of single-input nonlinear systems using piecewise constant controllers, *IEEE Transactions on Automatic Control*, vol. 52, no. 6, pp. 1150-1154, 2007.
- [44] R. Devanathan, Linearization condition through state feedback, *IEEE Transactions on Automatic Control*, vol. 46, no. 8, pp. 1257-1260, 2001.
- [45] J. Davila, L. Fridman and A. Levant, Second-order sliding-mode observer for mechanical systems, *IEEE Transactions on Automatic Control*, vol. 50, no. 11, pp. 1785-1789, 2005.
- [46] X. Lu and S. Spurgeon, Robust sliding mode control of uncertain nonlinear systems, *Systems & Control Letters*, vol. 32, no. 2, pp. 75-90, 1997.
- [47] X. Yan, C. Edwards and S. Spurgeon, Decentralised robust sliding mode control for a class of nonlinear interconnected systems by static output feedback, *Automatica*, vol. 40, no. 4, pp. 613-620, 2004.

- [48] T. Oguchi and J. Richard, Sliding-mode control of retarded nonlinear systems via finite spectrum assignment approach, *IEEE Transactions on Automatic Control*, vol. 51, no. 9, pp. 1527-1531, 2006.
- [49] J. Hammer, Internally stable nonlinear systems with disturbances: A parametrization, *IEEE Transactions on Automatic Control*, vol. 39, pp. 300-314, 1994.
- [50] A. C. Desoer and M. G. Kabuli, Right factorization of a class of time-varying nonlinear systems, *IEEE Transaction on Automatic Control*, vol. 33, no. 8, pp. 755-757, 1988.
- [51] A. Banos, Stabilization of nonlinear systems based on a generalized Bezout identity, *Automatica*, vol. 32, no. 4, pp. 591-595, 1996.
- [52] B. D. O. Anderson, M. R. James and D. J. N. Limebeer, Robust stabilization of nonlinear systems via normalized coprime factor representations, *Automatica*, vol. 34, no. 12, pp. 1593-1599, 1998.
- [53] A. Feintuch, Coprime factorization of discrete time-varying systems, *Systems & Control Letters*, vol. 7, no. 1, pp. 49-50, 1986.
- [54] G. Chen, A note on the coprime fractional representation of nonlinear feedback systems, *Systems & Control Letters*, vol. 14, no. 1, pp. 41-43, 1990.
- [55] S. Wen and M. Deng, Operator-based robust nonlinear control and fault detection for a Peltier actuated thermal process, *Mathematical and Computer Modelling*, vol. 57, no. 1-2, pp. 16-29, 2013.
- [56] R. Danow and G. Chen, A necessary and sufficient condition for right coprime factorization of nonlinear feedback control systems, *Circuit Systems Signal Process*, vol. 12, no. 3, pp. 489-492, 1993.

- [57] R. J. P. de Figueiredo and G. Chen, *Nonlinear feedback control systems: An operator theory approach*, New York: Academic Press, INC., 1993.
- [58] M. Deng, A. Inoue, K. Ishikawa and Y. Hirashima, Tracking of perturbed nonlinear plants using robust right coprime factorization approach, *Proceedings of the 2004 American Control Conference*, pp. 3666-3670, Boston, USA, 2004.
- [59] M. Deng, A. Inoue and K. Ishikawa, Operator-based nonlinear feedback control design using robust right coprime factorization, *IEEE Transactions on Automatic Control*, vol. 51, no. 4, pp. 645-648, 2006.
- [60] M. Deng, A. Inoue and K. Eda, Fault detection in a thermal process control system with input constraints using robust right coprime factorization approach, *Proc. IMechE, Part I: Journal of Systems and Control Engineering*, vol. 221, no. 6, pp. 819-831, 2007.
- [61] M. Deng, A. Inoue and Y. Baba, Operator-based nonlinear vibration control system design of a flexible arm with Piezoelectric actuator, *International Journal of Advanced Mechatronic Systems*, vol. 1, no. 1, pp. 71-76, 2008.
- [62] M. Deng and A. Inoue, Networked nonlinear control for an aluminum plate thermal process with time-delays, *International Journal of System Science*, vol. 39, no. 11, pp. 1075-1080, 2008.
- [63] T. Zhou, Nonparametric estimation for normalized coprime factors of a MIMO system, *Automatica*, vol. 41, no. 4, pp. 655-662, 2005.
- [64] M. Deng and T. Kawashima, Adaptive nonlinear sensorless control for an uncertain miniature pneumatic curling rubber actuator using passivity and robust right coprime factorization, *IEEE Transactions on Control Systems Technology*, vol. 24, no. 1, pp. 318-324, 2016.

