Operator-based robust nonlinear control design for uncertain systems with disturbances

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Summary

This dissertation considers robust nonlinear control design for uncertain systems with disturbances using operator-based right coprime factorization method, which complements the theoretical analysis and control design of nonlinear systems. That is, by using operator theory a unified control design scheme is provided for rejecting nonlinear systems with uncertainties and disturbances as well as robustly bounded input bounded output stable is realized.

With the increasing complexity requirement of the modern technology, a great number of systems possess nonlinear property and multivariable characteristic. Therefore, researches on the nonlinear systems have attracted many researchers' attention due to the important role they have played in real application. Especially, these issues, for instance, robust analysis, output tracking problem and uncertainties as well as disturbances reduction which are belong to the nonlinear systems still remain challenging owing to their complex structures and the nonlinear characteristics. Meanwhile, the uncertainties almost exist in many kinds of systems where the uncertainties are major concern of two types in the control of uncertain nonlinear systems-parametric perturbations, general perturbations yielding from modeling errors and external disturbances which are central considered in this dissertation due to making a tremendous affection within the control systems, hence, it is necessary to reduce the adverse effects from the uncertainties as well as disturbance.

In this dissertation, firstly, by introducing a nonlinear operator controller, operator-based right coprime factorization is employed to consider the nonlinear system with disturbances. Then, based on the proposed feasible design schemes, adverse effect resulting from disturbance in nonlinear system is reduced. Secondly, the nonlinear systems with uncertainties and disturbances

are considered by redesigning the feedback controller, which can deal with a broader class of nonlinear systems. Further, three cases respectively for illustrating the relationship between the proposed conditions and the internal uncertainties or disturbances. Meantime, by the proposed design scheme, both of robust stability and tracking performance are realized, which can get better performances. Thirdly, besides the above contributions, in this dissertation, bilinear operator-based right coprime factorization for a class of nonlinear systems with disturbance and perturbation is considered from the input-output view of point, which provides a quantitative analysis method for the appearing uncertainty and disturbance. The merit of the proposed method lies in that it utilizes the characteristic of bilinear operator to design two stable integral controller such that the disturbance can be reduced and meantime output maintains. Then, robust stability of the considered nonlinear systems is guaranteed using reset control method, which enriches the coprime factorization methods. Finally, simulation examples are provided to illustrate effectiveness of the proposed design scheme.

Contents

List of Figures

Chapter 1

Introduction

1.1 Background

Considering development of modern design engineering and control technological requirement on comfort creatures, reliability and safety for practical manufacture, simple and effective control method and design scheme in order to improve performance of systems and meet demand of customers are greatly necessary and rapidly over the past decades, leading to that much attentions focusing on simple and effective control design scheme are received from engineers and researchers $[1] - [17], [41] - [48], [59] - [68]$.

Concerning control design of systems, there have been significant developments from various perspectives for both linear and nonlinear control systems over the last four decades. Although the linear systems thanks to its inherent characteristic on simple structure which has obtained greatly advancements, in practice, a great number of systems possess nonlinear property and multivariable characteristic. That is, in practical application, most of dynamic systems posses nonlinear characteristics since the unavoidable factors exists. Therefore, for dealing with difficulties that linear control method cannot work, the nonlinear control and design methods have attracted many researchers' attention due to the important role they have played in real application [38] $-$ [58], and [69] $-$ [87]. In particular, these issues, robustness, output tracking, perturbation and disturbance belonging to the nonlinear systems still remain challenging owing to their complex structures and nonlinear characteristics [15], [37], [59], [61], [88] $-$ [93]. Meanwhile, perturbation and disturbance phenomenons almost exist in a great number of systems where disturbance has major concern of two types in the control of unknown disturbance and estimated disturbance. General perturbation and disturbance yielding from modeling errors and external environment are central considered in this dissertation due to making a tremendous affection within the control systems, hence, it is necessary to reduce the adverse effects from them.

For considering control design for nonlinear systems with perturbation and disturbance to guarantee robust stability of the overall systems, a great number of control methods have been proposed from different viewpoints such as Lyapunov-based control method, model predictive control method, gain scheduling method, fuzzy control method, adaptive control method, feedback linearization design method, sliding mode control method and so on [13] $-$ [20], [32] $-$ [45], [88] $-$ [93]. Among these methods, all of them are proposed based on ordinary differential equations expression of nonlinear systems or linear systems, which are of rather difficulty to measure state vectors directly on-line measurements, leading to some restrictions on applying these approaches [50] $-$ [54]. For dealing with robust phenomenons always appearing in systems and avoiding the existing unnoticeable and unavoidable adverse effect of real systems, one promising method, operator-based right coprime factorization control method, has been proposed on robust control design thanks to a convenient framework established using this method from input-output view of point according to operator theory [32] $-$ [37] and [55] $-$ [93].

As to operator-based right coprime factorization method, there are comparative and main merits, although each control design method for nonlinear systems has its own inherent merits and limitations on studying nonlinear systems. We summarized the main merits of operator-based right coprime factorization from the following aspects. Firstly, operator-based right coprime factorization is proposed for dealing with general cases, which merely requires input-output mapping function. Moreover, the input-output relationship can be a relatively easy work using directly methods like taking experimental data. On this point, compared to other techniques aforementioned of nonlinear systems, it is not necessary to get all the states information of systems and build ordinary differential equations. Therefore, it gives a convenient framework to consider nonlinear systems. Secondly, control design using opertor-based right coprime factorization is easy comparatively, whose requirement lies in building a Bezout identity based on the internal signal of systems in order to guarantee stability in context of bounded input bounded output stable definition. Finally but not least, for studying robustness of uncertain nonlinear systems, the operator-based right coprime factorization method has a great advantage over the other control methods. A simple and effective description for the uncertain nonlinear systems can be given based on this method, which avoids difficulties in analyzing uncertainties quantitatively.

In the following statements, a detailed and systematic summary on history of the operator-based nonlinear control with disturbances method is proposed $[18] - [80]$.

1.2 Developments of operator-based nonlinear control

In recent decades, there have been significant developments from various perspectives for both linear and nonlinear systems $[21] - [35], [59] - [63], [88]$.

As well known, the linear coprime factorization theory has being increasingly perfected. Particularly, in practice, almost all systems possess nonlinear property and multivariable characteristic, which have been attracting researchers' attention due to important role. For nonlinear systems, robust control, sensitivity and tracking issues [59], [63] still remain challenging due to inevitable factors appearing in systems, such as parametric perturbations, modeling errors and uncertainties. For dealing with these issues, a great number of effective methods are proposed, such as the adaptive control, the sliding mode control method, operator-based right coprime factorization method, the geometric approach and so on.

Since the early 1970s, Rosenbrock [18] was the first person who introduced the coprime factorization method into the multivariate system, which has played a decisive role in the study of control system on stabilization as well as robustness. The author considered an optimization controller on the basis of parameterizing all stable controllers by utilizing polynomial matrix expression defined in the linear time invariant setting. In [19], the authors proposed an available method based on a least-square Wiener-Hopf minimization of an appropriately chosen cost functional in which the method is so-called Youla-Kučera parametrization (aslo simply called as Youla parametrization) which is a formula that describes all possible stabilizing feedback controllers for a given plant, as function of a single parameter based on the physical assumptions. This formula greatly facilitates the study of robust stability and adaptive control throughout the viewpoint of left and right coprime factorization for the given plant and controllers. Further, in 1984, Nett took the attention on the class of elements for the existence of coprime fractional representations which from the Bezout domain as well as given the expression on left or right coprime factorization of the given plant. After that, in the aspect of linear control system, Mcfarlance [20] made a significant development on state space expression based on the introduced definition of normalization of mutual factor decomposition, which method could provide a convenient expression using normalization coprime factorization in the research of robust stability issues. Besides, some sufficient conditions for existence of a doubly coprime factorization belonging to a large class of infinite-dimensional systems have been proposed in [21]. Moreover, there were many practical methods on dealing with linear control system for reaching robust stability or robustness as well. However, in practical almost all the actual control systems are non-linear systems, the researchers have payed more and more attentions on it.

Recently, the results on coprime factorization method of nonlinear control system can be summarized into two categories, one is to study the relationship between factorization and composure by means of input-output operator, the other is to study the left and right coprime factorization from the viewpoint of state space. There many researchers have made great contributions on coprime factorization method, such as Hammer, Verma, Moore, Paice, Tay and Guanrong Chen and so on[21] − [37]. Most of them provided a convenient framework to research the nonlinear systems based on the idea of coprime factorization from a viewpoint of the input-output stability. Hammer [21] − [25] who was the first person considered the robustness of the nonlinear coprime factorization based on operator theory method from the factorization technology of input-output operator with set theory method defined in nonlinear discrete time system. Later in 1994, Hammer [25] investigated the internal stability of a class of discrete nonlinear systems with output interference taking advantage of the right coprime factorization, in which from the input or the interference to the output response is parameterized as well as combined with the fixed controller for stabilizing the internal system. During the decade Hammer has made a great improvement in coprime factorization, however, it was very difficulty to get the solutions on coprime factorization based on Bezout identity defined on the discrete-time system. In order to overcome the above issues, the author in [26], [27] redefined the continuous system based on the concept of construction from the view of input-output point, so that made right coprime factorization independent of the solution with Bezout identity. Verma had popularized right and left coprime factorizations into all of nonlinear system described by input-output viewpoint and made much more convenient application on stabilizing nonlinear system using coprime factorization. Later, concerning the state-space characterization with Youla-Kucera parameterization so as to generalize the Youla-Kucera parameterization into normal nonlinear control system, Paice, Moore and Tay el. [28] $-$ [31] have done a lot of attempts both right coprime factorization and left coprime factorization for nonlinear system. In [31], the authors considered the intention of constructing analytic tools for the solution of stabilizing a nonlinear system to construct a class of stable controllers to realize the whole stable system based on left coprime factorization method as well proposed a necessary and sufficient conditions for stabilizing the nonlinear system. Over the next few years, the formula of Youla parameterization which is completely consistent with the linear system is given by the left coprime factorization [28], that is for a given nonlinear plant, if it has a bound stable left coprime factorization, then for any bound input there exists a stable feedback-compensator, and parameterizing a class of such stabilizers in the context of a bounded-input bounded-output (BIBO) stable. On this basis, the robust stability of the system is studied using this parametric formula[30]. Further, this robust stability result was of great significance for the study of nonlinear adaptive control and simultaneous stabilization problems [29].

In 1993, Figueriedo and Chen [35] payed much more attention to robust control and robust stabilization of the nonlinear system based on operator right coprime factorization of nonlinear system which emphasized qualitative properties analysis rather than design as well as the left coprime factorization can be done in the same manner. In [35], the author considered in a general operator-based setting which can be regards as to be linear or nonlinear, finite dimensional or infinite dimensional, and can be either in the frequency domain or in the time domain. First, there two main mathematical background the classical nonlinear Lipschitz operator and the generalized nonlinear Lipschitz operator theory have been proposed by Chen which served as a foundation for the research topics in systems theory. Proceeding to the next step, the authors provided that main idea of right coprime factorization for nonlinear feedback systems from input-output state space based on operator theory. The original idea of right coprime factorization can be addressed as follows [37]: to factorize a given plant operator P as a composition of two different operators N and D such that $P = ND^{-1}$, where N is stable and D is stable and invertible; then, to design two suitable stable operators A, B satisfying the Bezout identity $AN+BD = M$, where M is an unimodular operator. Then, according to operator theory, the given plant P is said to have right coprime factorization, and the system is said the be stable. Based on the right coprime factorization approach, more and more attentions have been payed on robust and tracking control for the nonlinear system with unknown bounded perturbations in [37], [61], [63], and [69]. In [37], the authors investigated robust right coprime factorization to conduct the nonlinear systems with perturbation, which provided a fairly general operator-theoretic setting for system analysis, control and design. A sufficient condition for guaranteeing robust stability of nonlinear systems with perturbation is proposed using robust right coprime factorization. Later, in [62], a new condition was proposed based on a Lipschitz norm inequality to consider robust stability, whose merit lies in that the proposed design scheme of this dissertation could deal with a broader class of nonlinear system compared with the former method in [37]. Based on the robust sufficient condition of [63], in [70], the authors provided an operator-based isomorphism method to obtain factorization of nonlinear systems quantitatively. In 1989, the author Figueired and Chen who are the first introduced the disturbance rejection under the viewpoint of coprime factorization aspect. In [74], the authors proposed an internal model control to analyze effects from uncertainties of nonlinear systems. In real application, the robust right coprime factorization method has also been developed, such as robust controller design of uncertain discrete time-delay systems with input saturation and disturbance in [89], employed low-order modes to design the control scheme using the operator-based approach in [67] and so on.

As for the development of reset control, much attention has been given to focus on design and control aspects on linear systems or nonlinear systems [94] and [95]. In [94], a reset adaptive observer is considered, including an adaptive observer and a reset law that resets the output of an integrator depending on a predefined condition. In [95], a class of square continuous-time nonlinear controllers are designed based on a suitable resetting rule, which proves that the arising hybrid system with temporal regularization is passive in the conventional continuous-time sense. Moreover, based on the passivity property, the finite gain stability of the nonlinear systems is investigated. Reset control can achieve sensor noise suppression performance without degrading disturbance rejection, which makes reset control an important technique for performance improvement.

1.3 Motivations of the dissertation

Although it is simple and effective using the operator-based robust right coprime factorization method in the aspect of controlling and designing for nonlinear systems, and a great number of results are proposed in many fields. However, there are still some points worthy being studying, not only the purpose of enriching and refining the operator-based robust right coprime factorization method, but also dealing with more issues using it.

In this dissertation, main motivated concepts based on operator-based right coprime factorization are stated from the following three aspects. First, when it comes to that the operator-based right coprime factorization approach is used to deal with robust stability of uncertain nonlinear systems, the basic idea lies in how to guarantee robust Bezout identity. However, there is little research that authors consider the issue of reducing the adverse effect resulting from the existing uncertainties such that stability of the nominal system still remain. Based on this idea, in this dissertation, the Chapter 3 address an effective design scheme of combing operator-based right coprime factorization with a new nonlinear operator controller to deal with nonlinear systems with unknown disturbance for guaranteeing robust stability and reducing adverse effects. That is, with the framework established by using operator-based right coprime factorization, both of robust stability and reduction of disturbances are obtained by using the proposed nonlinear operator controller. Second, meantime with this issue, other motivated idea for dealing with both perturbation and disturbance is proposed in Chapter 4. Most traditional researches were aiming at one interference object in control system design, either internal perturbation or external disturbance, as has been stated above. However, in most practical nonlinear control systems there are a number of different kinds of external disturbance and internal perturbation subjected to the circumstance, temperature, coupling between different systems and so on. Therefore, in Chapter 4, for removing the adverse effect resulting from the external disturbance and internal perturbation, a feasible framework is proposed based on the designed scheme, which provides a convenient structure to consider the nonlinear system with external disturbance and internal perturbation. Moreover, based on the proposed design scheme, the adverse effects of internal perturbation and external disturbance are reduced, and output tracking performance is realized simultaneously. However, in Chapter 4, the quantitative analysis for the existing perturbation and disturbance is not considered. Therefore, in Chapter 5, a bilinear operator-based right coprime factorization for nonlinear system with perturbation and disturbance is introduced, which can consider adverse effect resulting from perturbation and disturbance quantitatively. Based on the proposed method, a feasible framework is established for considering robust control, sensitivity and tracking performance, which not only separates perturbation and disturbance, but also provides a fundamental base to design a controller for the considered system. After that, operator-based reset control for nonlinear systems with unknown bounded disturbance is addressed. That is, in the context of operator-based right coprime factorization, reset control is realized and robust stability of nonlinear systems with unknown bounded disturbance is guaranteed.

1.4 Contributions of the dissertation

The proposed nonlinear control scheme on uncertain nonlinear systems enrich the operator-based coprime factorization method. Meanwhile, bilinear operator-based right coprime factorization for nonlinear systems with perturbation and disturbance are discussed, providing a quantitative analysis method for the appearing perturbation and disturbance. The proposed control design scheme employs operator theory setting formulated under extended norm linear space, which is suitable for stability, causality, robustness, uniqueness of internal control signals as well as coprime factorization in nonlinear systems control theory and application. Extended norm linear space is important since all control signals in engineering are supposed to be time-limited. However, in the study of a control processing we do not know the time the process stops. That is the reason for providing the extended norm linear space definition, which can deal with the practical issue from mathematical theory, and many useful techniques and results can be carried over from the standard Banach space to the extended norm linear space, which is fundamental for a realizable physical control system.

Robust stability and tracking performance are necessary and critical for nonlinear systems, due to the fact that uncertainties always exist in the real systems, making an bad effect in the nonlinear systems. Based on operator-based right coprime factorization method, main principle of robust control is to design feedback controllers such that robust Bezout identity is satisfied even in the cases where the considered nonlinear systems exist uncertainties. Comparing to general control methods in nonlinear systems, such as linear matrix inequality, sliding mode control, adaptive control, operator-based right coprime factorization method is more simple and effective thanks to the simple framework obtained based on robust Bezout identity. This is one of the merits of operator-based right coprime factorization. Meantime, it is one main contributions of the proposed design schemes in this dissertation as well.

This dissertation is mainly focusing on considering the uncertain nonlinear systems by using operator-based right coprime factorization method. In detail, by introducing a nonlinear operator controller, opeartor-based right coprime factorization is employed to consider the nonlinear system with disturbance. Then, based on the proposed feasible design schemes, adverse effect resulting from disturbance in nonlinear system is reduced. Next, the nonlinear systems with perturbation and disturbance are considered by redesigning the feedback controller, which can deal with a broader class of nonlinear systems. Three cases respectively for illustrating the relationship between the proposed conditions and the internal perturbations or disturbances, that means which kind of cases would be corresponding to which conditions is shown respectively. Meantime, by the proposed design scheme, both of robust stability and tracking performance are realized, which can get better performances. Besides the above contributions, in this dissertation, bilinear operator-based right coprime factorization for a class of nonlinear systems with disturbance and perturbation is considered from the input-output view of point. Robust stability of the considered nonlinear systems is guaranteed, which enriches the coprime factorization methods.

In summary, this dissertation considers robust nonlinear control design for uncertain systems with disturbances using operator-based right coprime factorization, which complements the theoretical analysis and control design of nonlinear systems.

1.5 Organization of the dissertation

This dissertation is organized as follows. In Chapter 2, the mathematical preliminaries and problem statement are provided. Mathematical preliminaries will be recalled as the theoretical foundation for the research and also as the cornerstone leading to the following chapters in this dissertation.

Chapter 3 is devoted to investigate an effective design scheme of combining right coprime factorization with a new nonlinear operator controller to deal with nonlinear systems with unknown disturbance for guaranteeing robust stability and reducing the adverse effects of unknown disturbance. That is, with the robust right coprime factorization method, the equivalent framework of nonlinear systems is obtained, which provides a convenient viewpoint; then based on operator theory, a new nonlinear operator is proposed for dealing with the unknown disturbance of nonlinear systems to reduce adverse effects on nonlinear systems. Finally, a simulation example is provided to illustrate effectiveness of the proposed design scheme.

In Chapter 4, both internal perturbation and external disturbance of the nonlinear systems are considered together using a new design scheme based on redesigning the feedback controller. In detail, from error signal point of view, the adverse effects resulting from external disturbance and internal perturbation of the nonlinear systems are removed by the designed nonlinear operator. Three cases respectively for illustrating the relationship between the proposed conditions and the internal perturbations or disturbances, that means which kind of cases would be corresponding to which conditions, respectively is shown. Simultaneously, output tracking performance is realized using the proposed design scheme. Finally, a simulation example is provided to illustrate effectiveness of the proposed design scheme.

In Chapter 5, the bilinear operator-based right coprime factorization for nonlinear system with perturbation and disturbance is introduced, which can consider adverse effect resulting from perturbation and disturbance quantitatively. Based on the proposed method, a feasible framework is established for considering robust control, sensitivity and tracking performance, which not only separates perturbation and disturbance, but also provides a fundamental base to design a controller for the considered system. In terms of the insensitivity property, it is addressed for the case where perturbation and disturbance both exist in nonlinear systems. After that, operator-based reset control for nonlinear systems with disturbance is addressed. That is, in the context of operator-based right coprime factorization, reset control is realized and robust stability of nonlinear systems with disturbance is guaranteed. Finally, a simulation example is provided to illustrate effectiveness of the proposed design scheme.