- [65] M. Deng, S. Wen and A. Inoue, Operator-based robust nonlinear control for a Peltier actuated process, *Measurement and Control: The Journal of the Inst. of Measurement and Control*, vol. 44, no. 4, pp. 116-120, 2011.
- [66] C. A. Desoer and M. G. Kabuli, Right factorizations of a class of timevarying nonlinear systems, *IEEE Transactions on Automatic Control*, vol. 33, no. 8, pp. 755-757, 1988.
- [67] N. Linard, B. Anderson and F. De Bruyne, Coprime properties of nonlinear fractional system realization, *Systems & Control Letters*, vol. 34, no. 5, pp. 265-271, 1998.
- [68] A. D. B. Paice and A. J. van der Schaft, The class of stabilizing nonlinear plant controller pairs, *IEEE Transactions on Automatic Control*, vol. 41, no. 5, pp. 634-645, 1996.
- [69] E. D. Sontag, Smooth stabilization implies coprime factorization, *IEEE Transactions on Automatic Control*, vol. 34, no. 4, pp. 435-443, 1989.
- [70] J. M. A. Scherpen and A. J. van der Schaft, Normalized coprime factorizations and balancing for unstable nonlinear systems, *International Journal of Control*, vol. 60, no. 6, pp. 1193-1222, 1994.
- [71] A. D. B. Paice and J. B. Moore, On the Youla-Kucera parameterization for nonlinear systems, *Systems & Control Letters*, vol. 14, no. 2, pp. 121-129, 1990.
- [72] M. S. Verma and L. R. Hunt, Right coprime factorizations and stabilization for nonlinear systems, *IEEE Transactions on Automatic Control*, vol. 38, no. 2, pp. 222-231, 1993.

- [73] M. Vidyasagar, *Control Systems Synthesis: A Factorization Approach*, The MIT Press, Cambridge, MA, 1985.
- [74] G. Chen and Z. Han, Robust right coprime factorization and robust stabilization of nonlinear feedback control systems, *IEEE Transactions on Automatic Control*, vol. 43, no. 10, pp. 1505-1510, 1998.
- [75] K. Fujimoto and T. Sugie, State-space characterization of Youla parametrization for nonlinear systems based on input-to-state stability, *Proceedings of the 37th IEEE Conference on Decision and Control*, pp. 2479-2484, Florida, USA, 1998.
- [76] S. Bi and M. Deng, Operator-based robust control design for nonlinear plants with perturbation, *International Journal of Control*, vol. 84, no. 4, pp. 815-821, 2011.
- [77] Z. Han and G. Chen, Dynamic right coprime factorization for nonlinear systems, *Nonlinear Analysis*, vol. 30, no. 5, pp. 3113-3120, 1997.
- [78] D. A. Lawrence and W. J. Rugh, Gain scheduling dynamic linear controllers for a nonlinear plant, *Automatica*, vol. 31, no. 3, pp. 381-390, 1995.
- [79] X. Wang, C. Su and H. Hong, Robust adaptive control of a class of nonlinear systems with unknown dead-zone, *Automatica*, vol. 40, no. 3, pp. 407-413, 2004.
- [80] N. Bu and M. Deng, System design for nonlinear plants using operator-based robust right coprime factorization and isomorphism, *IEEE Transactions on Automatic Control*, vol. 56, no. 4, pp. 952-957, 2011.

- [81] M. S. Verma, Coprime fractional representations and stability of nonlinear feedback systems, *International Journal of Control*, vol. 48 no. 3, pp. 897-918, 1988.
- [82] J. Hammer, Nonlinear systems: Stability and rationality, *International Journal of Control* vol. 40, no. 1, pp. 1-35, 1984.
- [83] J. Hammer, Nonlinear systems, stabilization, and coprimeness, *International Journal of Control*, vol. 42 no. 1, pp. 1-20, 1985.
- [84] B. D. Paice and J. B. Moore, Robust stabilization of nonlinear plants via left coprime factorizations, *Systems & Control Letters*, vol. 15, no. 2, pp. 125-135, 1990.
- [85] T. T. Tay and J. B. Moore, Left coprime factorization and a class of stabilizing controllers for nonlinear systems, *International Journal of Control*, vol. 49, no. 4, pp. 1235-1248, 1989.
- [86] G. Chen and R. J. P. de Figueiredo, Construction of the left coprime fractional representation for a class of nonlinear control systems, *Systems & Control Letters*, vol, 14, no. 4, pp.353-361, 1990.
- [87] D. Wang, F. Li, S. Wen, X. Qi, P. Liu and M. Deng, Operator based sliding mode nonlinear control design for a process with constraint input, *Journal of Robotics and Mechatronics*, vol. 27, no. 1, pp. 83-90, 2015.
- [88] S. Bi, L. Wang, Y. Zhao and M. Deng, Operator-based robust control for nonlinear uncertain systems with unknown backlash-like hysteresis, *International Journal of Control, Automation, and Systems*, vol. 14, no. 2, pp. 479-477, 2016.

- [89] S. Bi, M. Deng, and Y. Xiao, Robust stability and tracking for operator-based nonlinear uncertain systems, *IEEE Transactions on Automation Science and Engineering*, vol.12, no. 3, pp. 1059-1066, 2015.
- [90] K. Glover and D. McFarlane, Robust stabilization of normalized coprime factor plant descriptions with H_∞ -bounded uncertainty, *IEEE Transaction on Automatic Control*, vol. 34, no. 8, pp. 821-830, 1989.
- [91] R. V. Kadison and J. R. Ringrose, *Fundamentals of the theory of operator algebras. Volume I: elementary theory*, 1st ed., American Mathematical Society, 1997.
- [92] H. Wang, X. Liu and K. Liu, Robust adaptive neural tracking control for a class of stochastic nonlinear interconnected systems, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 27, no. 3, pp. 510-523, 2016.
- [93] Y. Yu, C. Sun and Z. Jiao, Adaptive Fuzzy output tracking control of MIMO nonlinear uncertain systems, *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 2, pp. 287-300, 2007.
- [94] Y. Liu, J. Li, S. Tong and C. L. P. Chen, Neural network control-based adaptive learning design for nonlinear systems with full-state constraints, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 27, no. 7, pp. 1562-1571, 2007.
- [95] Y. Liu and S. Tong, Optimal control-based adaptive NN design for a class of nonlinear discrete-time block-triangular systems, *IEEE Transactions on Cybernetics*, vol. 46, no. 11, pp. 2670-2680, 2016.
- [96] Y. Liu, S. Tong, C. L. P. Chen and D. Li, Neural controller design-based adaptive control for nonlinear MIMO systems with unknown hysteresis inputs, *IEEE Transactions on Cybernetics*, vol. 46, no.1, pp. 9-19, 2016.

- [97] Y. Li, S. Tong and T. Li, Hybrid fuzzy adaptive output feedback control design for uncertain MIMO nonlinear systems with time-varying delays and input saturation, *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 4, pp. 841-853, 2016.
- [98] H. Wu and M. Deng, Robust adaptive control scheme for uncertain non-linear model reference adaptive control systems with time-varying delays, *IET Control Theory & Applications*, vol. 9, no. 8, pp. 1181-1189, 2015.
- [99] S. Ge and C. Wang, Adaptive neural control of uncertain MIMO nonlinear systems, *IEEE Transactions on Neural Networks*, vol. 15, no. 3, pp. 674-692, 2004.
- [100] C. L. P. Chen, Y. Liu and G. Wen, Fuzzy neural network-based adaptive control for a class of uncertain nonlinear stochastic systems, *IEEE Transactions on Cybernetics*, vol. 44, no. 5, pp. 583-593, 2016.
- [101] G. Zames, Feedback and optimal sensitivity: model reference transformations, multiplicative seminorms, and approximate inverses, *IEEE Transactions on Automatic Control*, vol. 26, no. 2, pp. 301-320, 1981.
- [102] V. Dolezal, Some results on sensitivity of general input-output systems, *Circuits, Systems, and Signal Processing*, vol. 10, no. 1, pp. 71-89, 1991.
- [103] G. Tao and F. Lewis, *Adaptive Control of Nonsmooth Dynamic Systems*, New York: Springer-Verlag, 2001.
- [104] J. Hammer, Fraction representations of nonlinear systems: A simplified approach, *International Journal of Control*, vol. 46, no. 2, pp. 455-472, 1987.