In Chapter 6, the proposed design methods in this dissertation for uncertain nonlinear systems are summarized, including operator-based nonlinear systems with unknown disturbance rejection using right coprime factorization, operator-based perturbed nonlinear systems with external disturbance rejection using right coprime factorization and bilinear operator-based right coprime factorization for robust control and sensitivity analysis of uncertain nonlinear systems.

Chapter 2

Mathematical preliminaries and problem statement

2.1 Introduction

In this chapter, the mathematical preliminaries and problem statement are provided. Mathematical preliminaries will be recalled as the theoretical foundation for the research and also as the cornerstone leading to the following chapters in this dissertation [32] – [37].

In Section 2.2, firstly, the definitions of spaces as the basis of the research including linear space, normed space, Banach space, Hilbert space, extended linear space which are all associated with Banach space are defined. Secondly, the definition of operator and some important operators are provided such as linear and nonlinear operator, invertible operator, stable operator, unimodular operator, Lipschitz operator and generalized Lipschitz operator which are all defined in Banach space. Based on the generalized Lipschitz operator, the causality is discussed about the relationship within the generalized Lipschitz operator. Thirdly, a special topic of factorization for nonlinear mappings which always shows the description on control systems will be provided. Coprime factorization as a basic tool for linear mapping has been well developed into nonlinear control systems. In here, as a key point, we recommended the right coprime factorization which has been an important techniques in the study of robust stabilization of the nonlinear system. Fourthly, the definition of robust right coprime factorization will be recalled, which are used to improve reference tracking and to enhance the rebustness of the compensated system in the face of plant uncertainties. Based on the operator theory, a simple necessary and sufficient condition on the existence of a right coprime factorization will be formulated to guarantee the coprimeness of the factorization for the nonlinear systems as well as to guarantee robust stability of the nonlinear systems with perturbations by a sufficient robust condition.

In Section 2.3, the main problem statements in this dissertation are discussed in order to develop the main results of this dissertation. In details, the extended robust right coprime factorization conditions associated to each kind of framework of nonlinear control systems with different form of uncertainties are described. At the same time, for separating the appearing internal perturbation and external disturbance in the systems, bilinear operator controller is proposed such that a feasible framework is established aiming to design and control of robust stability, sensitivity and tracking performance.

2.2 Mathematical preliminaries

In this section, the fundamental definitions and notations on kinds of spaces and operators are clarified throughout this dissertation aiming to enhance understanding the method that proposed in this control systems. Moreover, some important results are listed.

2.2.1 Definitions of spaces

In modern mathematics spaces are defined as sets with some added structure. They are frequently described as different types of manifolds, which are spaces that locally approximate to Euclidean space, and where the properties are defined largely on local connectedness of points that lie on the manifold. There are however, many diverse mathematical objects that are called spaces. For example, vector spaces such as function spaces may have infinite numbers of independent dimensions and a notion of distance very different from Euclidean space, and topological spaces replace the concept of distance with a more abstract idea of nearness. In here, first, there are two basic space will be introduced: linear space that is also called vector space, and topological space. A vector space (also called a linear space) is a collection of objects called vectors, which may be added together and multiplied by numbers, called scalars. Moreover, in the number sense, the linear space is made up of real linear spaces what over the field of real numbers, complex linear spaces what over the field of complex numbers and more general linear spaces over any field. A topological space may be defined as a set of points, along with a set of neighbourhoods for each point, satisfying a set of axioms relating points and neighbourhoods. In this dissertation, the used space is based on linear space that is also named vector spaces which played as a fundamental role during the research.

Linear spaces

A nonempty set $\mathcal{V}$ that is an arbitrary field is called a linear space if there exist any pair of elements $f, g \in \mathcal{V}$ can satisfy:

(1) added together by an operation can get an element $f + g \in \mathcal{V}$ that is called the property of addition;

(2) $f + g = g + f$;

(3) $f + (g + h) = (f + g) + h$, such that, for any elements $f, g, h \in \mathcal{V}$ are hold;

(4) for all $f \in \mathcal{V}$, such that $f + 0 = f$ always hold, since there exists a unique element $0 \in \mathcal{V}$;

(5) for each element $f \in \mathcal{V}$ such that $f + (-f) = 0$, if and only if $-f \in \mathcal{V}$;

(6) multiplied by any coefficient α of a field $\mathcal{R}$ of real numbers can get an element $\alpha \cdot f \in \mathcal{V}$ that is called the property of multiplication;

(7) $\alpha(f + g) = \alpha f + \alpha g$;

(8) $(\alpha + \beta)f = \alpha f + \beta f$, where $\beta \in \mathcal{R}$;

(9) $(\alpha\beta)f = \alpha(\beta f)$;

(10) $1 \times f = f$.

Moreover, a complex vector space is a vector space whose field of scalars is the complex numbers. A nonempty subset U of a linear space $\mathcal{V}$ is called a subspace of $\mathcal{V}$ if it is satisfied with the addition and scalar multiplication in $\mathcal{V}$ from $(1) - (10)$, which can be expressed in the form of $U \subset \mathcal{V}$.

Normed linear spaces

A normed linear space (also called normed vector space) is a vector space which defined in norm from the viewpoint of mathematics. A normed linear space is a pair $V, \| \cdot \|$ where V is a vector space and $\| \cdot \|$ is a norm on V. $\| \cdot \|$ is called the length of vector, that has the following properties in such a vector space:

(1) $\| x \| \geq 0$; and $\| x \| = 0$ if and only if $x = 0$;

where x is any elements in V, 0 is the zero vector associated to zero length; every other vector has a positive length.

(2) $\| ax \| = | a | \| x \|$, for any scalar a;

in which a vector multiply by random positive number just changing its length without changing its direction.

(3) $\| x + y \| \leq \| x \| + \| y \|$;

whenever $x,\ y \in V$; which is called the triangle inequality. That is the distance from point A through B to C is never shorter than going directly from A to C, or the shortest distance between any two points is a straight line.

Banach space

In mathematics, a Banach space is a complete normed vector space more specifically in functional analysis. In details, a Banach space is a vector space over the field R of real numbers which is respect to a norm, or a Banach space is a vector space over the field C of complex numbers, which is complete associated to norm. From the view of geometrical point, a Banach space is a vector space with a metric that allows the computation of vector length and distance between vectors.

From the view of sequence point, in the sense that a Cauchy sequence of vectors always converges to a well defined limit which is within the space. That is, for every Cauchy sequence x_n belongs to a vector space X, there always exists an element $x \in X$ such that

$\lim_{n\to\infty} x_n = x \Longleftrightarrow \lim_{n\to\infty} \| x_n - x \|_X = 0.$

Extended linear space

In general, an extended linear space also called an extended normed linear space is not complete in norm indicating that it is determined by a relative Banach space [35]. Let M be a linear space which is the family of real-valued measurable functions defined on $[0,\ \infty)$. Let F_T be the projection operator mapping from M to M_T which is another linear space defined in measurable function, for each constant $T \in [0,\ \infty)$, such that

$$f_T(t) := F_T(f)(t) = \begin{cases} f(t), & t \le T \\ 0, & t > T \end{cases}$$

where $f_T(t) \in M_T$ is called the truncation of $f(t)$ associated to T. Then, for any given Banach space X of measurable functions, set

$$X^e = \{f \in M : \| f_T \| < \infty \text{ for all } T\ < \infty\} \tag{2.1}$$

Obviously, X^e is a linear subspace of X. The space X^e is called the extended linear space related to the Banach space.

It is worth mentioning that the extended linear space is usually not completeness in norm. As a matter of fact, there only is local norm boundedness corresponding to an element in X^e, such that even a norm cannot be well defined in X^e. The reason for considering extended linear space is that all the control signals are time-limited in practical as well as many useful approaches and results can be bring from the standard Banach space X to the extended space X^e if the norm is defined in a suitable way.

2.2.2 Definitions of operators

Let U and Y be linear spaces defined in the field of scalar numbers, and let U_s and Y_s be two normed linear spaces, called the stable subspaces of U and Y, respectively, defined suitably by two normed linear spaces under certain norm denoted $U_s = \{u \in U :\| u \|< \infty\}$ and $Y_s = \{y \in Y :\| y \|< \infty\}$ [61].

Operator

In mathematics, an operator $S : U \to Y$ is generally a mapping that acts on the elements of input space U to produce other elements of the output space Y. And the framework of the operator S can be shown in Figure 2.1, moreover, its expression form can be written as

$$y(t) = S(u)(t)$$

form the viewpoint of mathematical, where $u(t)$ and $y(t)$ are the element of U and Y denoted the input single and output single, respectively.

Linear and nonlinear operator

Let $S : U \to Y$ be an operator mapping from input space U to the output space Y denoted by $\mathcal{D}(S)$ and $\mathcal{R}(S)$ as the domain and range of S, respectively. Provided that S is satisfied with the following condition (Addition

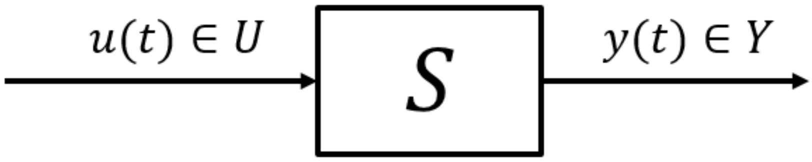

Figure 2.1: An operator diagram

Rule and Multiplication Rule)

$$S : au_1 + bu_2 \to aS(u_1) + bS(u_2)$$

for all u_1, $u_2 \in \mathcal{D}(S)$ and all $a, b \in$ R, then S is said to be linear operator; otherwise, it is called to be nonlinear operator. According to the definition of linear operator, it is noted that a linear operator is satisfied with addition rule and multiplication rule for different elements belonging to domain space. It can be found that linearity is a special case of nonlinearity. In what follows, nonlinear will always mean not necessarily linear unless otherwise indicated.

Bounded input bounded output (BIBO) stability

Let S be a nonlinear operator that acts on its domain $\mathcal{D}(S) \subseteq U$ and range $\mathcal{R}(S) \subseteq Y$. S is said to be input-output stable, if $S(U) \subseteq Y$. Another crucial definition is bounder input bounded output (BIBO) stability [61]. From the viewpoint of signal processing, the BIBO is form of stability for linear signals and systems with taking inputs. If S maps all input functions from U_s into the output space Y_s, such that $S(U_s) \subseteq Y_s$, then operator S is said to be bounded input bounded output (BIBO) stable or simply, stable. That is, the output will be bounded for every input to the system. Otherwise, S is said to be unstable, when S maps some inputs from U_s to $Y^e \backslash Y_s$ (if not empty). For any stable operators defined here and later, in this dissertation they always mean BIBO stable.

Invertible

An operator S is called invertible if there exists an operator P such that

$$S \circ P = P \circ S = I$$

where I denotes the identity operator, P is said to be the inverse of S expressed in the form of P^{-1}, in which $\circ$ denotes the operation defined in the operator theory which can be simple presented as SP.

Unimodular operator

Let $\mathcal{S}(U, Y)$ be the set of stable operators from U to Y. Then $\mathcal{U}(U, Y)$ is a subset which defined under $\mathcal{S}(U, Y)$ in the form of

$$\begin{aligned} \mathcal{U}(U, Y) =& \{F : F \in \mathcal{S}(U, Y), \\ & F \text{ is invertible with } F^{-1} \in \mathcal{S}(U, Y)\}. \end{aligned}$$

Hence, every elements of $\mathcal{U}(U, Y)$ are said to be unimodular operators [37].

Lipschitz operator

Let $\mathcal{L}(X_s, Y_s)$ denote the family of two normed linear operators over the complex numbers from X_s to Y_s, where X_s and Y_s are two normed linear spaces. Let $\mathcal{N}(X_s, Y_s)$ be the family of all nonlinear operators mapping from X_s into Y_s, obviously, $\mathcal{L}(X_s, Y_s) \subseteq \mathcal{N}(X_s, Y_s)$. In the case that $X_s = Y_s$, we use the notation $\mathcal{L}(X_s)$ and $\mathcal{N}(X_s)$, respectively, instead of $\mathcal{L}(X_s, Y_s)$ and $\mathcal{N}(X_s, Y_s)$ for simplicity.

Let $\mathcal{F}(U_s, Y_s)$ be the family of operators S, where U_s is a subset of X_s, and $\mathcal{F}(U_s, Y_s) \in \mathcal{N}(X_s, Y_s)$. Let $Lip(U_s, Y_s)$ be the subset of $\mathcal{F}(U_s, Y_s)$ with all its elements S satisfying $\| S \| < \infty$. Each $S \in Lip(U_s, Y_s)$ is called a Lipschitz operator mapping from U_s to Y_s, and the number $\|S\|$ is introduced by

$$\|S\| := \sup_{\substack{x_1, x_2 \in U_s \\ x_1 \neq x_2}} \frac{\|S(x_1) - S(x_2)\|_{Y_s}}{\|x_1 - x_2\|_{X_s}}$$

is called the Lipschitz semi-norm of the operator S on U_s [35].

Note that, in general, $\| S \| = 0$ does not necessarily imply $S = 0$. In fact, $\| S \| = 0$ if and only if S is a constant-operator (need not be zero) that maps all elements from U_s to the same element in Y_s.

For any fixed $x_0 \in U_s$, the number

$$\| S \|_{Lip} := \| S(x_0) \|_{Y_s} + \sup_{\substack{x_1, x_2 \in U_s \\ x_1 \neq x_2}} \frac{\| S(x_1) - S(x_2) \|_{Y_s}}{\| x_1 - x_2 \|_{X_s}} \tag{2.2}$$

defines a norm for all $S \in Lip(U_s, Y_s)$. Then, $\| S \|_{Lip}$ is called the Lipschitz norm of S defined by $x_0 \in U_s$. It is worth reminding that, it amounts to showing that $\| S \|_{Lip} = 0$ implies $S = 0$, which called zero operator. It is also evident that a Lipschitz operator is both bounded and continuous on its domain.

Generalized Lipschitz operator

Let $\mathcal{L}(X, Y)$ denote the family of two normed linear operators over the complex numbers from X to Y. Let $\mathcal{N}(X, Y)$ be the family of all nonlinear operators mapping from X into Y, which are two Banach spaces. Obviously, $\mathcal{L}(X, Y) \subseteq \mathcal{N}(X, Y)$. In the case that $X = Y$, we use the notation $\mathcal{L}(X)$ and $\mathcal{N}(X)$, respectively, instead of $\mathcal{L}(X, Y)$ and $\mathcal{N}(X, Y)$ for simplicity.

Let X_u and Y_u be two extended linear spaces, which are associated with two given Banach spaces X and Y of real-valued measurable functions defined on the time domain $[0, \infty)$, respectively. Let U be a subset of X_u. If there exists a constant L such that

$$\| [S(x_1)]_T - [S(x_2)]_T \|_{Y_u} \leq L \| [x_1]_T - [x_2]_T \|_{X_u} \tag{2.3}$$

for all $x_1, x_2 \in U$ and for all $T \in [0; \infty)$. The nonlinear operator $S : U \longrightarrow Y_u$ is called a generalized Lipschitz operator on U, and its actual norm can be given by

$$\begin{aligned} \| S \|_{g-Lip} = \| S(x_0) \|_{Y_u} + \| S \| = \| S(x_0) \|_{Y_u} \\ + \sup_{T\in[0,\infty)} \sup_{\substack{x_1,x_2\in U \\ x_1\neq x_2}} \frac{\| [S(x_1)]_T - [S(x_2)]_T \|_{Y_u}}{\| [x_1]_T - [x_2]_T \|_{X_u}} \end{aligned} \tag{2.4}$$

for any fixed $x_0 \in U$.

Note that the least such constants L shown in (2.3) is given by

$$\| S \| := \sup_{T\in[0,\infty)} \sup_{\substack{u_1,u_2\in D^e \\ u_1\neq u_2}} \frac{\| [Q(u_1)]_T - [Q(u_2)]_T \|}{\| [u_1]_T - [u_2]_T \|} \tag{2.5}$$

which is a semi-norm for general nonlinear operators.

There are some remarks need to be mentioned, it is since that the standard Lipschitz operator and generalized Lipschitz operator have different domains and ranges, so that the family of standard Lipschitz operator and generalized Lipschitz operator are not comparable. However, it can be verified that many standard Lipschitz operators are also extended Lipschitz. And it can be also verified that generalized Lipschitz operator is more widely useful than standard Lipschitz operator for nonlinear systems in the aspects of control design and engineering such as stability, robustness, uniqueness of internal control signals. For any operator defined throughout this section always assumed to be generalized Lipschitz operator.

Causal

Let U^e be the extended linear space depended on a given Banach space U, and let $S : U^e \to U^e$ be a nonlinear operator defined on a nonlinear control system. Then, S is called causal if and only if

$$P_T S P_T = P_T S$$

for all $T \in [0, \infty)$, where P_T is a projection operator.

Form the viewpoint of physical the definition of causality is addressed as follows. The idea that the outputs of the systems at any time depends only on the present and past values of the corresponding system inputs, then we have $SP_T(u) = Q(u)$ for all input signals u belonging to the domain of S, so that $P_T S P_T = P_T S$. Conversely, if $P_T S P_T = P_T S$ for all $T \in [0,\ \infty)$, then we have $P_T S(I - P_T)(u) = 0$ for all input u in the domain of Q, which implies that any value of a system input in the future, $(I - P_T)(u)$, does not affect the present and past values of the corresponding system output given by $P_T S(\cdot)$, or in other words, system outputs depend only on the present and past values of the corresponding system inputs.

Lemma 2.1 *A nonlinear operator $S : U_e \to U_e$ is causal if and only if for any $x_1, x_2 \in U_e$ and $T \in [0, \infty)$, $x_{1T} = x_{2T}$ implies $[S(x_1)]_T = [S(x_2)]_T$.*

Proof. The proof is given in Appendix A.1 [35].

Lemma 2.2 *If $S : U_e \to U_e$ is a generalized Lipschitz operator, then S is causal.*

Proof. The proof is given in Appendix A.2 [35].

Lemma 2.3 *A nonlinear generalized Lipschitz operator produces a unique output from an input, that is, if the input x and output y are related by a generalized Lipschitz operator S such that $y = S(x)$, then $x_T = \tilde{x}_T$ implies that $y_T = \tilde{y}_T$ for all $T \in [0,\ \infty)$.*

It is worth mentioning that a nonlinear operator may produce nonunique outputs from an input for a set-valued mapping. It is clear that from **Lemma 2.2** and **Lemma 2.3** imply that the uniqueness requirement can be guaranteed by introduced the generalized Lipschitz operator. That is, in real systems, the internal signals of the systems are required to be unique.

2.2.3 Definition of right coprime factorization

A nominal operator based nonlinear control system is shown in Figure 2.2, in which the given plant $P : U \to Y$ is from the input space U to the output space Y, where the signals u and y denote the control input and system output, respectively.

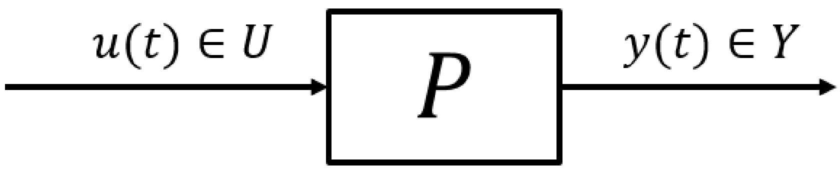

Figure 2.2: A nominal operator diagram

Right factorization

For the given normal system operator $P : U \to Y$ shown in Figure 2.3, where U and Y are the input space and the output space [61]. If there exist a linear space W and two stable operators N and D, such that the operator P as a composition of N and D in form of $P = ND^{-1}$, where $N : W \to Y$ and $D : W \to U$ is invertible, then the operator P is said to have a right factorization, the linear space W is called a quasi-state space of P .