- [105] R. Iyer, X. Tan, and P. Krishnaprasad, Approximate inversion of the Preisach hysteresis operator with application to control of smart actuators, *IEEE Transactions on Automatic Control*, vol. 50, no. 6, pp. 798-810, 2005.
- [106] S. Bi, M. Deng, and A. Inoue, Operator based robust nonlinear control system design of MIMO nonlinear feedback control systems. *Proceedings of International Conference on Control 2008, Manchester, UK*, CD-ROM, 2008.
- [107] S. Bi, M. Deng, and A. Inoue, Operator based robust stability and tracking performance of MIMO nonlinear systems. *International Journal of Innovative Computing, Information and Control*, v. 5 no. 10, pp. 3351-3358, 2009.
- [108] S. Bi, M. Deng, and S. Wen, Operator based output tracking control for nonlinear uncertain systems with unknown time-varying delays. *IET Control Theory & Applications*, vol. 5, no. 5, pp. 693-699, 2011.
- [109] Z. Chen, S. Ge, Y. Zhang, and Y. Li, Adaptive neural control of MIMO nonlinear systems with a block-triangular pure-feedback control structure. *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 11, pp. 2017-2029, 2014.
- [110] B. Chen, X. Liu, and S. Tong, Adaptive fuzzy output tracking control of MIMO nonlinear uncertain systems. *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 2, pp. 287-300, 2007.
- [111] M. Deng and S. Bi, Operator-based robust nonlinear control system design for MIMO nonlinear plants with unknown coupling effects, *International Journal of Control*, vol. 83, no. 9, pp. 1939-1946, 2010.

- [112] Y. Gao and Y. Liu, Adaptive fuzzy optimal control using direct heuristic dynamic programming for chaotic discrete-time system. *Journal of Vibration and Control*, vol. 22, no.2, pp. 595-603, 2016.
- [113] S. Huang, K. Tan, and T. Lee, Further result on a dynamic recurrent neural-network-based adaptive observer for a class of nonlinear systems. *Automatica*, vol. 41, no. 12, pp. 2161-2162. 2005.
- [114] E. Muramatsu, M. Ikeda, and T. Tamai, Stabilization of multi-input multi-output systems with variable operating conditions. *Nonlinear Analysis, Theory, Methods and Applications*, vol. 30, no. 4, pp. 2291-2296,1997.
- [115] R. Nelson and R. Freeman, Decentralized H_∞ filtering in a multi-agent system. *Proceedings of 2009 American Control Conference, St. Louis, MO*, pp. 5755-5760, 2009.
- [116] S. Shah, Z. Iwai, I. Mizumoto, and M. Deng, Simple adaptive control of processes with time-delay. *Journal of Process Control*, vol. 7, no. 6, pp. 439-499, 1997.
- [117] W. Zhang, F. Allgower, and T. Liub, Controller parameterization for SISO and MIMO plants with time delay. *Systems & Control Letters*, vol. 55, no. 10, pp. 794-802, 2006.
- [118] R. Zoub and M. Malabre, Almost disturbance decoupling and pole placement. *Automatica*, vol. 45, no. 11, pp. 2685-2691, 2009.

Appendix A

Proof

A.1 Proof of Lemma 2.1

Suppose that $Q : U^e \rightarrow U^e$ is causal. Then by definition we have that $P_T Q P_T = P_T Q$, so that if $x_T = y_T$, then

$$\begin{aligned} [Q(x)]_T &= P_T Q(x) = P_T Q P_T(x) = P_T Q(x_T) = P_T Q(y_T) \\ &= P_T Q P_T(y) = P_T Q(y) = [Q(y)]_T \end{aligned} \tag{A.1}$$

Conversely, suppose that $x_T = y_T$ implies $[Q(x)]_T = [Q(y)]_T$ for all $x, y \in U^e$ and all $T \in [0, \infty)$. Fix a $T \in [0, \infty)$, for any $x \in U^e$, let $y = x_T$, then $x_T = y_T$, so that $[Q(x)]_T = [Q(y)]_T$. Consequently, we have that

$$\begin{aligned} P_T Q P_T(x) &= P_T Q(x_T) = P_T Q(y) \\ &= [Q(y)]_T = [Q(x)]_T = P_T [Q(x)] \end{aligned} \tag{A.2}$$

Since $x \in U^e$ and $T \in [0, \infty)$ are arbitrary, it follows that $P_T Q P_T = P_T Q$ for all $T \in [0, \infty)$, which implies that Q is causal.

A.2 Proof of Lemma 2.2

Since

$$\| [Q(x)]_T - [Q(y)]_T \| \leq \| Q \| \| x_T - y_T \| \quad (\text{A.3})$$

for all $x, y \in U^e$ and all $T \in [0, \infty)$. Hence, $x_T = y_T$ implies that $[Q(x)]_T = [Q(y)]_T$ for all $x, y \in U^e$ and all $T \in [0, \infty)$.