Right coprime factorization

Provided that P exists a right factorization (N, D), furthermore, the two stable operator N and D satisfy the Bezout identity $AN+BD = M$, for some stable operators A and B, where $A : Y \to U$ and $B : U \to U$ is invertible, Bezout identity as shown in the form of

$$AN + BD = M, \text{ for } M \in \mathcal{U}(W,\ U),$$

where M is unimodular, then the factorization is said to be coprime, that is, operator P is said to have a right coprime factorization [61]. Generally, P is unstable and $(N,\ D,\ A,\ B)$ are stable to be determined to be design in the system issue.

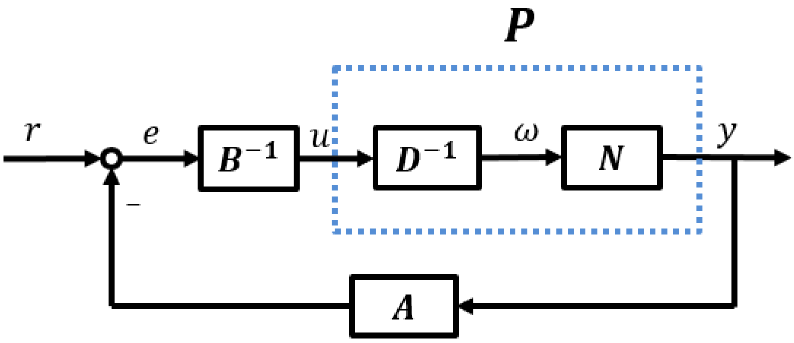

Figure 2.3: A nonlinear system with right coprime factorization

We remake that the transformative Bezout identity introduced here is defined on the linear space X. Moreover, if $X = U$, the M can be usually replaced by the identity operator I. Note that the initial state should be consistent with the Bezout identity, that is, $AN(w_0, t_0)+BD(w_0, t_0) = M(w_0, t_0)$ should be satisfied. Furthermore, in this dissertation, we select $t_0 = 0$ and $w_0 = 0$ without loss of generality.

Well-posedness

The nonlinear control system shown in Figure 2.3 is said to be well-posed, if for every input signal $r \in U$ determine an unique corresponding signals in the system (i.e. e, u, w, b and y) are uniquely determined.

Overall stable

The nonlinear system shown in Figure 2.3 is said to be overall stable, provided that $r \in U$, implies that $u \in U,\ y \in Y,\ w \in W,\ e \in U$ and $b \in U$.

Lemma 2.4 *Assume that the system shown in Figure 2.3 is well-posed and the system has a right factorization in the form of* $P = ND^{-1}$. *Then the system is said to be overall stable if and only if the operator* M *in Bezout identity is a unimodular operator.*

Proof. The proof is given in Appendix A.3 [35].

2.2.4 Definition of robust right coprime factorization

Considering the nonlinear system with perturbation show in Figure 2.4. Suppose that the system is denoted as $\overline{P} = P + \Delta P$, in which P is denoted the normal system and the perturbed system are given as $\overline{P}$. ΔP denotes the case $N \to N + \Delta N$, in other words, the perturbation can be considered as the results caused by ΔN. The right factorization of the nominal system P and the overall system $\overline{P}$ can be rewritten as

$$P = ND^{-1}$$

and

$$P + \Delta P = (N + \Delta N)D^{-1}$$

respectively, where N and D are stable operators, D is invertible and ΔN is denoted as the bounded perturbations.

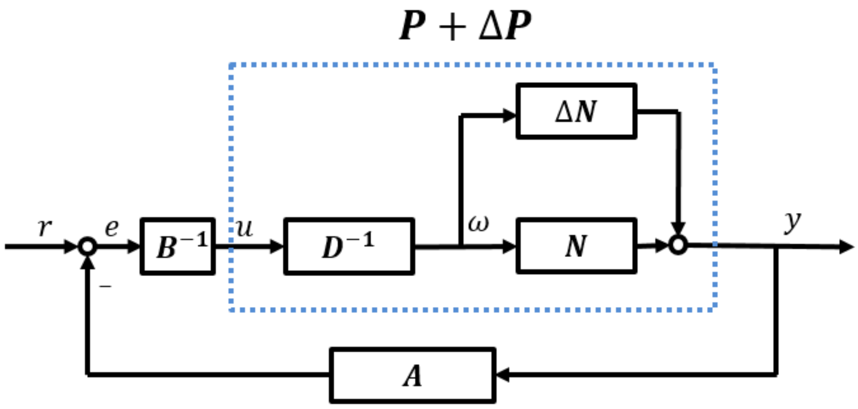

Figure 2.4: A nonlinear system with bounded perturbations

It is worth mentioning that under what conditions the nonlinear system with unknown bounded perturbations is said to have a robust right coprime factorization, when the perturbed nonlinear system still remains a right coprime factorization, that is, what conditions can guarantee the nonlinear system still having the robust stability property.

In the discussion of this problem, according to the definition of null set, in [37], if and only if the following condition is satisfied, so as to guarantee the nonlinear system with unknown bounded perturbations to be robustly stable,

$$A(N + \Delta N) - AN = 0. \tag{2.6}$$

under the condition of satisfaction of $\mathcal{R}(\Delta N) \subseteq \mathbf{N}(A)$, where $\mathbf{N}(A)$ is the null set defined by

$$\mathbf{N}(A) = \{x:\ x \in \mathcal{D}(A) \text{ and } A(x+y) = A(y) \text{ for all } y \in \mathcal{D}(A)\}$$

Based on the proposed sufficient condition, the fact that

$$A(N + \Delta N) + BD = AN + BD = M$$

is obtained, which guarantee the robust stability of the nonlinear systems with unknown bounded perturbations.

However, because of the condition in [37] is harsh to satisfy, so that the proposed design scheme for the nonlinear systems with unknown bounded perturbations is crucial to realize. Therefore, a generalized sufficient condition is proposed in [35] which compared with [37] in order to improve and extend the condition.

Lemma 2.5 *Let D_e be a linear subspace of the extended linear space U_e associated with a given Banach space U, moreover denoted $(A(N + \Delta N) - AN)M^{-1} \in Lip(D_e)$. Denote the Bezout identity of the nominal system and the perturbed system respected to ΔN in the form of $AN + BD = M, A(N + \Delta N) + BD = \tilde{M}$, respectively. If the condition as follows*

$$\| (A(N + \Delta N) - AN)M^{-1} \| < 1$$

is satisfied, then the system shown in Figure 2.4 is robust stable

Proof. The proof is given in Appendix A.4 [35].

Then remarking the above Lemma, considering the system shown in Figure 2.4, assume that right factorization of the unstable system is given as $P + \Delta P = (N + \Delta N)D^{-1}$, where $N + \Delta N$ is an unimodular operator. Then M in the nominal Bezout identity can be equivalent ot $M + \Delta M$ as a result of $N \to N + \Delta N$. If two designed operators A and B satisfy the Bezout identity $A(N + \Delta N) + BD = M + \Delta M$, moreover, $(N + \Delta N)(M + \Delta M)^{-1} = I$, then the output can track to the reference input while the nonlinear system is overall stable.

2.3 Problem statement

In terms of operator-based right coprime factorization, there exist a great number of significant developments of this method, which are mainly employed to consider nonlinear systems. This method considers a nonlinear system as an operator from mathematical point of view, then based on operator theory, control design for the nonlinear system is considered. Among the existing method, robust right coprime factorization to conduct nonlinear systems with perturbation, which provided a fairly general operator theoretic setting for system analysis, control and design. A new sufficient condition for nonlinear systems with unknown perturbation was proposed based on a Lipschitz norm inequality to consider robust stability, whose merit lies in that the proposed design scheme could deal with a broader class of nonlinear system compared with the former method. However,there is little research that considers the issue of reducing the adverse effects on a system resulting from unknown disturbances. Therefore, in order to solve this method, an effective design scheme of combining right coprime factorization with a new nonlinear operator controller is proposed to deal with nonlinear systems with unknown disturbance for guaranteeing robust stability and reducing the adverse effects of unknown disturbance.

For the record, most traditional researches are aimed at one interference object in control system design, either internal perturbation or external disturbance, as has been stated above. However, there still exist some fundamental and critical issues which need to be considered. In most practical nonlinear control systems there are a number of different kinds of external disturbance and internal perturbation subjected to the circumstance, temperature, coupling between different systems and so on. Besides that the previous methods of dealing with internal perturbation or external disturbance employ the same controller to guarantee robust stability, which restricts application range of the proposed method because the considered perturbation or disturbance in the former researches is indirectly determined by controllers. Therefore, the next main important issue in this dissertation, both internal perturbation and external disturbance of the nonlinear systems are considered using new design scheme. The main benefits of method lie in: 1) Both external disturbance and internal perturbation of nonlinear systems are considered. Compared with the former works, the proposed design scheme using the designed compensator not only can deal with the external disturbance, but also can handle the existing perturbation simultaneously, based on which robust stability of the considered nonlinear systems is guaranteed; 2) For removing the adverse effect resulting from the external disturbance and internal perturbation, a feasible framework is proposed based on the designed scheme, which provides a convenient structure to consider the nonlinear system with external disturbance and internal perturbation; 3) A nonlinear operator controller for removing internal perturbation and external disturbance is designed, which provides a united scheme for dealing with the adverse effect. Based on the proposed design scheme, output tracking performance is realized simultaneously.

Further, even though the external disturbance and internal perturbation have been removed by the proposed united design scheme, but the adverse effects haven't separated effectively for many practical control system. Best on this motivation, the bilinear operator-based right coprime factorization for nonlinear system with perturbation and disturbance is introduced, which can consider adverse effect resulting from perturbation and disturbance quantitatively. The bilinear operator controller is estabilished in a special way, which means that the controller has a more freedom to be satisfied with practial requirement. Based on the proposed method, a feasible framework is established for considering robust control, sensitivity and tracking performance, which not only separates perturbation and disturbance, but also provides a fundamental base to design a controller for the considered system. After that, robust stability for the uncertain nonlinear systems is guaranteed under the proposed framework.

2.4 Conclusion

In this chapter, the mathematical preliminaries including the basic definitions and notations are introduced, which are necessary for developing main results of this dissertation. In details, such as the definition of extended linear spaces, the definition of generalized Lipschitz operators which play an foundation role for this dissertation. Next for considering nonlinear systems, the concept of right coprime factorization and robust right coprime factorization are described. Moreover, two main sufficient conditions are given in a fairly general operator setting for guaranteeing robust stability of the nonlinear systems with perturbations, which served as the tool of the theoretical basis for developing the main results in this dissertation. Finally, the concerned problems are also summarized in this chapter.

Chapter 3

Operator-based nonlinear systems with unknown disturbance rejection

3.1 Introduction

In terms of control design of systems, there have been significant developments from various perspectives for both linear and nonlinear control systems over the past decades. Nevertheless, in practice, a great number of systems possess nonlinear property and multivariable characteristic. Therefore, researches on the nonlinear systems have attracted many researchers' attention due to the important role they have played in real application. Especially, these issues, for instance, robust analysis, output tracking problem and disturbance reduction which are belong to the nonlinear systems still remain challenging owing to their complex structures and the nonlinear characteristics. Meanwhile, the disturbance almost exist in many kinds of systems where the disturbance has major concern of two types in the control of unknown disturbance and estimated disturbance , general disturbance yielding from modeling errors and external environment which are central considered in this chapter due to making a tremendous affection within the control systems, hence, it is necessary to reduce the adverse effects from the disturbance.

One of methods in studying nonlinear systems is operator-based right coprime factorization. There exist a great number of significant developments of this method, which are mainly employed to consider nonlinear systems. This method considers a nonlinear system as an operator from mathematical point of view, then based on operator theory, control design for the nonlinear system is considered. In terms of operator-based right coprime factorization, there exist many relevant results on robust stability of nonlinear systems with unknown disturbance [88] – [93]. In [37], the authors investigated robust right coprime factorization to conduct nonlinear systems with perturbation, which provided a fairly general operator theoretic setting for system analysis, control and design. However, there is little research that considers the issue of reducing the adverse effects on a system resulting from unknown disturbances.

Therefore, this chapter is devoted to investigate an effective design scheme of combining right coprime factorization with a new nonlinear operator controller to deal with nonlinear systems with unknown disturbance for guaranteeing robust stability and reducing the adverse effects of unknown disturbance. That is, with the robust right coprime factorization method, the equivalent framework of nonlinear systems is obtained, which provides a convenient viewpoint to consider the above issue; then based on operator theory, a new nonlinear operator is proposed for dealing with the unknown disturbance of nonlinear systems to reduce adverse effects on nonlinear systems.

In Section 3.2, based on the developments in the previous chapter, this position to study some general theories and strategies in qualitatively for now compensator design of the nonlinear system. At first, the framework of the considered nonlinear systems will be introduced. Second, in order to describe the design scheme more precisely three admissible classes related to source inputs, outputs and error signals of the given control systems have been clarified, what these considerations are all formulated in a general extended linear space setting in the time domain. Meanwhile, according to the proposed framework of the nonlinear system and Bezout identity using in reference, the robust stability for nonlinear system with right coprime factorization will be discussed from different viewpoints. In Section 3.3, based on the provided brief motivation for the issues in above section, the unknown disturbance rejection for the nonlinear systems on account of operator theorem with right coprime factorization has been discussed in detail. Firstly, a precise description on the generalized inner inverse operator has been given for the proposed problem. And then for the unknown disturbance rejecting problem, a general constructive procedure for realizing the object has be discuss in a mathematical formulation viewpoint by providing an equivalent operator controller on the nonlinear systems. That is, robust stability of the nonlinear systems with unknown disturbance is guaranteed by combining right coprime factorization with the proposed controller. Simultaneously, adverse effects resulting from the disturbance are removed by using the proposed nonlinear operator controller. Last, a simulation example is given based on the basic results using right coprime factorization to show the effectiveness of the proposed design scheme. In Section 3.4, operator-based nonlinear systems with unknown disturbance rejection using right coprime factorization methods are summarized.

3.2 Construction on the considered nonlinear systems based on operator theorem

3.2.1 Construction on the considered nonlinear systems

In this section, a general multi-input and multi-output nonlinear system is considered, which is defined in an extended linear space setting in the time domain. Before describing the proposed feasible method for solving the main design problem more precisely, a available lemma, at least qualitatively, for investigating the proposed method has to be provided from mathematical viewpoint.

Lemma 3.1 *Let H be a linear subspace defined in the extended linear space X_e which is associated with a given Banach space X, and let $S \in Lip(H)$ with $\| S \| < 1$, where $\| \cdot \|$ is the semi-norm for the generalized Lipschitz operator as defined in (2.5). Then, the operator $I - S$ is invertible on H with satisfying*

$$\| (I-S)^{-1} \|_{Lip} \leq \| (I-S)^{-1}(x_0) \|_X + (1 - \| S \|)^{-1}$$

for any $x_0 \in H$. Moreover, defined $C_0 := I$ and inductive $C_n := I + SC_{n-1}$ for $n = 1, 2, ...$, then for each fixed $T \in [0, \infty)$ and all $\bar{x} \in H$, we can get

$$\lim_{n \to \infty} [C_n(\bar{x})]_T = [(I-S)^{-1}(\bar{x})]_T$$

obviously, the bound error within the above equation is that,

$$\| [(I-S)^{-1}(\bar{x})]_T - [C_n(\bar{x})]_T \|_X \leq \frac{\| S \|^n \| [S(\bar{x})]_T \|_X}{1 - \| S \|} \qquad (n = 0, 1, 2, ...)$$

Consequently, for each $T \in [0, \infty)$, when satisfying $\| S \|_{Lip} < 1$, can be got

$$\begin{aligned} &\| [(I-S)^{-1}(\bar{x})]_T - [C_n(\bar{x})]_T \|_X \\ &\qquad \leq \frac{\| S \|^n \| [S(\bar{x})]_T \|_X}{1 - \| S \|} \leq \frac{\| S \|_{Lip}^n \| [S(\bar{x})]_T \|_X}{1 - \| S \|_{Lip}} \end{aligned}$$

Proof. For any $\bar{x}_1, \bar{x}_2 \in H$, obtained

$$
\begin{aligned}
&\| [(I-S)(\bar{x}_1)]_T - [(I-S)(\bar{x}_2)]_T \|_X \\
&\quad \geq \| [\bar{x}_1]_T - [\bar{x}_2]_T \|_X - \| [S(\bar{x}_1)]_T - S(\bar{x}_2)]_T \|_X \\
&\quad \geq (1 - \| S \|) \| [\bar{x}_1]_T - [\bar{x}_2]_T \|_X
\end{aligned}
$$

Then, dividing both sides by the non-zero $\| [\bar{x}_1]_T - [\bar{x}_2]_T \|_X$ and taking supremum over $[0, \infty)$, it is obvious that $I - S$ is injective on H with $[\bar{x}_1]_T \neq [\bar{x}_2]_T$. Hence, in order to prove that $I - S$ is invertible belonging to $Lip(H)$, this problem has been transformed into prove that it is surjective on H with $(I-S)^{-1}$ is Lipschitz on H.

Continuously, fix $T \in [0, \infty)$. First step is to prove that $(I-S)^{-1}$ is existing and it is in $Lip(H)$. For any $x_1, x_2 \in H$, corresponding $\bar{x}_1, \bar{x}_2 \in H$ associated with $x_1(t) = (I-S)(\bar{x}_1)(t)$ and $x_2(t) = (I-S)(\bar{x}_2)(t)$ for all $[0, \infty)$. Then, if $(I-S)^{-1}$ exists, follows the above inequality

$$
\begin{aligned}
&\| [(I-S)^{-1}(x_1)]_T - [(I-S)^{-1}(x_2)]_T \|_X \\
&\quad = \| [\bar{x}_1]_T - [\bar{x}_2]_T \|_X \\
&\quad \leq (1 - \| S \|)^{-1} \| [(I-S)(\bar{x}_1)]_T - [(I-S)(\bar{x}_2)]_T \|_X \\
&\quad = (1 - \| S \|)^{-1} \| [x_1]_T - [x_2]_T \|_X
\end{aligned}
$$

since $\| S \| \leq 1$, it implies that the inverse mapping of $I - S$ is in $Lip(H)$ with

$$\| (I-S)^{-1} \|_{Lip} \leq \| (I-S)^{-1}(x_0) \|_X + (1 - \| S \|)^{-1}$$

Second step is to verify that $I - S$ is surjective on H such that $(I-S)^{-1} \in Lip(H)$. By the definition of the operators C_n, we can obtain that

$$\| [C_{n+1}(\bar{x})]_T - [C_n(\bar{x})]_T \|_X \leq \| S \|^n \| [S(\bar{x})]_T \|_X \qquad n = 1, 2, ...$$

where fixed $T \in [0, \infty)$, $\bar{x} \in H$.

a) Indeed, it is obviously true, when $n = 0$.

b) Suppose that for $n = k - 1$, this above inequality is held. Then, we can get that

$$\begin{aligned}
&\| [C_{k+1}(\bar{x})]_T - [C_k(\bar{x})]_T \|_X \\
&\quad = \| [SC_k(\bar{x})]_T - [SC_{k-1}(\bar{x})]_T \|_X \\
&\quad \leq \| S \| \| [C_k(\bar{x})]_T - [C_{k-1}(\bar{x})]_T \|_X \\
&\quad \leq \| S \| \| S \|^{k-1} \| [S(\bar{x})]_T \|_X
\end{aligned}$$

so that the inequality is satisfied for all $n = 0, 1, 2, ...$

c) Consequently, for any positive integer m, always exists that

$$\begin{aligned}
&\| [C_{n+m}(\bar{x})]_T - [C_n(\bar{x})]_T \|_X \\
&\quad = \| \sum_{k=0}^{m-1} ([C_{n+k+1}(\bar{x})]_T - [C_{n+k}(\bar{x})]_T \|_X \\
&\quad \leq \sum_{k=0}^{m-1} \| [C_{n+k+1}(\bar{x})]_T - [C_{n+k}(\bar{x})]_T \|_X \\
&\quad \leq \sum_{k=0}^{m-1} \| S \|^{n+k} \| [S(\bar{x})]_T \|_X \\
&\quad \leq \frac{\| S \|^n \| [S(\bar{x})]_T \|_X}{1 - \| S \|}
\end{aligned}$$

Then the above inequality implies that for arbitrary $\bar{x}$ in H, C has domain H with independent of T. Moreover, we can get $\lim_{n \to \infty} [C_n(\bar{x})]_T = [C(\bar{x})]_T$ for all $\bar{x} \in H$. Further, follow the above inequality that

$$\begin{aligned}
\| [C(\bar{x})]_T - [C_n(\bar{x})]_T \|_X &= \lim_{m \to \infty} \| [C_{n+m}(\bar{x})]_T - [C_n(\bar{x})]_T \|_X \\
&\leq \frac{\| S \|^n \| [S(\bar{x})]_T \|_X}{1 - \| S \|}
\end{aligned}$$

and note that A is continuous Lipschitz operator. Consequently, we can get that

$$\begin{aligned}[C(\bar{x})]_T &= \lim_{n\to\infty}[C_n(\bar{x})]_T \\ &= \lim_{n\to\infty}[(I+SC_{n-1})(\bar{x})]_T \\ &= \bar{x}_T + [SC(\bar{x})]_T\end{aligned}$$

where the convergence belongs to norm. Since $C = I+SC$, so that $(I-S)C = I$, which implies that C is the inverse of $I-S$. Above all imply that $I-S$ is surjective. Completing the proof of the Lemma.

In the following investigation, the problem statement is proposed. To some extent, a flexible design is difficult to obtain because of many difficulties in dealing with unknown disturbance of nonlinear systems. How to effectively finding a corresponding framework to analyze adverse effect of unknown disturbance of nonlinear systems is a crucial step. Therefore, in this chapter, a new design scheme will be considered to solve robust stability and unknown disturbance issues based on the operator-based right coprime factorization method. It is worth to emphasize that all space sets associated with the following issues will be re-defined in the Banach space with Lipschitz norm which are different from above appeared.

At this position, a general multi-input and multi-output nonlinear system is considered shown in Figure 3.1, which is defined in an extended linear space setting in the time domain. The system equations are given as

$$\begin{cases} e(t) = r(t) - y(t) \\ y(t) = PC(e)(t) + v(t) \\ r(t) = W(u)(t) \end{cases} \tag{3.1}$$

where r denotes the reference input signal as the output of a controller W driven by an external signal u; e denotes the error signal between the reference input r and the system output signal y that means y is required to follow the given reference signal r; v as a input signal denotes the external disturbance; P and C denote the plant and the controller, respectively. Note that the plant

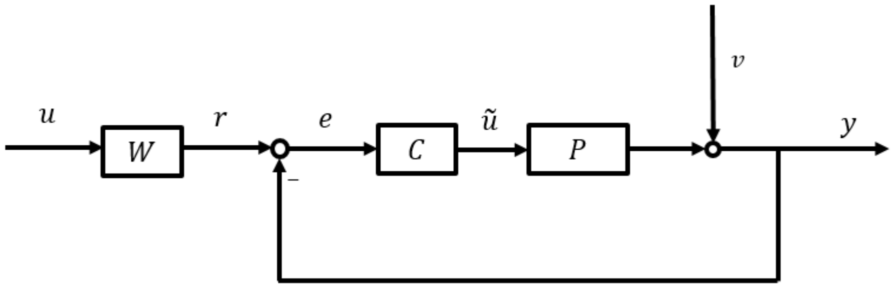

Figure 3.1: The nonlinear system with unknown disturbance

P is a nonlinear operator which is assumed to be given, and the controller C is another nonlinear operator which is to be design for removing the disturbance as well as stabilizes the whole system stability.

Let U, Y and D denote input space, output space and uncertain space, respectively, which are three extended linear spaces of $l-, p-$ and $q-$ dimensional complex-valued measurable functions defined on the time domain $[0, \infty)$ where $T \leq \infty$ and $1 \leq l, p, q < \infty$, so that can be found $u \in U$; $y, r, e \in Y$; and $v \in D$. Moreover, let $\mathfrak{D}(\cdot)$ and $\mathfrak{R}(\cdot)$ denote domains and ranges for the operators in this configuration respectively.

In order to describe the design scheme more precisely, firstly, we need to clarify three admissible classes related to source inputs, outputs and error signals of the given control systems as follows:

$$U = \{u :\| u_T \|_U \leq M_u < \infty \; for \; all \; T \in \; [0, \infty)\} \tag{3.2}$$

$$Y = \{y :\| y_T \|_Y \leq M_y < \infty \; for \; all \; T \in \; [0, \infty)\} \tag{3.3}$$

and $Y_0 \subseteq Y$; *where*

$$Y_0 = \{e : e = r - y,\ r,\ y \in Y\} \tag{3.4}$$

Based on practical considerations these assumptions make sense that the choice of M_u and M_y could be dictated by the allowable maximum dynamic range.

It is worth to mention that our first objective is to make the system to be input-output stable under consideration, that is for any input in U the corresponding system output must be in Y. However, as is well known, even if the plant P and the designed compensator C are both bounded and continuous, their boundedness and continuity cannot imply the stability of the overall system from input space to output space under the closed-loop configuration due to mapping a signal to somewhere outside the output space.

3.2.2 Robust stability

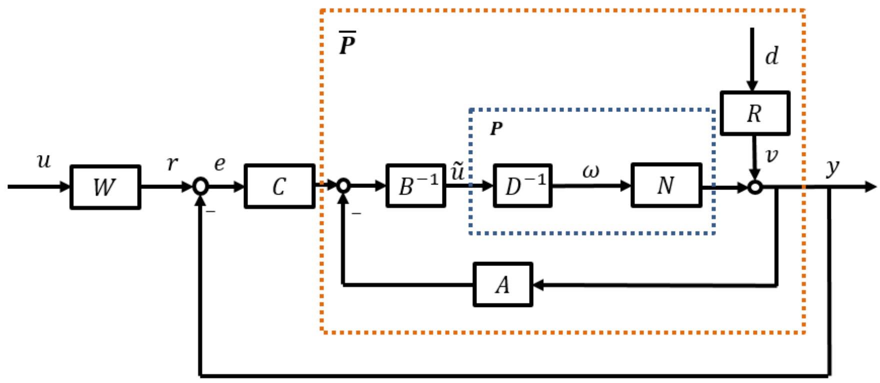

Figure 3.2: The proposed nonlinear system with unknown disturbance

It follows from the above statements. We first have to give a new viewing angle to consider the unstable nonlinear system using operator-based right coprime factorization method to guarantee the stability of the uncertain nonlinear system, wherein the real plant $\bar{P}$ has a nonlinear disturbance with respect to the nonlinear nominal plant P shown in Figure 3.2, where a stable operator R is designed to transform the external source d into the disturbance v, that is, the disturbance v is driven by d, which is assumed to be associated with the plant input $\tilde{u}$. Moreover, we suppose that the class of disturbances is defined by D as shown in follows, which is a bounded subset in the corresponding extended linear space.

$$D = \{d : \| d_T \|_D \leq M_d < \infty \; for \; all \; T \in \; [0, \infty)\} \tag{3.5}$$

In order to impose on this system for its well-posed, some necessary assumptions need to be clarified. Assume that $W : U \to Y$ and $R : D \to Y$ are both bounded linear operators as well as $P : \mathfrak{R}(C) \to Y$ and $C : Y \to \mathfrak{D}(P)$ are two nonlinear operators. For the consistency of the overall system, assuming that $\mathfrak{R}(C) \subset \mathfrak{D}(P)$ and $\mathfrak{R}(W)\dot{+}(\mathfrak{R}(R)\dot{+}\mathfrak{R}(P)) \subset \mathfrak{D}(C)$, where $\dot{+}$ is the geometric sum of two sets of vectors.

Further, from the physical viewpoint, can be easily to find the $\mathfrak{R}(W) \subseteq \mathfrak{R}(R)$, that is, the range of disturbance inputs is usually larger than that the input signals especially taking into account of random disturbance rejection. Under all of these assumptions, the stability and the uniqueness of internal signals will be considered using right coprime factorization.

Assume that the given model plant P has a right coprime factorization, $P = ND^{-1}$, satisfying with a Bezout identity, where N is stable, D is stable and invertible, and R has a right factorization in the form $R = GD^{-1}$, where G is stable and D is stable and invertible, if there exists a transformative Bezout identity related to operators A and B, which is satisfied

$$A(N + G) + BD = \tilde{M} \tag{3.6}$$

where A and B are stable, B is invertible as well as $\tilde{M}$ is unimodular. Then the system is overall stable.

We remake that the transformative Bezout identity introduced here is defined on the linear space Ω. Moreover, if $\Omega = U$, the $\tilde{M}$ can be usually replaced by the identity operator I. Note that the initial state should be consistent with the Bezout identity, that is, $A(N+G)(w_0, t_0) + BD(w_0, t_0) = \tilde{M}(w_0, t_0)$ should be satisfied. Furthermore, in this section, we select $t_0 = 0$ and $w_0 = 0$ without loss of generality.

3.3 Nonlinear systems with unknown disturbances rejection

3.3.1 The admissible class of compensator

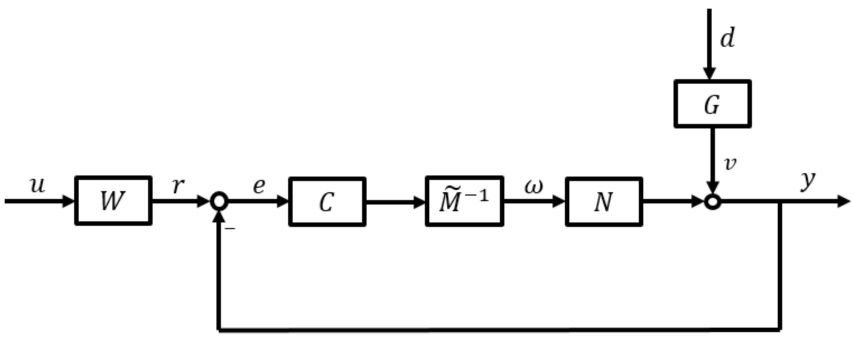

Figure 3.3: The equivalent system with unknown disturbance

The nonlinear system with unknown disturbance shown in Figure 3.2 can be equivalently transferred into Figure 3.3. Therefore, for the obtained equivalent system, the nonlinear system with disturbance design problem can be stated as follows,

$$\begin{cases} e(t) = r(t) - y(t) \\ y(t) = N\tilde{M}^{-1}C(e)(t) + G(d)(t) \\ r(t) = W(u)(t) \end{cases} \tag{3.7}$$

where r denotes the reference input signal and v the disturbance signal with respect to the external source d. The system output y is required to follow the given reference signal r. One of objectives of the section is to design a controller C in order to reduce the adverse effects coming out of the disturbance d for the overall nonlinear system.

Here, it should be noticed that since the set of defined admissible respect to disturbance is no longer a linear subspace, leading to the subset constituted by the actual systems outputs is not necessarily a linear subspace. Therefore, the nonlinear system with unknown disturbance design problem can be stated as follows. Given $\tilde{M}$, N, W and G as described above, one of objectives of this section is to design a controller C in order to remove the adverse effect of disturbance v.

From mathematical view point, we choose the quasi-sate space $\Omega = U$ in this section, a certain admissible class S^* will be firstly proposed, which is composed of all nonlinear compensator operators, as follows:

$$S^* = \{C \in Lip(Y) : NC \in Lip(Y)\} \tag{3.8}$$

that S^* is an infinite-dimensional Banach space. Note that, Y denotes the output space for the system as well as the input space for the compensator C, so that $Lip(Y)$ denotes the family of generalized Lipschitz operators mapping from Y to itself. Using S^* defined in (3.8) as an underlying operator space for the admissible class of compensators, a very large framework for the design purposes is obtained. The merits of the Banach space, S^*, for the underlying space of controllers will be elaborated by the following Lemmas.

Lemma 3.2 *Let N and S^* be given as described above. If the following subset S_0 consisting of the compensators that belong to the family S^* is satisfied, then the overall system is said to be stable:*

$$S_0 = \{C \in S^* : \quad \| NC \| < 1\} \tag{3.9}$$

where $\| \cdot \|$ is a generalized Lipschitz semi-norm of the operator.

Proof. First, we remark that in order to formulate this problem, the nonlinear system as shown in Figure 3.3 will be considered without the disturbance signals and the forward filter W form the physically viewpoint. As is well known, if the overall feedback system is stable provided that all the internal signals are mathematically well connected in the since that the overall feedback system is causal with unique internal signals to output signals. Then a detail proof will be given from the mathematical well-connection viewpoint under an infinite-dimensional Banach space.

It follows from the Figure 3.3 without considering the forward filter and disturbance signals that:

$$e(t) + N\tilde{M}^{-1}C(e)(t) = r(t)$$

which is a vector-valued nonlinear equation for each fixed input R corresponding an unique error signal e. In which, without loss of generality, the $\tilde{M}$ replaced by the identity operator I has been defined. Then composite operator NC is Lipschitz under the condition in Equation (3.9) in the sense that the norm is strictly less than 1 uniformly on bounded subsets of Y. Furthermore, NC formulates a mapping from Y to itself, that is, for each $r \in Y$, there exists a unique corresponding $e \in Y$ satisfying the above equation. This implies that the overall system is output-input stable.This completes the proof.

Lemma 3.3 *If the condition that $\| NC \| < 1$ is satisfied, then the nonlinear operator*

$$I + NC : Y \to Y$$

is invertible, and its inverse, denoted by $(I + NC)^{-1}$*, is also a generalized Lipschitz operator in* $Lip(Y)$ *satisfied with a fixed* x_0 *as*

$$\| (I + NC)^{-1} \|_{Lip} \leq \| (I + NC)^{-1}(x_0) \|_Y + (1 - \| NC \|)^{-1}$$

Proof . First, we observe that for any $a_1, a_2 \in Y$,

$$\begin{aligned} &\| (I + NC)(a_1) - (I + NC)(a_2) \|_Y \\ &\qquad \geq \| a_1 - a_2 \|_Y - \| (NC)(a_1) - (NC)(a_2) \|_Y \\ &\qquad \geq (1 - \| NC \|) \| a_1 - a_2 \|_Y \end{aligned} \tag{3.10}$$

By satisfying the condition that $\| NC \| < 1$. Hence, implying $I + NC$ is an injective mapping, namely, $(I + NC)(a_1) = (I + NC)(a_2)$ implies that $a_1 = a_2$.

Finally, for x_1, x_2 in the range of $I + NC$, from the above Equation (9) and the definition of the norm we have

$$\begin{aligned} &\| (I + NC)^{-1} \|_{Lip} = \| (I + NC)^{-1}(x_0) \|_Y \\ &\quad + \sup_{\substack{x_1, x_2 \in Y \\ x_1 \neq x_2}} \frac{\| (I + NC)^{-1}(x_1) - (I + NC)^{-1}(x_2) \|_Y}{\| x_1 - x_2 \|_Y} \\ &\quad \leq \| (I + NC)^{-1}(x_0) \|_Y + (1 - \| NC \|)^{-1} \end{aligned}$$

thus, this completes the proof.

Consider, therefore, relationships within Equations (3.7) and all the conditions stated above are satisfied. Then, the following equation can be obtained,

$$e(t) + NC(e)(t) = W(u)(t) - G(d)(t)$$

then, by Lemma 1, we can get

$$e(t) = (I + NC)^{-1}(W(u) - G(d))(t) \tag{3.11}$$

According to the notation found in Eq.(10), we will discuss the adverse effect of unknown disturbance by separating d and u to reduce the unknown disturbance of the nonlinear systems. However, there are two issues of crucial importance. First, the two external inputs u and d has a complex relationship, which plays a negative role in reducing the unknown disturbance. Second, $(I + NC)^{-1}$ is not a simple calculation due to nonlinear property. Hence, in the following, a new design scheme will be proposed in detail to reduce the disturbance d based on the operator theory.

Consequently, a generalized inner inverse will be proposed in a mathematical manner for solving the objective problems which above posed.

3.3.2 Rejection scheme analysis

It is easily seen from the system Equation (3.11), since $(I + NC)^{-1}$ is nonlinear, which is different from linear case, not fitting the left distributive law, so that it is difficult to design a general framework for the nonlinear system with disturbance to separate $W(u) - G(d)$ in such a general setting. At this position, the definition of the inner inverse will be introduced.

Definition 3.1. Let X_1, X_2 be two Banach spaces and $\mathfrak{B}(X_1, X_2)$ denote the Banach space consisting of all bounded linear operators mapping from X_1 to X_2. For each $T \in \mathfrak{B}(X_1, X_2)$, the notions $\mathfrak{N}(T)$ and $\mathfrak{R}(T)$ denoted the null space and range of T.

There exists $J \in \mathfrak{B}(X_1, X_2)$, if condition $TJT = T$ is satisfied, then the operator J is said to be an inner inverse of T; as well as if condition $JTJ = J$ is satisfied, then the operator J is said to be an outer inverse of T. Moreover, if J is both an inner inverse and outer inverse of T, then the operator J is said to be a generalized inverse of T. It is noted that the generalized inverse would not exist as well as without ensureness the property of unique even if it exist.

There are some properties of the generalized inverse should be introduced as follows:

For any given operator $T \in \mathfrak{B}(X_1, X_2)$, there exists a bounded generalized inverse T^+, having:

(1) There exist TT^+ and T^+T are both bounded projectors, such that properties $\mathfrak{R}(TT^+) = \mathfrak{R}(T)$, $\mathfrak{R}(T^+T) = \mathfrak{R}(T^+)$, $\mathfrak{N}(T^+T) = \mathfrak{N}(T)$, and $\mathfrak{N}(TT^+) = \mathfrak{N}(T^+)$;

(2) Denoted the topological direct sum decompositions: $X_1 = \mathfrak{N}(T) \oplus \mathfrak{R}(T^+)$, $X_2 = \mathfrak{N}(T^+) \oplus \mathfrak{R}(T)$.

Now, Let go back to consider the problem on how to separate the argument $W(u)(t) - G(d)(t)$ with $(I + NC)^{-1}$ completely. First, denote the G^+ to be inner inverse of G. It is obvious that $G : D \to Y_0$ is stable defined on the closed subset D, its generalized inner inverse $G^+ : Y_0 \to D$ is also a bounded linear operator. Here, the generalized inner inverse operator G^+ is defined to be one such that

$$GG^+G = G$$

which defined on the time domain of G implies that

$$GG^+G(d) = G(d) \qquad for\ any \quad d \in D$$

and also have $GG^+ = I$ on the range of G, where I is the identity operator, that is, namely,

$$GG^+(v) = v \qquad for\ any \quad v \in \mathfrak{R}(G)$$

Moreover, the operator norm of this inner inverse operator has the upper bound as shown below

$$\parallel G^+ \parallel := \sup_{\|v\|_Y = 1} \parallel G^+ v \parallel_D \leq M_d \tag{3.12}$$

because of $G^+(v) \in D$ for any $v \in \mathfrak{R}(G)$, what M_d has been defined in Equation (3.5). Consequently, it follows from the Equation (3.11), a theorem will be proposed as below:

Theorem 3.1. *According to Equation (3.12), the norm of e can be expressed by:*

$$\begin{aligned}\| e \|_Y \leq & \| (I+NC)^{-1} \| \| G \| M_d \| W \| \| u \|_U \\ & + \| (I+NC)^{-1} G \| \| d \|_D \end{aligned} \tag{3.13}$$

which implies that not only the boundedness in terms of $\| e \|_Y$, but also being separated into two parts.

Proof. First establish the norm of error signal e as shown in follows:

$$\| e \|_Y = \| (I+NC)^{-1}(Wu - Gd) \|_Y$$

Therefore, based on (3.12),

$$\begin{aligned}\| e \|_Y \leq & \| (I+NC)^{-1} G \| \| G^{+} Wu - d \|_D \\ \leq & \| (I+NC)^{-1} G \| M_d \| W \| \| u \|_U + \| d \|_D) \end{aligned}$$

hence, we can get

$$\begin{aligned}\| e \|_Y \leq & \| (I+NC)^{-1} \| \| G \| M_d \| W \| \| u \|_U \\ & + \| (I+NC)^{-1} G \| \| d \|_D \end{aligned}$$

The proof of the theorem is completed.