A.3 Proof of Lemma 2.4

Sufficiency: Since $M \in \mu(W, U)$, for any $r \in U_s$, we have

$$r(t) = (AN + BD)w(t)$$

that is $w(t) = M^{-1}r(t) \in W_s$. Moreover, since $y(t) = N(w(t))$, $e(t) = BD(w(t))$, and $b(t) = A(y(t)) = AN(w(t))$, the stability of A, B, N and D implies that $y \in Y_s$, $e \in U_s$ and $b \in U_s$. Thus, the system is overall stable.

Necessity: First, it follows from the well-posedness and through the path of N and A that $M : W \rightarrow U$ is invertible. Then, it can be verified that both M and M^{-1} are stable. As a result, $M \in \mu(W, U)$.

A.4 Proof of Lemma 2.5

According to M is unimodular, we can get it is invertible. Also based on

$$AN + BD = M \quad (\text{A.4})$$

$$A(N + \Delta N) + BD = \bar{M} \quad (\text{A.5})$$

we have

$$\begin{aligned}
\bar{M} &= M + A(N + \Delta N) - AN \\
&= [I + (A(N + \Delta N) - AN)M^{-1}]M
\end{aligned} \tag{A.6}$$

combining with $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$, then $I + (A(N + \Delta N) - AN)M^{-1}$ is invertible, where I is the identity operator. Consequently,

$$\bar{M}^{-1} = M^{-1}(I + A(N + \Delta N)M^{-1} - ANM^{-1})^{-1} \tag{A.7}$$

Meanwhile, since $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$ and $M \in \mu(W, U)$, then $\bar{M} \in \mu(W, U)$ provided that the system is well-posed. As a result, for any $r \in U_s$, $w = \bar{M}^{-1}r \in W_s$. Further, since $y = (N + \Delta N)(w)$, $e = BD(w)$ and $b = A(N + \Delta N)(w)$, the stability of A , B , N , D and ΔN implies that $y \in Y_s$, $e \in U_s$ and $b \in U_s$. Then, the system is overall stable.

Appendix B

Publications

Journal papers

1. **F. Tao** and M. Deng, Adjoint Operator-based Isomorphism Realization and Control Design for Nonlinear Systems, *IET Control Theory & Applications*, Vol. 10, No. 8, pp. 919-925, 2016.
2. **F. Tao** and M. Deng, Perturbed Nonlinear Systems Control using Extended Robust Right Coprime Factorization, *International Journal of Control*, DOI: 10.1080/00207179.2016.1234718, 2016 (accepted).
3. **F. Tao** and M. Deng, Robust Stability of MIMO Nonlinear Uncertain Systems using Operator-based Right Coprime Factorisation, *International Journal of Systems Science*, DOI: 10.1080/00207721.2016.1277406, 2016 (accepted).

Proceedings papers

1. **F. Tao** and M. Deng, L_α Operator-based the Right Coprime Factorization for Nonlinear Systems, *Proceedings of the 2014 International Conference on Advanced Mechatronic Systems*, pp. 430-435, August 10-12, 2014, Kumamoto, Japan.

2. **F. Tao**, M. Deng and S. Bi, Practical Design Scheme for Nonlinear Systems with Unknown Bounded Perturbations using Robust Right Coprime Factorization, *Proceedings of 2015 American Control Conference*, pp. 5942-5947, July 1-3, 2015, Chicago, USA.
3. **F. Tao** and M. Deng, Operator-based Left Coprime Factorization for Nonlinear Systems, *Proceedings of the 2015 International Conference on Advanced Mechatronic Systems*, pp. 66-69, August 22-24, 2015, Beijing, China.

Other papers

1. M. Deng and **F. Tao**, Operator-based Optimal Design for Nonlinear Feedback Tracking Systems, *Proceedings of The 11th World Congress on Intelligent Control and Automation*, pp. 2607-2612, June 29-July 4, 2014, Shenyang, China.
2. M. Deng, A. Inoue and **F. Tao**, Fault Diagnose and Control for Plant's Safe Control of Integrated Environment, *Chemical Engineering*, Vol. 59, No. 10, pp. 53-60, 2014 (in Japanese).
3. **F. Tao** and M. Deng, Operator-based Robust Left Coprime Factorization for Nonlinear Systems, *IEE Japan Technical Meeting on Control*, pp. 21-26, October 31, 2015, Tokyo, Japan.
4. **F. Tao** and M. Deng, Inner Product-based Robust Right Coprime Factorization for Nonlinear Control, *IEE Japan Technical Meeting on Control*, pp. 25-30, December 19, 2015, Gunma, Japan.