In the position of this point, the main problem is that to simultaneously reject the disturbance and minimize the norm of the error response of the system. That is, in this case, the disturbance rejection problem has been posed as to minimize first part of the second term in the right hand side of (3.13) for remaining less tan or equal to a prescribed bound enough small.

For simplicity to describe the design scheme, we let

$$\Phi(t) = (I+NC)^{-1} G(d(t)) \tag{3.14}$$

However, since the inverse of $I+NC$ in the objective functional is difficult to calculated, so that this problem is difficult to solve from the technical perspective. We will propose a new nonlinear control operator in the following theorem.

Theorem 3.2. *In terms of Equation (3.14), there exists a nonlinear control operator Q as shown in Equation (3.16) such that $\Phi(t)$ tends to be zero for reducing the unknown disturbance for the nonlinear system shown in Fig. 3.3.*

Proof : For any generalized Lipschitz operator $Q \in Lip(Y)$ satisfying $NQ \in Lip(Y)$ with $\| NQ \| < \frac{1}{2}$, $(I - NQ)^{-1}$ exists and is also in $Lip(Y)$, then we define

$$C := Q(I - NQ)^{-1}$$

so that

$$\begin{aligned} \| NC \| &= \| NQ(I - NQ)^{-1} \| \\ &\leq \| NQ \| \| (I - NQ)^{-1} \| < 1 \end{aligned}$$

which implies that $\| NC \| < 1$ is satisfied. Hence, we observe that

$$I + NC = (I - NQ)^{-1} \tag{3.15}$$

In order to reduce the effection of the unknown disturbance d on the overall nonlinear system, the nonlinear operator Q with the time-varying gain is designed as follows,

$$Q(v)(t) = v(t)N^{-1} \int_0^t e^{-v(\tau)} \cdot 2\sin(v(\tau)) \cdot \dot{v}(\tau) d\tau \tag{3.16}$$

According to (3.15) and $v(t) = G(d(t))$, $\Phi(t)$ is shown as follows,

$$\begin{aligned} \Phi(t) &= (I + NC)^{-1} G(d(t)) \\ &= (I - NQ)(v(t)) \\ &= v(t) - NQ(v(t)) \end{aligned} \tag{3.17}$$

Based on (3.16) and (3.18),

$$\Phi(t) = v(t) - v(t)\int_0^t e^{-v(\tau)} \cdot 2\sin(v(\tau)) \cdot \dot{v}(\tau)d\tau \tag{3.18}$$

as $t \to \infty$,

$$\begin{aligned}
&\int_0^t e^{-v(\tau)} \cdot 2\sin(v(\tau)) \cdot \dot{v}(\tau)d\tau \\
&\quad = (-2\sin(v(\tau)) \cdot e^{-v(\tau)})|_0^t \\
&\quad + \int_0^t e^{-v(\tau)} \cdot 2\cos(v(\tau)) \cdot \dot{v}(\tau)d\tau \\
&\quad = \lim_{t\to\infty} e^{-v(\tau)}(-\sin(v(\tau)) - \cos(v(\tau))\Big|_0^t \\
&\quad \to 1
\end{aligned} \tag{3.19}$$

under (3.19),

$$v(t) - v(t)\int_0^t e^{-v(\tau)} \cdot 2\sin(v(\tau)) \cdot \dot{v}(\tau)d\tau \to 0 \tag{3.20}$$

that is,

$$\Phi(t) = (I + NC)^{-1}G(d(t)) \to 0 \tag{3.21}$$

Then the error response of the system, which is composed of two parts has been transformed into only one part by reducing the unknown disturbance based on the proposed design scheme. In the next section, a simulation example will be shown to confirm effectiveness of the proposed design scheme.

3.3.3 Simulation examples

In this section, we will show two examples, in order to illustrate the effectiveness of the proposed design scheme. Let $C_{[0,\infty)}$ be the space of continuous functions, and $C^1_{[0,\infty)}$ consists of all the functions having a continuous first derivative, both are defined on $[0, \infty)$.

This is the first example including two cases which represent two kinds of disturbances. Case one, the nominal plant and right factorization are given as follows:

$$\begin{aligned} P(\tilde{u})(t) = 2(2t + e^t)\tilde{u}(t) \\ - \int_0^t e^{-2\tau} \cdot (2\tau + e^\tau) \cdot \tilde{u}(\tau) d\tau \end{aligned} \tag{3.22}$$

$$N(\omega)(t) = 2\omega(t) - \int_0^t e^{-2\tau} \omega(\tau) d\tau \tag{3.23}$$

$$D(\omega)(t) = \frac{1}{2t + e^t}\omega(t) \tag{3.24}$$

According to the proposed design, the perturbed operator and its right factorization are given as follows:

$$R(d)(t) = \int_0^t \sin^2((2\tau + e^\tau) \cdot d(\tau)) d\tau \tag{3.25}$$

$$G(d)(t) = \int_0^t \sin^2(d(\tau)) d\tau \tag{3.26}$$

The disturbance is assumed to be $d(t) = 3.2te^{-2t} + 1$. It is easy to find that P is unstable with stabilized N, D. Based on the proposed design scheme, the controllers A and B can be defined as follows:

$$A(y)(t) = \frac{1}{2}y(t) \tag{3.27}$$

$$
\begin{aligned}
B(\tilde{u})(t) = & \int_0^t \frac{2\tau e^{-2\tau} + e^{-\tau}}{2} \cdot \tilde{u}(\tau) d\tau \\
& - \frac{1}{2} \int_0^t \sin^2((2\tau + e^{\tau})\tilde{u}(\tau)) d\tau
\end{aligned} \tag{3.28}
$$

Thus,

$$
\begin{aligned}
& (A(N+G) + BD)(\omega)(t) \\
& \qquad = \omega(t) - \frac{1}{2} \int_0^t e^{-2\tau} \omega(\tau) d\tau \\
& \qquad + \frac{1}{2} \int_0^t \sin^2(\omega(\tau)) d\tau + \frac{1}{2} \int_0^t e^{-2\tau} \cdot \omega(\tau) d\tau \\
& \qquad - \frac{1}{2} \int_0^t \sin^2(\omega(\tau)) d\tau \\
& \qquad = \omega(t)
\end{aligned}
$$

It can be verified that A and B are satisfied with Equation (3.6).

Based on the proposed design scheme, the proposed controller Q is given as follows.

$$
\begin{aligned}
Q(v)(t) = & v(t) \cdot e^{(-\frac{1}{4}e^{-2t})} \cdot \int_0^t \dot{y_0}(\tau) e^{\frac{1}{4}e^{-2\tau}} d\tau \\
& \cdot \int_0^t e^{-v(\tau)} \cdot \sin(v(\tau)) \cdot \dot{v}(\tau) d\tau \}
\end{aligned} \tag{3.29}
$$

In order to show the example more explicitly, simulation results are given. The reference input is chosen as $r = 1.6te^{-t} + 0.05$. Then the simulation results are given in Figures 3.4-3.7, where the reference input r is shown in Figure 3.4, the control input is shown in Figure 3.5, the plant output $y(t)$ under designed scheme is shown in Figure 3.6, in contrast the plant output without control is shown in Figure 3.7. From the comparison of Figure 3.6 and Figure 3.7, we can find the proposed design scheme is effective

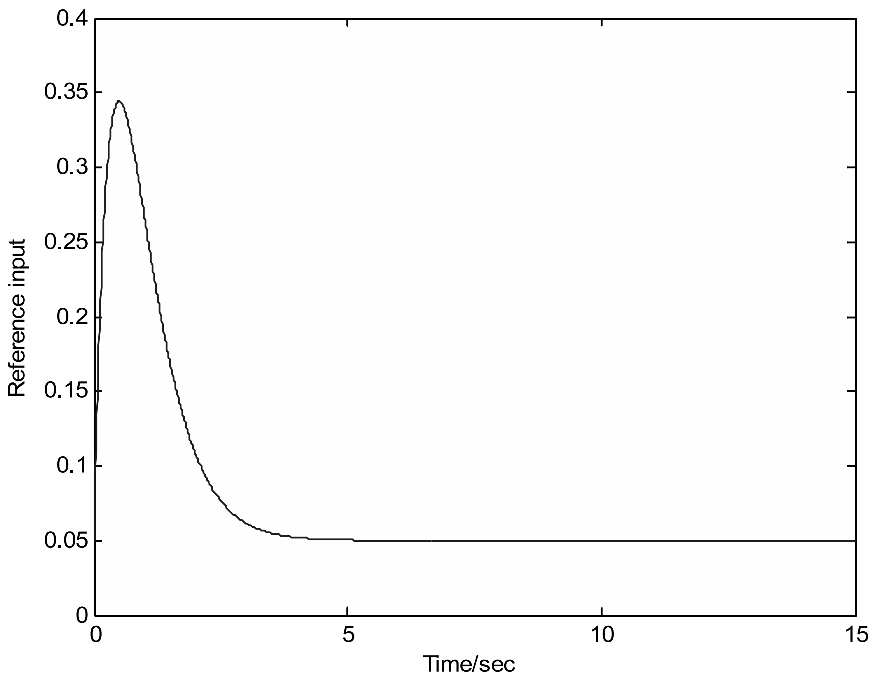

Figure 3.4: Reference input

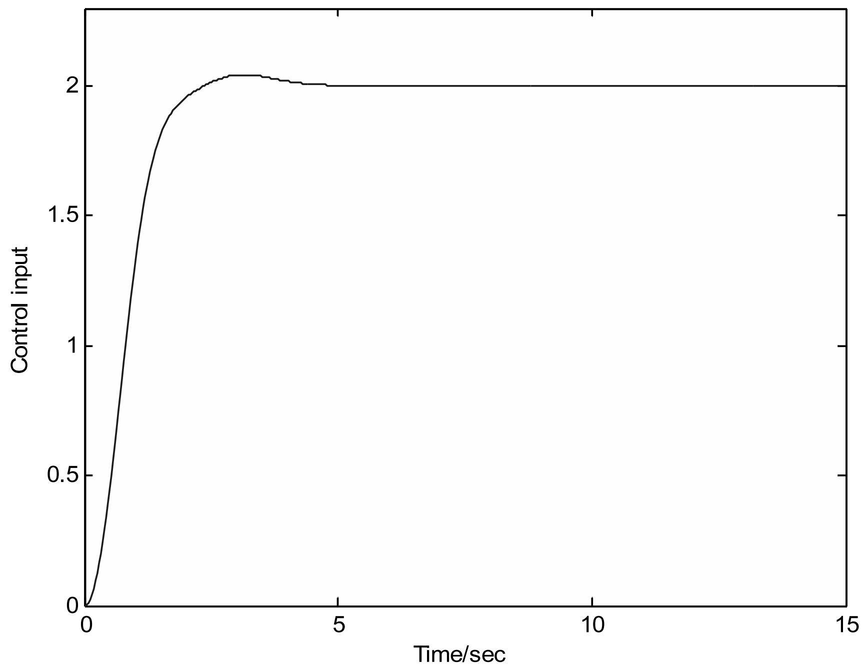

Figure 3.5: Control input

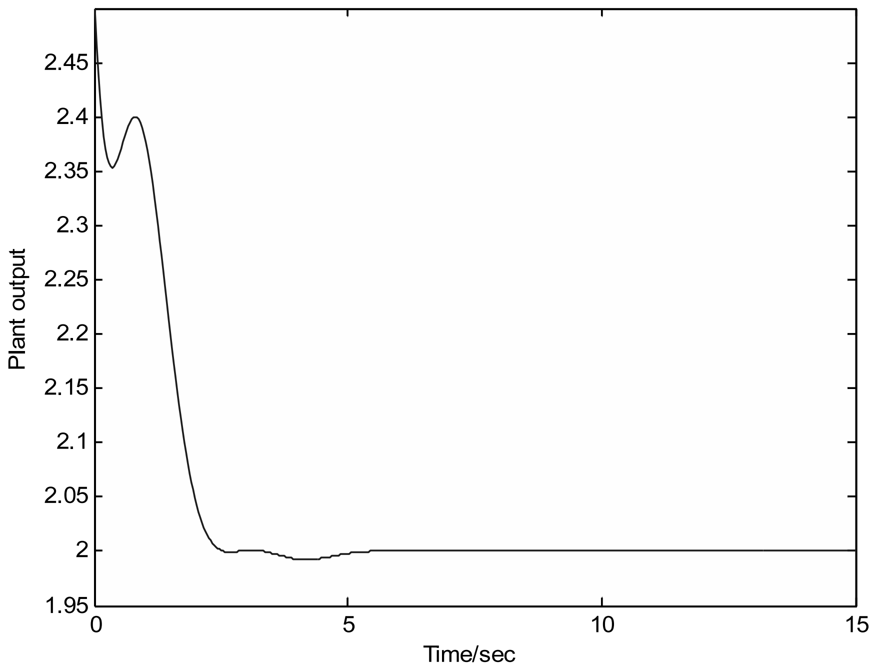

Figure 3.6: Plant output with Q

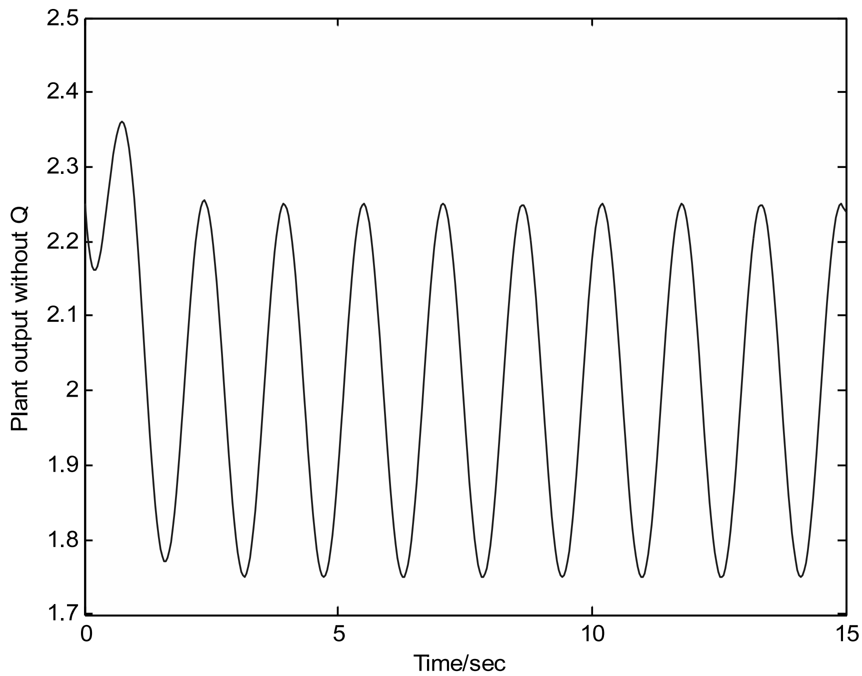

Figure 3.7: Plant output without Q

in reducing the disturbance. Therefore, the simulation results demonstrate the fact that the unknown disturbance is effectively reduced and the overall robust stability is guaranteed by the proposed method.

Case two, in order to show the method feasibly, the other kind of disturbance which is called random disturbance is chosen as $\frac{16}{5}te^{-2t}+rand(size(0.32te^{(}-2t)+2))$. Then nominal plant which satisfies the right factorization with N and D is given the same as above, as well as the designed controllers.Then the random disturbance is given in the same dimension as shown in Figure 3.8. Then the simulation result is given in Figure 3.9, where the plant output with control is shown. The simulation results demonstrate the fact that the disturbance is effectively rejected by the proposed method.

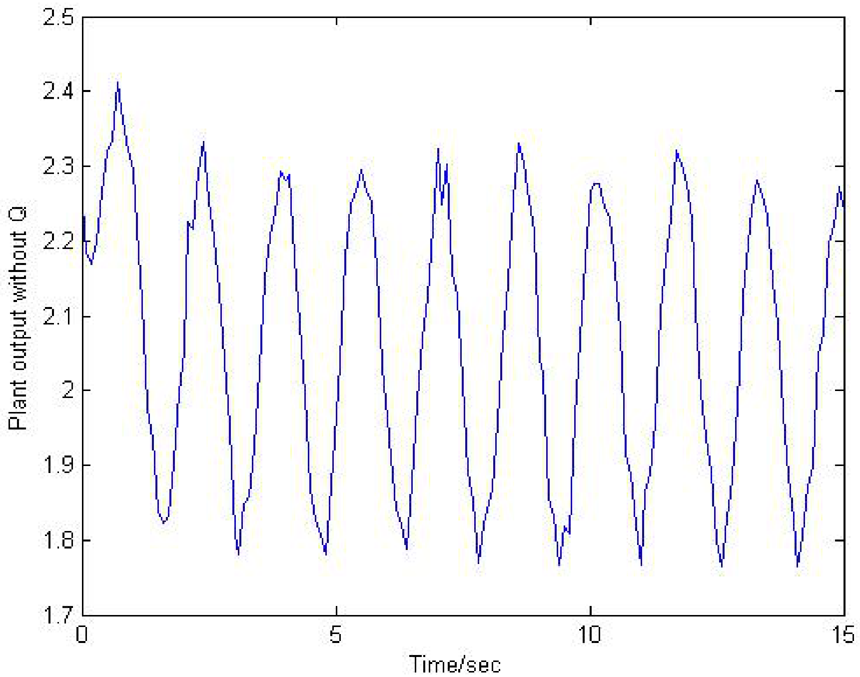

Figure 3.8: Random disturbance

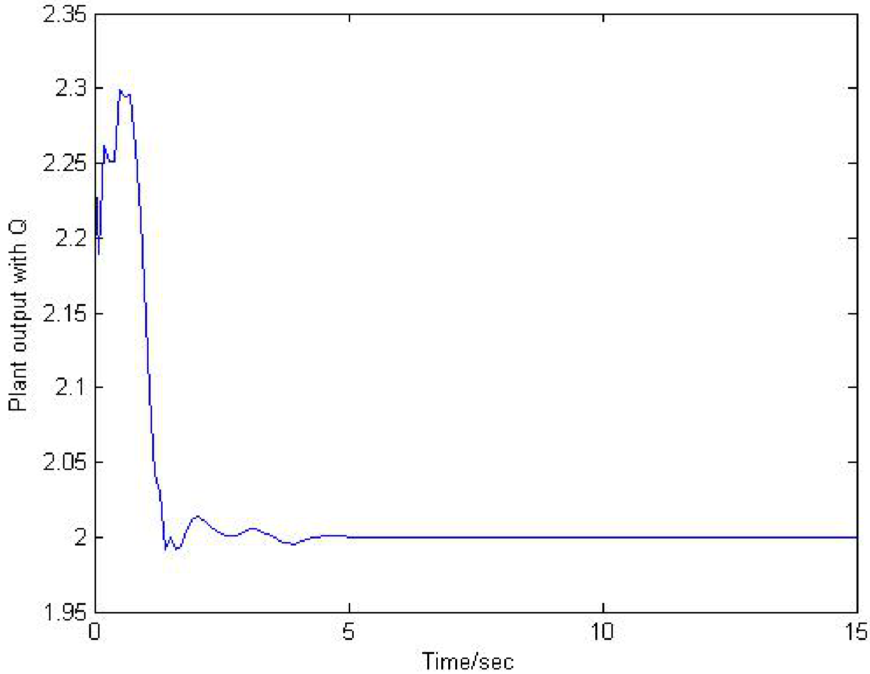

Figure 3.9: Plant output with control

The other simple example is discussed as follows. The plant and related controllers are given as follows.

$$P(\tilde{u}(t)) = \int_0^t (e^{\tilde{u}(\tau)} + \sin(\tilde{u}(\tau)) + \tilde{u}(\tau))d\tau$$

$$W(u(t)) = 2e^{-2u(t)} + 0.5$$

$$R(\tilde{d}(t)) = \int_0^t \sin(5\tilde{d}(\tau))d\tau$$

Based on the proposed design scheme, the controllers Q is given as follows:

$$\begin{aligned} Q(r(t)) = & \frac{r(t)}{\int_0^t (e^{\omega(\tau)} \sin(\omega(\tau)) + \omega(\tau))d\tau} \\ & \cdot \int_0^t 2e^{-r(\tau)} \sin(r(\tau))\dot{r}(\tau)d\tau \end{aligned}$$

In order to show the example more explicitly, simulation is done. The reference input is chosen as $r = 2e^{-2t} + 0.5$. The disturbance is chosen as $R(\tilde{d}(t)) = \int_0^t \sin(5\tilde{d}(\tau))d\tau$. Then the simulation results are given in Figures 3.10-3.12, where control input with Q is shown in Figure 3.10, control output $y(t)$ without Q is shown in Figure 3.11, control output $y(t)$ with Q is shown in Figure 3.12. The simulation results demonstrate the fact that the disturbance is effectively rejected by the proposed method.

3.4 Conclusion

In this chapter, robust stability of nonlinear systems with unknown disturbance is guaranteed based on the proposed design scheme. The merits of the proposed design scheme lie in that the convenient frameworks as shown in Figures 3.2 and 3.3 are obtained by using the right coprime factorization

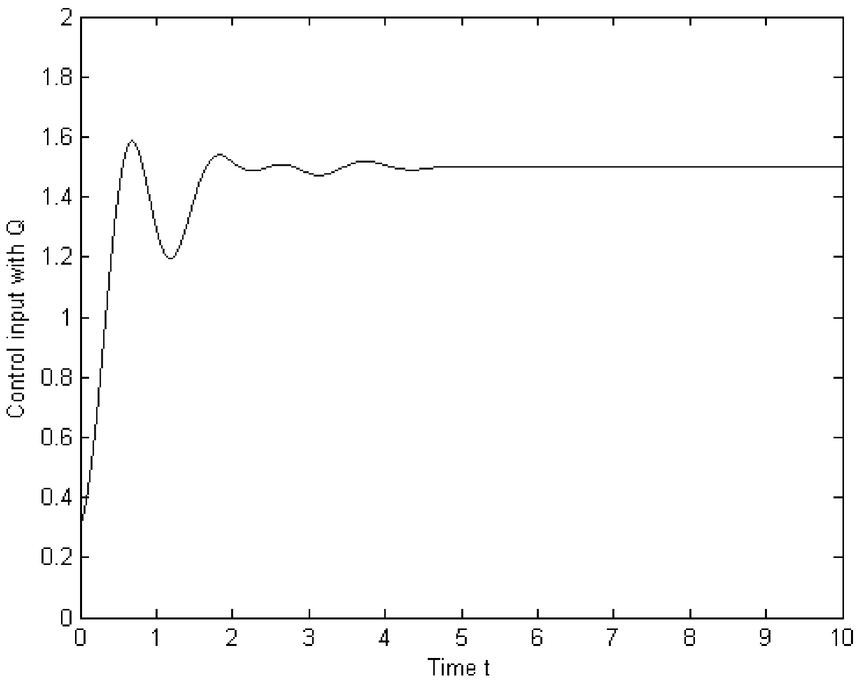

Figure 3.10: Control input with Q

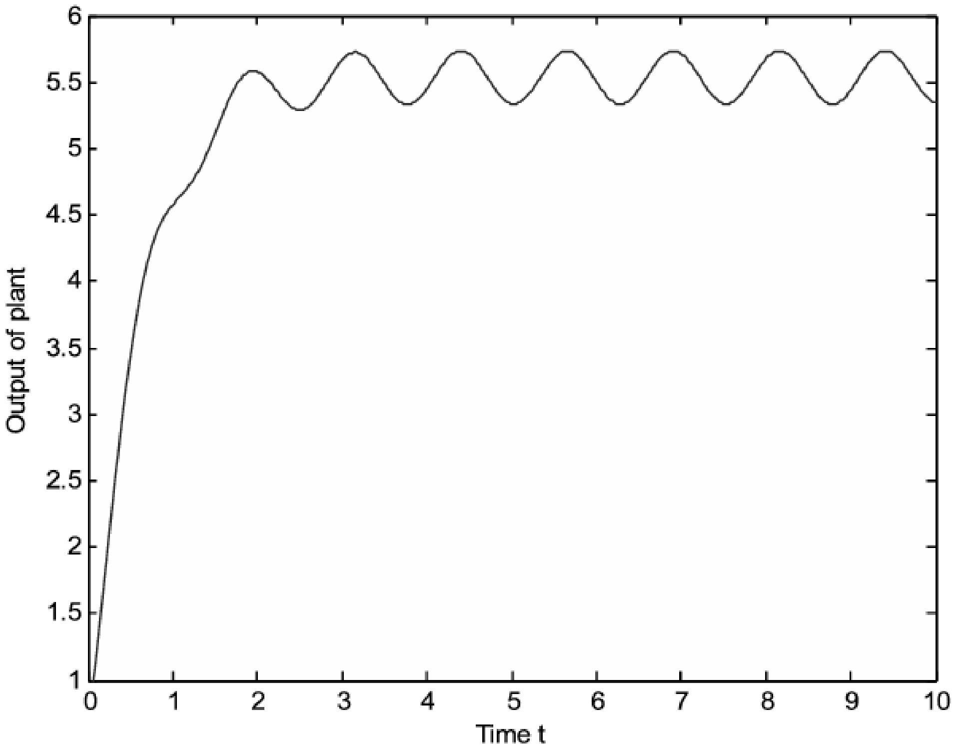

Figure 3.11: Plant output without Q

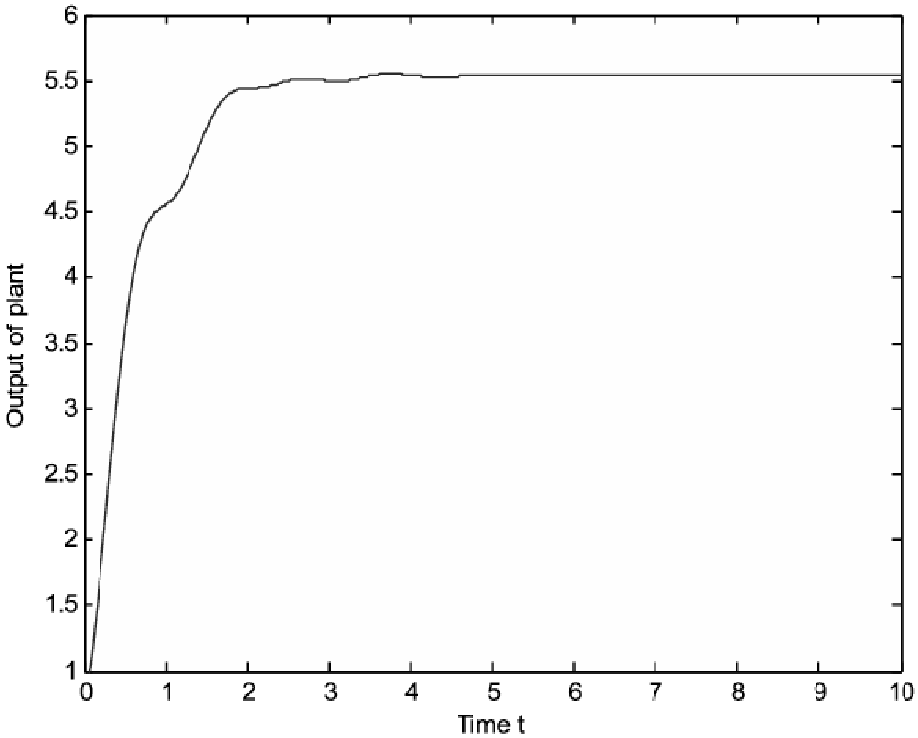

Figure 3.12: Plant output with Q

technique and the proposed controller C including Q is designed to reduce the unknown disturbance, which can not only guarantee robust stability of the overall nonlinear system and but also reduce the unknown disturbance. Firstly, some basic knowledge with respect to nonlinear operators were reviewed. Secondly, a convenient framework was obtained by using the right coprime factorization technique, based on which a nonlinear operator controller was designed to remove the adverse effects of the unknown disturbance of the nonlinear systems. The proposed design scheme can not only achieve the robust stability of the overall system but also reduce the unknown disturbance. Finally, a simulation example was given to confirm the effectiveness of the proposed design scheme.

Chapter 4

Operator-based nonlinear uncertain systems with external disturbance rejection using robust right coprime factorization

4.1 Introduction

Robust stability control design is a central problem and there have been significant developments for both linear and nonlinear control systems over the past decades based on a great number of control methods. Especially, these issues, such as robust analysis, output tracking problem, perturbation and disturbance, which are involved in the nonlinear systems still remain challenging owing to their complex structures and nonlinear characteristics. In terms of internal perturbation and external disturbance, it is extremely hard to avoid in practical nonlinear systems, because there are many kinds of reasons leading to them, such as modeling errors, unknown parameters and super added unknown part of control input, which are inevitable in some cases. It is necessary to reduce the adverse effect resulting from internal perturbation and external disturbance in order to improve system performance.

For considering the control design for nonlinear systems with perturbation to guarantee the stability of the overall systems. Many traditional methods have been proposed from different viewpoints, which is rather difficult to measure state vectors directly on-line measurements, which leads to some restrictions on applying these approaches. In Chapter 3, unknown disturbance rejection in nonlinear systems has been considered using robust right coprime factorization based on operator theorem. According to the proposed method, robust right coprime factorization method has been applied from the input-output point of view to consider the nonlinear systems and proved to be an effective method in dealing with control design issues of nonlinear systems. Moreover, the disturbance rejection has been discussed by designing a new controller with integrator to realize anti-jamming capability. For the record, most traditional researches are aimed at one interference object in control system design, either internal perturbation or external disturbance, as has been stated in Chapter 3. However, in most practical nonlinear control systems there are a number of different kinds of external disturbance and internal perturbation that always exist together subjected to the circumstance, temperature, coupling between different systems and so on. Therefore, in this Chapter, both internal perturbation and external disturbance of the nonlinear systems are considered together using new design scheme. In detail, from error signal point of view, the adverse effects resulting from external disturbance and internal perturbation of the nonlinear systems are removed by the designed nonlinear operator, simultaneously, output tracking performance is realized using the proposed design scheme.

In Section 4.2, based on the developments in the previous chapter, at this position to propose a new strategies from the qualitative perspectively via some general theories in order to design a compensator with integrator for resisting the adverse effects from the internal perturbation and external disturbance to ensure the whole system working stable. First, the problem statements will be provided, in which the motivations as well as merits will be expounded for catching the main purports of this chapter. Second, as it well known, Chen and Deng have proposed some sufficient condition in a fairly general operator-theoretic setting for guaranteeing robust stability of nonlinear systems with perturbation. In this Section, I will show three cases respectively for illustrating the relationship between the proposed conditions and the internal perturbations or disturbances, that means which kind of cases would be corresponding to which conditions, respectively. Third, an numerical example will be shown for proving the feasibility of the proposed condition in the concern of the input-output relation.

In Section 4.3, based on the conception of nonlinear right coprime factorizations(input/output approach), the existence of the robustness of nonlinear right coprime factorizations is discussed as well as the stability of overall system has been realized. On the basis of the proposed design scheme, a convenient framework is obtained for discussing rejection issues for external disturbance and internal perturbation. In details, as emphasized before, it is extremely hard to avoid uncertainties existing in practical industry effecting from unknown parameters, modelling errors, coupling and so on. In this section, these kinds of uncertainties will be generalized into two main types, one is internal perturbations which always exist within the system, another is external disturbance which alway yield from the outside factors of system. Moreover, compared with the former works, the proposed design scheme using the designed compensator not only can deal with the external disturbance, but also can handle the existing perturbation as well. Last, with the proposed convenient structure tracking performance has been realized simultaneously.

In Section 4.4, operator-based perturbed nonlinear systems with unknown disturbance rejection using right coprime factorization methods are summa-

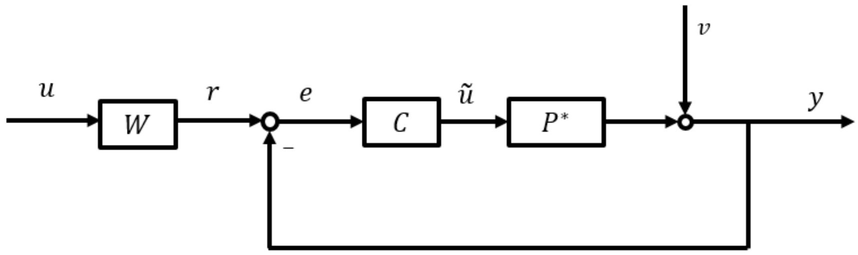

Figure 4.1: The nonlinear system with internal perturbation and external disturbance

rized.

4.2 Robust stability of nonlinear uncertain systems with disturbance

4.2.1 Problem statement

In this chapter, nonlinear systems with both internal perturbation and external disturbance are considered which is defined in an extended linear space setting in the time domain as shown in Figure 4.1, where $r \in Y$ denotes reference input as the output of a filter W driven by an external signal $u \in U$ in which Y is the output space respect to extended linear space ,U is the external input space also respect to extended linear space; $e \in Y_0$ denotes error signal between reference input r and system output $y \in Y$, in which Y_0 is a subset of Y; $v \in D$ as a input signal denotes external disturbance, in which D is the admissible class of disturbances, $\tilde{u}$ is the plant input. In general, C is a nonlinear controller which is associated with the plant input signal.

In order to describe the design scheme more precisely, firstly, we need to clarify three admissible classes related to source inputs, outputs and error signals. The two linear subspaces as input and output spaces are considered as follows:

$$U = \{u :\| u_T \|_U \leq M_u < \infty \text{ for all } T \in \ [0, \infty)\} \tag{4.1}$$

$$Y = \{y :\| y_T \|_Y \leq M_y < \infty \text{ for all } T \in \ [0, \infty)\} \tag{4.2}$$

Let $Y_0 \subseteq Y;\ where$

$$Y_0 = \{e : e = r - y,\ r,\ y \in Y\} \tag{4.3}$$

The objectives of this chapter are mainly to deal with internal perturbation and external disturbance of the nonlinear system to guarantee robust stability of the overall system and to design tracking control scheme such that output of the nonlinear system tracks reference input. Firstly, a theorem on robust right coprime factorization is proposed by using a Lipschitz inequation with a reconstructed controller for the nonlinear system with internal perturbation and external disturbance. Secondly, by combining the robust right coprime factorization method with a new nonlinear operator controller, tracking performance is realized, while internal perturbation and external disturbance of the nonlinear system are reduced. The proposed design scheme not only deals with internal perturbation and external disturbance but also achieves the tracking performance of the overall system.

4.2.2 Three cases of robustly stable conditions

This chapter considers a nonlinear system with internal perturbation and external disturbance. First of all, it is worth to mention again that the

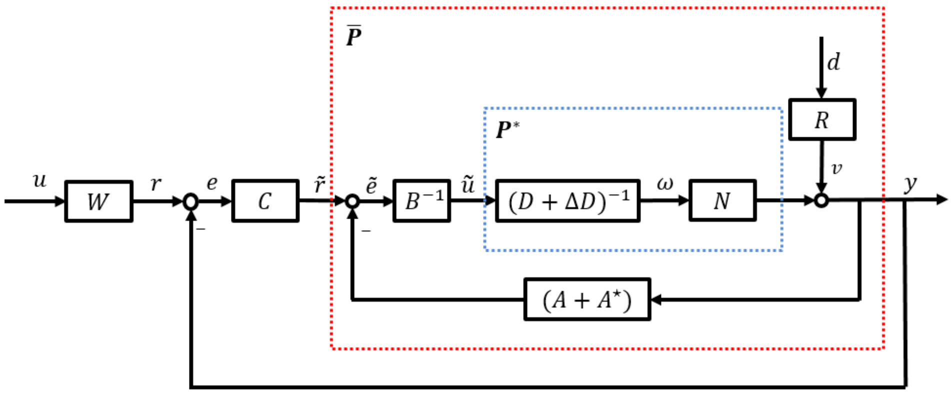

Figure 4.2: The proposed nonlinear system with perturbation and disturbance

Bezout identity is given for the nominal plant in the form of $AN+BD=M$. If the Bezout identity is satisfied that the nominal plant is said to have a right coprime factorization. Suppose that the plant P has a perturbation ΔP, that is, $P \to P+\Delta P$ denoted as $P^* = P+\Delta P$, where ΔP denotes the case $D \to D+\Delta D$ and $D+\Delta D$ is an invertible operator, in other words, the perturbation can be considered as the results caused by ΔD. The robust right factorization can be rewritten to be $P+\Delta P = N(D+\Delta D)^{-1}$, where in this chapter, we consider the case of plant with uncertainty from the input side, shown in Figure 4.2, where the system P^* which is composed of the normal plant P and ΔP as internal perturbation of this nonlinear system; a stable operator R is assumed to transform the external source d into the disturbance v. In the considered system, ΔD is took into account as internal perturbation of the nonlinear system, such that, P^* can be factorized into two parts N and $(D+\Delta D)^{-1}$. Moreover, in the view of mathematical, we suppose that the class of disturbances is defined by $D_d \subseteq U$ as shown as follows, which is a bounded subset in the corresponding extended linear space.

$$D_d = \{d :\| d \|_{D_d} \leq M_{\tilde{d}} < \infty \text{ for all } T \in [0, \infty)\} \tag{4.4}$$

At this position, the objective problems can be summarized that: 1) the consideration is that under what conditions can guarantee the perturbed plant of the nonlinear system still holding a robust right coprime factorization. 2) the consideration is that satisfying what conditions can maintain the perturbed system to be overall stability. Based on the above analysis for the nonlinear system with internal perturbation and external disturbance shown in Figure 4.2, a theorem on robust right coprime factorization will be discussed in order to guarantee robust stability by using a Lipschitz inequation with a new constructed controller.

Theorem 4.1. *Let the given normal system P have a right coprime factorization $P = ND^{-1}$, where N is stable; D is stable and invertible. For the real system $P^* = P + \Delta P$ with external disturbance v, the assumed stable operator R has a right factorization in the form of $R = G(D + \Delta D)^{-1}$, where G is stable, $D + \Delta D$ is stable and invertible. If there exists a stable operator $A^\star$ and B, such that*

$$\|A^\star(N + G) + B(D + \Delta D) - BD\| < 1 \tag{4.5}$$

then the nonlinear system with internal perturbation and external disturbance has a robust right coprime factorization, that is, robust stability of the overall system is guaranteed, where $A^\star$ and B are stable, ΔD is bounded, $\| \cdot \|$ is Lipschitz norm.

Proof. As for the exact system shown in Figure 4.2, $D+\Delta D$ and $N+G$ are stable such that the system in the form of $(A+A^\star)(N+G)+B(D+\Delta D) = \tilde{M}$ is well-posed. According to the fact that for each d, the signal v is uniquely determined by the system input $\tilde{u}$, denoted as $d = \varphi(\tilde{u})$. For any $\tilde{r} \in U$, we can get

$$\begin{aligned}
\tilde{r} &= B(D + \Delta D)(\omega) + (A + A^{\star})(y) \\
&= B(D + \Delta D)(\omega) + (A + A^{\star}) \\
&\quad\ [G(D + \Delta D)^{-1}(d) + N(\omega)] \\
&= B(D + \Delta D)(\omega) + (A + A^{\star})[G(D + \Delta D)^{-1} \\
&\quad\ (D + \Delta D)(\omega) + N(\omega)] \\
&= [A(G + N) + A^{\star}(G + N) + B(D + \Delta D)](\omega) \\
&= [I - BD + A^{\star}(G + N) + B(D + \Delta D)](\omega)
\end{aligned}$$

whenever $\|A^{\star}(N + G) + B(D + \Delta D) - BD\| < 1$, then the operator $I - BD + A^{\star}(G + N) + B(D + \Delta D)$ has a stable invertible. Moreover, since $A^{\star}$, B, D, N, and G, ΔD are all stable, hence, $G + N$,$D + \Delta D$ and $I - BD + A^{\star}(G + N) + B(D + \Delta D)$ is stable that can be get. All of these imply that $I - BD + A^{\star}(G + N) + B(D + \Delta D)$ is unimodular. As a result, based on the proposition of [37] the nonlinear system with internal perturbation and external disturbance has a robust right coprime factorization, and the overall system is guaranteed to be robustly stable. The proof of theorem is completed.

Remark. Something to highlight for you is that, based on the **Theorem 4.1**, the exact system with internal perturbation and external disturbance can be illustrated in the form of $(A + A^{\star})(G + N) + B(D + \Delta D) = \tilde{M}$, which is a Bezout identity associated with the considered nonlinear system in this chapter, where $\tilde{M}$ is unimodular. Note that the initial state should be consistent with the Bezout identity, that is, $(A + A^{\star})(N + G)(w_0, t_0) + B(D + \Delta D)(w_0, t_0) = \tilde{M}(w_0, t_0)$ should be satisfied. Furthermore, in this chapter, we again select $t_0 = 0$, $w_0 = 0$ but $\tilde{M} \neq I$, which is much different from former, a more general practical form can be used into the robust right coprime factorization.

Theorem 4.2. Let the given normal system P have a right coprime factorization $P = ND^{-1}$, where N is stable; D is stable and invertible. For the real system $P^\star = P + \Delta P$, $D + \Delta D$ is stable and invertible. If there exists a stable operator $A^\star$ and B, such that

$$\|A^\star N + B(D + \Delta D) - BD\| < 1 \tag{4.6}$$

then the nonlinear system with internal perturbation has a robust right coprime factorization. That is, robust stability of the overall system is guaranteed.

Proof. First, some detail explanations are given for the proposed theorem. That is, the real system with internal perturbation but without external disturbance, namely, $\Delta D \neq 0$ $v = 0$.

For any $\tilde{r} \in U$, we can get

$$\begin{aligned}
\tilde{r} &= B(D + \Delta D)(\omega) + (A + A^\star)(y) \\
&= B(D + \Delta D)(\omega) + (A + A^\star)N(\omega) \\
&= [B(D + \Delta D) + I - BD + A^\star N](\omega) \\
&= [I + A^\star N + B(D + \Delta D) - BD](\omega)
\end{aligned}$$

whenever $\|A^\star N+B(D+\Delta D)-BD\| < 1$, then the operator $[I-BD+A^\star N+B(D + \Delta D)]$ has a stable invertible. Moreover, since $A^\star$, B, D, N, and ΔD are all stable, hence, $I-BD+A^\star N+B(D+\Delta D)$ is stable that can be get. All of these imply that $I - BD + A^\star N + B(D + \Delta D)$ is unimodular. As a result, the nonlinear system with internal perturbation has a robust right coprime factorization, and the overall system is guaranteed to be robust stable.

Theorem 4.3. *Let the given normal system P have a right coprime factorization $P = ND^{-1}$, where N is stable; D is stable and invertible. For the real system P with external disturbance v, the assumed stable operator R*

has a right factorization in the form of $R = GD^{-1}$*, where* G *is stable,* D *is stable and invertible. If there exists a stable operator* $A^\star$ *and* B*, such that*

$$\|A^\star(G+N)\| < 1 \tag{4.7}$$

then the nonlinear system with external disturbance has a robust right coprime factorization, that is, robust stability of the overall system is guaranteed.

Proof. First, some detail explanations are given for the proposed theorem. That is, the real system with internal perturbation but without external disturbance, namely, $\Delta D = 0$ $v \neq 0$.

For any $\tilde{r} \in U$, we can get

$$\begin{aligned}
\tilde{r} &= BD(\omega) + (A + A^\star)(y) \\
&= BD(\omega) + (A + A^\star)[GD^{-1}(d) + N(\omega)] \\
&= BD(\omega) + (A + A^\star)[GD^{-1}D(\omega) + N(\omega)] \\
&= [A(G+N) + A^\star(G+N) + BD](\omega) \\
&= [I + A^\star(G+N)](\omega)
\end{aligned}$$

whenever $\|A^\star(N+G)\| < 1$, then the operator $[I + A^\star(G+N)]$ has a stable invertible. Moreover, since $A^\star$, N and G are all stable, hence, $I + A^\star(G+N)$ is stable that can be get. All of these imply that $I + A^\star(G+N)$ is unimodular. As a result, the nonlinear system with internal perturbation and external disturbance has a robust right coprime factorization, and the overall system is guaranteed to be robust stable.

4.2.3 An example for showing the necessity of the proposed method

A numerical example is given to demonstrate the limitation of the former condition in guaranteeing the robust stability of nonlinear systems. In this case, we suppose $P,\ N,\ D$ as follows.

$$P(\tilde{u})(t) = \frac{t+1}{(2t+1)t}(\tilde{u})(t) + \frac{t}{2t+1}$$

which can be factorizated into tow parts shown in below with satisfying the right coprime factorization.

$$N(\omega)(t) = \frac{1}{2t+1}\omega(t) + \frac{t}{2t+1}$$
$$D(\omega)(t) = \frac{t}{t+1}(\omega)(t)$$

As for the plant, the stable controllers $A,\ B$ can be defined as follows,

$$A(y)(t) = \frac{2t+1}{t+1}(y)(t)$$

and

$$B(\tilde{u})(t) = \tilde{u}(t) - \frac{t}{t+1}$$

which satisfy the Bezout identity.

As for the real plant, however, the uncertainties always be existed, in this case, whether the controllers A and B also can make the overall system stable is a problem which need to be proof. So that, we suppose that ΔD and G represent internal perturbation and external disturbance as follows. $\Delta D(\omega)(t) = \frac{2}{t+1}(\omega)(t)$ and $G(\omega)(t) = \frac{1}{2t+1}(\omega)(t) + 1$.

Based on the previous condition on the robust stability we can find

$$\begin{aligned}
&\| (A(N+G) - AN + B(D+\Delta D) - BD)M^{-1} \| \\
=&\| \frac{1}{t+1}\omega(t) + \frac{2t+1}{t+1} + \frac{2}{t+1}\omega(t) - \frac{t}{t+1} \| \\
=&\| \frac{3}{t+1}\omega(t) + 1 \| > 1
\end{aligned}$$

which verified that the controller A and B didn't play a desired role to ensure the system stable. So that the new theorem rises in response to the proper time and conditions.

Then taking into account on **Theorem 4.1**, a new controller $A^\star$ is given in the form of

$$A^\star(y)(t) = -y(t) + \frac{5t^2+5t+1}{(2t+1)(t+1)}$$

, so that the condition can be verified

$$\begin{aligned}
&\| A^\star(N+G) + B(D+\Delta D) - BD \| \\
=&\| -\frac{2}{2t+1}\omega(t) - \frac{3t+1}{2t+1} + \frac{5t^2+5t+1}{(2t+1)(t+1)} + \frac{2}{t+1}\omega(t) - \frac{t}{t+1} \| \\
=&\| \frac{2t}{2t^2+3t+1}\omega(t) \| < 1
\end{aligned}$$

which illustrates that the proposed design scheme of **Theorem 4.1** is effective for dealing with internal perturbation and external disturbance. That is, based on the former method, the robust stability of the nonlinear systems with internal perturbation and external disturbance cannot be guaranteed because the norm value of the inequation is larger than 1. However, $A^\star$ plays a necessary role on robust stability of the overall system when the nonlinear systems with internal perturbation and external disturbance.

4.3 Rejection scheme analysis and output tracking issue

4.3.1 Equivalent problem

The robustness of the right coprime factorization of the nonlinear system with internal perturbation and external disturbance shown in the Figure 4.2 has been investigated, which results in the robust stabilization of the entire feedback control system based on right coprime factorization in reasonably general operator theoretic setting. We are in a position to address the problem that the relationships around the control system still remain unchanged under the internal perturbation due to the whole complex structure, so that

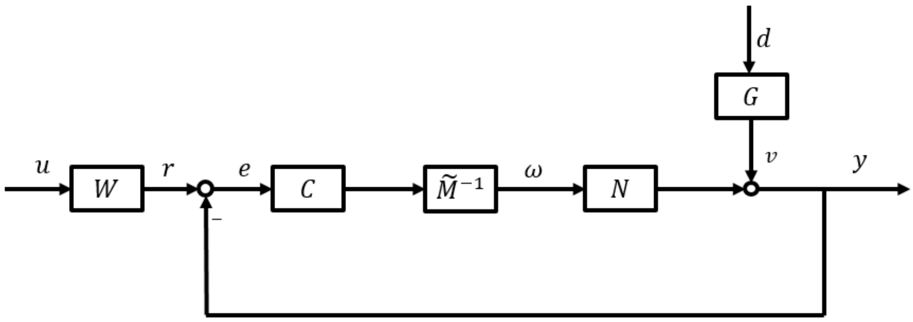

Figure 4.3: Equivalent system

it is difficult to design the compensator belonging to a certain admissible so as to remove the adverse effects from the uncertainties. Then an equivalent problem statement will be discussed.

Corollary 4.1 Assume that the nominal plant with right coprime factorization shown in Figure 2.3 is well-posed. If for any $\tilde{r} \in U_e$, can be got $\omega = \tilde{M}^{-1}(\tilde{r}) \in W_e$ in Figure 4.2, when if and only if $\tilde{M}$ is unimodular operator, such that the equivalent feedback control system for Figure 4.2 can be obtained in the sense that show in Figure 4.3.

To state the corollary, denote some notions again. Let the plant input space, output space and quasi-state space be $U_e \subset U$, $Y_e \subset Y$ and W, respectively. $P^* = N(D + \Delta D)^{-1} : W \to D_d$, $N : W \to Y_e$, $G : D_d \to Y_e$, $D + \Delta D : W \to U_e$ and $\tilde{M^{-1}} : U_e \to W$ is an unimodular operator.

(*Sufficiency*). Since the feedback system shown in Figure 4.2 is well-posed, then we define an implicit unimodular operator $\tilde{M} : W \to U_e$, in the sense that we do not know its explicit construct at present, for any $\tilde{r} \in U_e$, so that $\omega = \tilde{M}^{-1}\tilde{r}$. Further, since $y = (N + G)\omega$. From the above two equations, we can get

$$y = (N + G)\tilde{M}^{-1}\tilde{r}$$

which the corresponding figure has been shown in Figure 4.3.

(*Necessity*). To start with, following the Figure 4.2 with the well-posedness feedback control system. We have $\tilde{r} = \tilde{e} + (A + A^{\star})y$, $\tilde{e} = B(D + \Delta D)\omega$ and $y = (N + G)\omega$, such that,

$$\begin{aligned}\tilde{r} &= B(D + \Delta D)\omega + (A + A^{\star})(N + G)\omega \\ &= [B(D + \Delta D) + (A + A^{\star})(N + G)]\omega\end{aligned}$$

from the **Theorem 4.1** can be got that $(A+A^{\star})(G+N)+B(D+\Delta D) = \tilde{M}$, which is a Bezout identity associated with the considered nonlinear system.

So as to, $\tilde{r} = \tilde{M}\omega$. From the corollary, not only get the equivalent framework of Figure 4.2, but also obtain the quantitative form of $\tilde{M}$.

4.3.2 The admissible class for controller design

Based on **Corollary 4.1**, the nonlinear system with internal perturbation and external disturbance shown in Figure 4.2 can be equivalently transferred to Figure 4.3. In terns of the obtained equivalent system, it can be described as follows,

$$\begin{cases} e(t) = r(t) - y(t) \\ y(t) = N\tilde{M}^{-1}C(e)(t) + G(d)(t) \\ r(t) = W(u)(t) \end{cases} \tag{4.8}$$

where r denotes the reference input and v is external disturbance with respected to the external source d.

Therefore, the nonlinear system with internal perturbation and external disturbance design problem can be stated as follows. Given $\tilde{M}$, N, W and G as described above, one of objectives of this section is to design a controller C in order to remove the adverse effect of internal perturbation ΔD and external disturbance v such that output tracking performance is realized.

In order to pose the problem mathematically, a certain admissible class S^* will be firstly proposed which is composed of all nonlinear compensator operators, as follows:

$$S^* = \{C \in Lip(Y) : N\tilde{M}^{-1}C \in Lip(Y)\} \tag{4.9}$$

that S^* is an infinite-dimensional Banach space. Note that, Y denotes the output space for the system and input space for the compensator C, so that $Lip(Y)$ denotes the family of generalized Lipschitz operators mapping from Y to itself.

Using S^* defined in Equation (4.9) as an underlying operator space for the admissible class of controllers, a very large framework for the design purposes is obtained. The merits of the Banach space, S^*, for the underlying space of controllers will be elaborated by the following. That is, comparing with the last Chapter with assuming $\tilde{M} = I$, the structured admissible class has a general wider practicability without emphasizing the $\tilde{M}$ quantitatively, in the sense that $\tilde{M} \neq I$.

Lemma 4.1. *If the condition that* $\| N\tilde{M}^{-1}C \| < 1$ *is satisfied, then the nonlinear operator*

$$I + N\tilde{M}^{-1}C : Y \to Y$$

is invertible, and its inverse, denoted by $(I+N\tilde{M}^{-1}C)^{-1}$*, is also a generalized Lipschitz operator in* $Lip(Y)$ *satisfied with a fixed* x_0 *as*

$$\begin{aligned} &\| (I+N\tilde{M}^{-1}C)^{-1} \|_{Lip} \leq \\ &\qquad \| (I + N\tilde{M}^{-1}C)^{-1}(x_0) \|_Y + (1- \| N\tilde{M}^{-1}C \|)^{-1} \end{aligned}$$

Proof . First, we observe that for any $a_1, a_2 \in Y$,

$$\begin{aligned} &\| (I + N\tilde{M}^{-1}C)(a_1) - (I + N\tilde{M}^{-1}C)(a_2) \| \\ &\quad \geq \| a_1 - a_2 \| - \| (N\tilde{M}^{-1}C)(a_1) - (N\tilde{M}^{-1}C)(a_2) \| \\ &\quad \geq (1- \| N\tilde{M}^{-1}C \|) \| a_1 - a_2 \| \end{aligned} \tag{4.10}$$

By satisfying the condition that $\| N\tilde{M}^{-1}C \| < 1$. Hence, implying $I + N\tilde{M}^{-1}C$ is an injective mapping, namely, $(I+N\tilde{M}^{-1}C)(a_1) = (I+N\tilde{M}^{-1}C)(a_2)$ implies that $a_1 = a_2$.

Finally, for x_1, x_2 in the range of $I + N\tilde{M}^{-1}C$, from the above Equation (4.10) and the definition of the norm we have

$$\begin{aligned}
&\| (I + N\tilde{M}^{-1}C)^{-1} \| = \| (I + N\tilde{M}^{-1}C)^{-1}(x_0) \| \\
&+ \sup_{x_1,x_2 \in Y} \frac{\| (I + N\tilde{M}^{-1}C)^{-1}(x_1) - (I + N\tilde{M}^{-1}C)^{-1}(x_2) \|}{\| x_1 - x_2 \|} \\
&\leq \| (I + N\tilde{M}^{-1}C)^{-1}(x_0) \| + (1 - \| N\tilde{M}^{-1}C \|)^{-1}
\end{aligned}$$

so that, this completes the proof.

It is worth to mention that this theorem has played an significant role because in the sense that all the operators C, $N\tilde{M}^{-1}C$, and $(I + N\tilde{M}^{-1}C)^{-1}$ in the nonlinear control system all belong to the same family $Lip(Y)$, which is an infinite-dimensional Banach space. Therefore, the overall closed-loop structure can be well defined in mathematically with unique internal signals.

4.3.3 Internal perturbation and external disturbance rejection and output tracking issue

Considering relationships within Equation (4.8) and all the conditions stated above are satisfied. Then, the following equation can be obtained,

$$e + N\tilde{M}^{-1}C(e) = W(u) - G(d)$$

then, by **Lemma 4.1**, we can get

$$e(t) = (I + N\tilde{M}^{-1}C)^{-1}(W(u) - G(d))(t) \tag{4.11}$$

Since $(I + N\tilde{M}^{-1}C)^{-1}$ is nonlinear, which is different with linear case, not fitting the left distributive law, there exist some obstacles to design a general framework for the nonlinear system with internal perturbation and external disturbance to separate $W(u) - G(d)$. However, considering that $G : D_d \to Y_0$ is stable defined on the bounded closed subset D_d, its generalized inner inverse $G^+ : Y_0 \to D_d$ can be defined based on the closed linear operator theory which is also a bounded linear operator. Indeed, the operator norm of this inner inverse operator satisfies

$$\parallel G^+ \parallel := \sup_{\|\delta\|_Y = 1} \parallel G^+ v \parallel_{D_d} \leq M_{\tilde{d}} \tag{4.12}$$

Consequently, the norm of e can be obtained

$$\begin{aligned}\parallel e \parallel_Y =& \parallel (I + N\tilde{M}^{-1}C)^{-1}(W(u) - G(d)) \parallel_Y \\ \leq& \parallel (I + N\tilde{M}^{-1}C)^{-1}G \parallel \parallel G^+Wu - d \parallel_{D_d} \\ \leq& \parallel (I + N\tilde{M}^{-1}C)^{-1}G \parallel (M_{\tilde{d}} \parallel W \parallel \parallel u \parallel_U + \parallel d \parallel_{D_d}) \\ \leq& \parallel (I + N\tilde{M}^{-1}C)^{-1} \parallel \parallel G \parallel M_{\tilde{d}} \parallel W \parallel \parallel u \parallel_U \\ &+ \parallel (I + N\tilde{M}^{-1}C)^{-1}G \parallel \parallel d \parallel_{D_d}\end{aligned} \tag{4.13}$$

In order to simultaneously reject the adverse effect from internal perturbation and external disturbance, namely, design a compensator to minimizing the norm of the error response of the system as well as realize the tracking performance from the reference signal to the output signal. In the concern of the solvability of the problems, we formulate the problem precisely in a mathematical manner to provide minimizing the first term in the right-hand side of Equation (4.13) as shown in follows:

$$\min_{C \in S^*} \parallel (I + N\tilde{M}^{-1}C)^{-1} \parallel_{Lip} \tag{4.14}$$

Theorem 4.4. *In terms of Equations (4.15) and (4.16), there exists a control operator Q as shown in Figure 4.4, whenever t tends to be zero such*

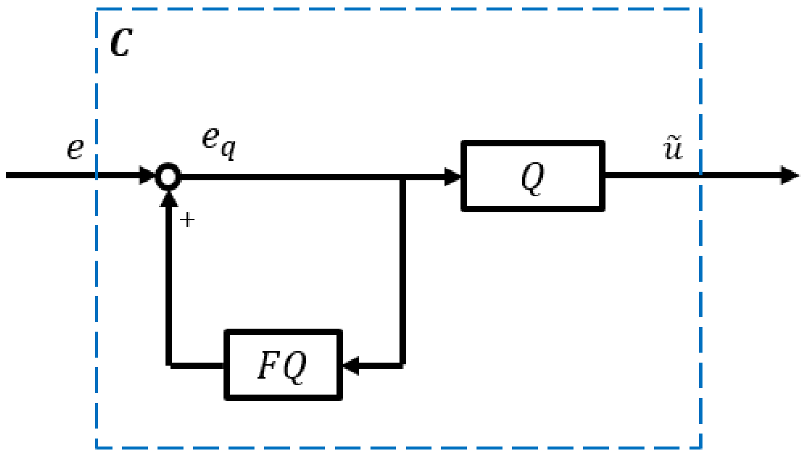

Figure 4.4: The design diagram of controller C

that uncertainties tend to be zero simultaneously for reducing the internal perturbation and external disturbance for the nonlinear system.

Proof . For any generalized Lipschitz operator $Q \in Lip(Y)$ satisfying $FQ \in Lip(Y)$ with

$$\| FQ \| < \frac{1}{2}, \text{and}, (I - FQ)^{-1}(0) = 0$$

exists and is also in $Lip(Y)$, then we define

$$C := Q(I - FQ)^{-1}$$

where $F = N\tilde{M}^{-1}$, that the design diagram as shown in Figure 4.4.

From *Lemma 3.1*, we have $\| (I - N\tilde{M}^{-1}Q)^{-1} \| < 2$, so as to

$$\begin{aligned} \| N\tilde{M}^{-1}C \| &= \| N\tilde{M}^{-1}Q(I - FQ)^{-1} \| \\ &\leq \| N\tilde{M}^{-1}Q \| \| (I - FQ)^{-1} \| < 1 \end{aligned}$$

which implies that $\| N\tilde{M}^{-1}C \| < 1$ is satisfied. Hence, we observe that

$$\begin{aligned} I + N\tilde{M}^{-1}C &= I + N\tilde{M}^{-1}Q(I - FQ)^{-1} \\ &= [(I - FQ) + N\tilde{M}^{-1}Q](I - FQ)^{-1} \\ &= [I - FQ + FQ](I - FQ)^{-1} \\ &= (I - FQ)^{-1} \end{aligned} \tag{4.15}$$

In order to remove the adverse effect of internal perturbation and external disturbance of the nonlinear system, the nonlinear operator Q with the time-varying gain is designed as follows,

$$\begin{aligned} Q(x)(t) = x(t)E \int_0^t (x(\tau)e^{-x^2(\tau)} + e^{-x(\tau)} \\ \cdot \sin(x(\tau))) \cdot \dot{x}(\tau)d\tau \end{aligned} \tag{4.16}$$

where x is a variable, and $FE = I$.

Then, from Figure 4.3, we can get

$$\begin{aligned} e(t) &= (I + N\tilde{M}^{-1}C)^{-1}(W(u) - G(d))(t) \\ &= (I - FQ)(r(t) - v(t)) \end{aligned} \tag{4.17}$$

Furthermore, combing Figure 4.3 and Figure 4.4 we can get

$$\begin{aligned} y(t) &= N(\omega)(t) + G(d)(t) \\ &= N\tilde{M}^{-1}(\tilde{u})(t) + v(t) \\ &= F(\tilde{u})(t) + v(t) \\ &= FQ(r - v)(t) + v(t) \end{aligned} \tag{4.18}$$

From Equation (4.16), $y(t)$ is shown as follows,

$$\begin{aligned} y(t) = FE \cdot (r - v)(t) \int_0^t ((r - v)(\tau)e^{-(r-v)^2(\tau)} + e^{-(r-v)(\tau)} \\ \cdot \sin((r - v)(\tau))) \cdot (\dot{r} - \dot{v})(\tau)d\tau + v(t) \end{aligned} \tag{4.19}$$

Since $FE = I$, and as $t \to \infty$,

$$\int_0^t ((r-v)(\tau)e^{-(r-v)^2(\tau)} + e^{-(r-v)(\tau)} \cdot \\ \sin((r - v(\tau))) \cdot (\dot{r} - \dot{v})(\tau)d\tau \to 1 \tag{4.20}$$

Therefore,

$$\begin{aligned} y(t) &= (r-v)(t) + v(t) \\ &= r(t) \end{aligned} \tag{4.21}$$

the tracking performance is guaranteed.

Considering the above analysis implies that internal perturbation and external disturbance can be removed by using the proposed design scheme based on the operator-based control structure.

4.3.4 Simulation example

In this section, we will provide a simulation example in order to confirm the effectiveness of the proposed design scheme.

Let $C_{[0,\infty)}$ be the space of continuous functions, and $C^1_{[0,\infty)}$ consists of all the functions having a continuous first derivative, both are defined on $[0, \infty)$.

Considered a nonlinear plant $P^* = P + \Delta P : U \to Y$ as shown in Figure 4.2 is defined as follows, in which the input space and the output space is U and Y, respectively.

$$\begin{aligned} P^*(\tilde{u}) &= (P + \Delta P)(\tilde{u})(t) \\ &= 4e^t \cdot \tilde{u}(t) - 2\int_0^t e^{-\tau} \cdot \tilde{u}(\tau)d\tau \end{aligned}$$

Based on the proposed P^*, the operators N, D, and ΔD are factorized as following,

$$N(\omega)(t) = 2\omega(t) - \int_0^t e^{-2\tau}\omega(\tau)d\tau$$

$$D(\omega)(t) = (2t + e^t)^{-1}\omega(t)$$

$$\Delta D(\omega)(t) = \frac{1}{2}(2t - e^t)e^{-t}(e^t + 2t)^{-1} \cdot \omega(t)$$

where $\Delta D(\omega)(t)$ is denoted as the internal perturbation. Moreover, from the obtained right factorization, we can get

$$(D + \Delta D)^{-1}(\tilde{u})(t) = 2e^t \cdot \tilde{u}(t)$$

which is obviously unstable.

According to the proposed design, the external disturbance and its right factorization are given as follows:

$$R(d)(t) = \int_0^t -2e^{2\tau} \cdot d(\tau)d\tau$$

$$G(\omega)(t) = \int_0^t -e^{\tau} \cdot \omega(\tau)d\tau$$

$$(D + \Delta D)^{-1}(\tilde{u})(t) = 2e^t \cdot \tilde{u}(t)$$

where R and G are stable, and $(D + \Delta D)^{-1}$ is unstable, driven signal of external disturbance $d(t)$ is assume to be $2\tilde{u}(t)$.

In this example, we choose the quasi-state $W = U$. It is easy to find that $P^\star$ is unstable. Based on **Theorem 4.1**, the controllers A, $A^\star$ and B can be designed as follows:

$$A(y)(t) = \frac{1}{2}(1 - e^{-2t})(y)(t)$$

$$A^\star(y)(t) = (\frac{1}{4}e^{-2t} - \frac{1}{2}te^{-3t})(y)(t)$$

$$B(\tilde{u})(t) = (2te^{-2t} + e^{-t})\tilde{u}(t)$$

Moreover, the two conditions are verified as follows.

$$\begin{aligned}
&((A + A^{\star})(N + G) + B(D + \Delta D))(\omega)(t) \\
&\quad = \frac{1}{2}(1 - \frac{1}{2}e^{-2t} - te^{-3t}) \cdot (2)(\omega)(t) \\
&\qquad + (2te^{-2t} + e^{-t})(\frac{1}{2}e^{-t}(\omega)(t)) \\
&\quad = (1 - \frac{1}{2}e^{-2t} - te^{-3t})(\omega)(t) \\
&\qquad + (te^{-3t} + \frac{1}{2}e^{-2t})(\omega)(t) \\
&\quad = \omega(t)
\end{aligned} \tag{4.22}$$

$$\begin{aligned}
&\| A^{\star}(N + G) + B(D + \Delta D) - BD \| \\
&\quad =\| (\frac{1}{4}e^{-2t} - \frac{1}{2}te^{-3t})(2(\omega)(t)) + (2te^{-2t} + e^{-t}) \\
&\qquad \cdot (\frac{1}{2}e^{-t})(\omega)(t) - (2te^{-2t} + e^{-t})(\frac{1}{2t + e^{t}})(\omega)(t) \| \\
&\quad =\| ((e^{-2t}(\frac{1}{2} - te^{-t})) + \frac{2te^{-t} + 1}{2e^{2t}} \\
&\qquad - \frac{2te^{-t} + 1}{e^{t}(2t + e^{t})})(\omega)(t) \| < 1
\end{aligned}$$

Based on Equation (4.22), the controller E is designed to be the inverse of N. That is, $NE = I$. Therefore, from the designed controllers, the perturbed Bezout identity for the nonlinear system with internal perturbation and external disturbance can be verified to be satisfied. That is, robust stability of the overall nonlinear control system is guaranteed.

In order to show the example more explicitly, simulation results are given. The reference input is chosen as $r = 10e^{-1.15t}\sin^2(t) + te^{-t}$. According to

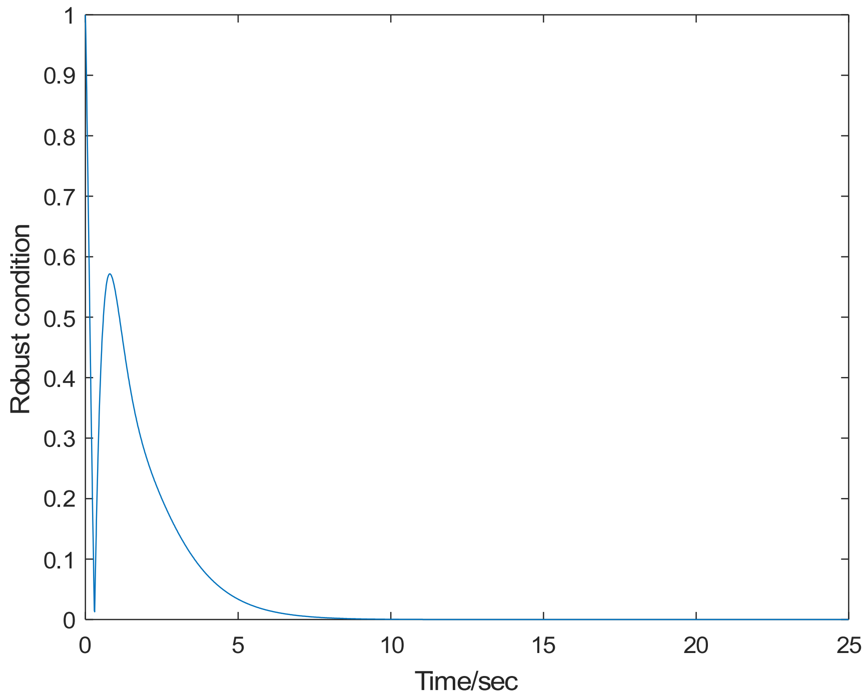

Figure 4.5: Effectiveness of robust condition

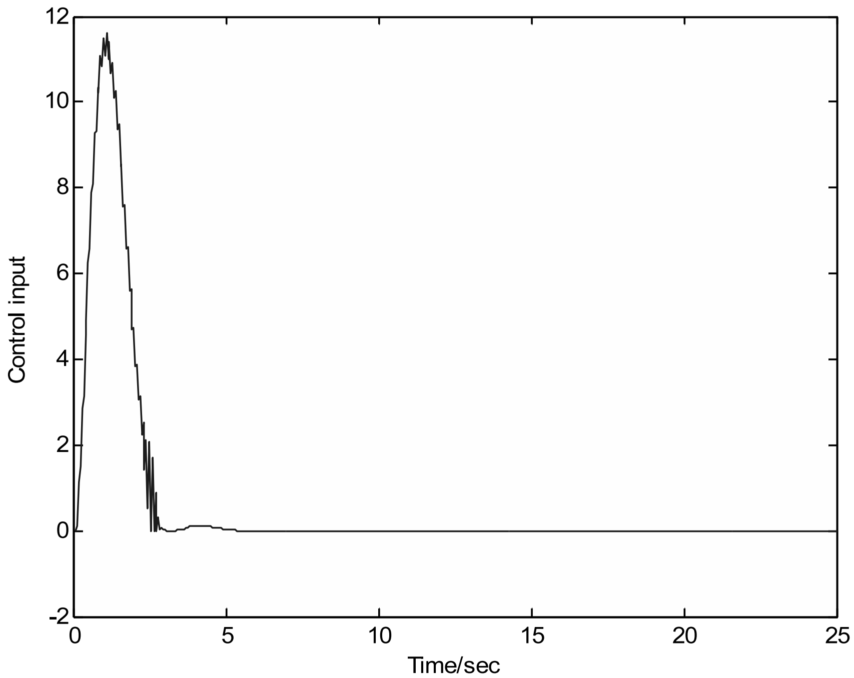

Figure 4.6: Control input

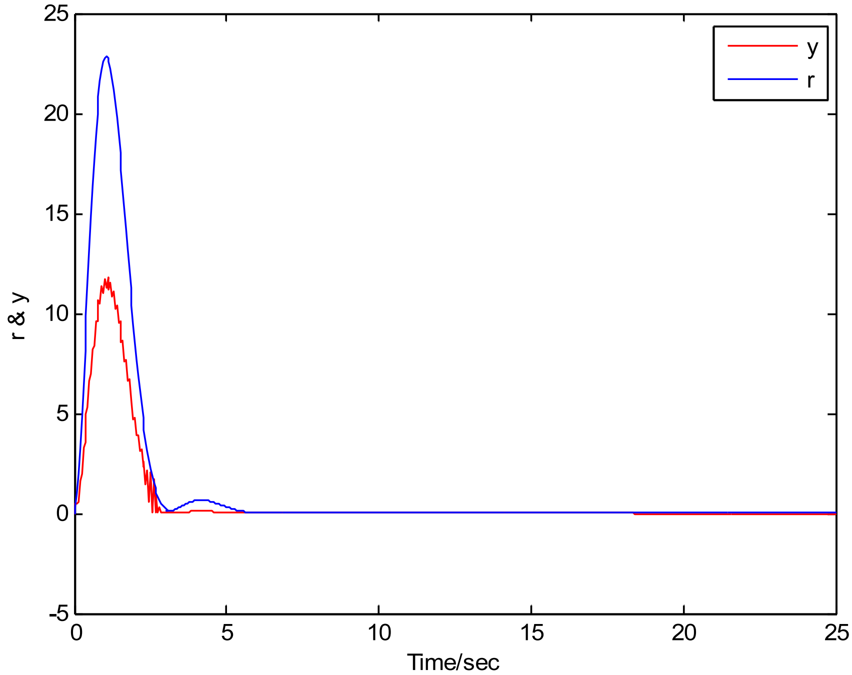

Figure 4.7: Reference input r & plant output y

the Equation (4.22), the perturbed Bezout identity is verified based on the proposed design scheme for guaranteeing robust stability of the nonlinear systems with internal perturbation and external disturbance. In detail, the obtained perturbed Bezout identity shows the relationship between the reference input $r(t)$ and internal signal $\omega(t)$, which means that a bounded reference input can lead to bounded internal signal. Meanwhile, combining the stable part of the right factorization, N, robust stability of the overall system is guaranteed under the definition of BIBO stability. Then the simulation results are given in Figures 4.6 and 4.7, where the reference input r and plant ouput y are shown in Figure 4.7, the control input is shown in Figure 4.6.

Moreover, for showing the effectiveness of the proposed sufficiently robust condition Equation (4.5), the simulation result is provide as shown in Figure 4.5. From Figure 4.5, the norm of Equation (4.5) is less than 1, which furtherly shows the proposed design scheme in this section is effective. Therefore, the simulation results demonstrate the fact that internal perturbation and external disturbance of the considered nonlinear system are effectively removed and output tracking performance is realized.

4.4 Conclusion

In this chapter, robust stability and tracking performance of nonlinear systems with internal perturbation and external disturbance are discussed by using operator-based robust right coprime factorization. Firstly, robust stability was considered based on a Lipschitz norm inequation by the proposed new controller. Secondly, based on the proposed design scheme o robust stability, a convenient and feasible framework was obtained in order to reject internal perturbation and external disturbance. Thirdly, the adverse effect resulting from internal perturbation and external disturbance of the nonlinear system was removed through the designed nonlinear operator controller. Simultaneously, output tracking performance was realized using the proposed design scheme. Finally, a simulation example was given to confirm effectiveness of the proposed design scheme of this method.

Chapter 5

Operator-based nonlinear robust control and sensitivity analysis of uncertain nonlinear systems

5.1 Introduction

As we addressed in the former chapters, in practice, almost all systems possess nonlinear property and multivariable characteristic, which have been attracting researchers' attention due to important role. For nonlinear systems, robust control, sensitivity and tracking issues [59], [63] still remain challenging due to inevitable factors appearing in systems, such as parametric perturbations, modeling errors and uncertainties. For dealing with these issues, a great number of effective methods are proposed, such as the adaptive control, the sliding mode control method, operator-based right coprime factorization method, the geometric approach and so on.

In Chapter 3 and Chapter 4, the uncertainties rejection including the internal perturbation and the external disturbance as well as the nonlinear control problem has been discussed for nonlinear systems based on generalize Lipschitz operator in an extended linear space setting form the physical meaning and formulating mathematically perspective. However, the previous two chapters payed more attention to remove the adverse affects in unitary formulation, nevertheless lost sight of the interplay within the internal perturbation and external disturbance. Motivated by this issue, in this chapter, the bilinear operator-based right coprime factorization for nonlinear system with perturbation and disturbance is introduced, which can consider adverse effect resulting from perturbation and disturbance quantitatively. Based on the proposed method, a feasible framework is established for considering robust control, sensitivity and tracking performance, which not only separates perturbation and disturbance, but also provides a fundamental base to design a controller for the considered system. After that, robust stability for the uncertain nonlinear systems is guaranteed under the proposed framework. In terms of the insensitivity property, it is addressed for the case where perturbation and disturbance both exist in nonlinear systems, which extend the former results of [85], [86]. Sequentially, tracking performance is obtained by using a simple and effective controller.

In Section 5.2, Start to consider the mentioned problems, the bilinear operator controller based on right coprime factorization for nonlinear systems with perturbation and disturbance is established in a special way, what means that the controller is not general expression for a broader class of nonlinear systems, lacking of design freedom in practical applications. In this section, we propose a general design scheme for the bilinear operator controller, which can consider quantitatively adverse effect resulting from perturbation and disturbance, respectively, as well as making the proposed design scheme could have a more freedom to be satisfied with practical requirement. After that, based on the proposed method, a feasible framework is established for considering robust control, which not only separates perturbation and disturbance, but also provide a fundamental base to design controllers for the considered system.

In Section 5.3, in terms of the insensitivity property, it is addressed for the case from theoretical perspective, where perturbation exists in the nonlinear system, which extend the former results of and proves a relationship between robust stability and insensitivity. Compared with the former results, the obtained results in this section extend the application fields of operator-based right coprime factorization and improve the design scheme for quantitatively analyzing the effect of disturbance and perturbation and tracking performance. Finally, a simulation example is given to confirm the effectiveness of the proposed design scheme.

In Section 5.4, main result on operator-based reset control for nonlinear systems with unknown bounded disturbance is addressed. That is, in the context of operator-based right coprime factorization, reset control is realized and robust stability of nonlinear systems with unknown bounded disturbance is guaranteed.

In Section 5.5, the summary of robust control and sensitivity analysis of uncertain nonlinear system using bilinear operator-based right coprime factorization is given.

5.2 Bilinear operator-based nonlinear robust control

5.2.1 Problem statement

Motivated by former research, the bilinear operator-based right coprime factorization for nonlinear systems with perturbation and disturbance is introduced, which can consider quantitatively adverse effect resulting from perturbation and disturbance, respectively. Based on the proposed method, a feasible framework is established for considering robust control and sensitivity property, which not only separates perturbation and disturbance, but also provide a fundamental base to design controllers for the considered system. After that, robust stability for the uncertain nonlinear system is discussed under the proposed framework. In terms of the insensitivity property, it is addressed for the case where perturbation and disturbance exists in the nonlinear system, which extend the former results of [86] and proves a relationship between robust stability and insensitivity.

Considering the increasing demand for system reliability and the requirement on dealing with perturbation and disturbance of nonlinear systems, in this chapter, sensitivity analysis and robust control of nonlinear systems with perturbation and disturbance are considered based on the robust right coprime factorization. First, the adverse effect resulting from the perturbation and disturbance of the nonlinear systems are analyzed by the sensitivity index. By using the proposed design scheme, the quantitative analysis on the sensitivity depending on the appearing perturbation and disturbance is discussed. Second, for guaranteeing the perfect performance as the nominal systems works, robust control is designed using the proposed method. Third, the uniformly insensitivity property has been certified as well as built the connection between the robustness and sensitivity has been established.

5.2.2 Bilinear operator

In this section, the uncertain nonlinear system shown in Figure 5.1 is considered by using bilinear operator-based right coprime factorization. For addressing main results of this section firstly, bilinear operator is introduced for dealing with perturbation and disturbance. Moreover, the bilinear operator controller is proposed to guarantee robust stability of the uncertain nonlinear system.

Definition 5.1. In mathematics, a bilinear operator is a mapping yielding an element of a third vector space driven by two elements of two vector spaces. In details, supposed that $\varPhi : \mathfrak{V} \times \mathfrak{W} \to \mathfrak{X}$ is a map, where $\mathfrak{V}$, $\mathfrak{W}$

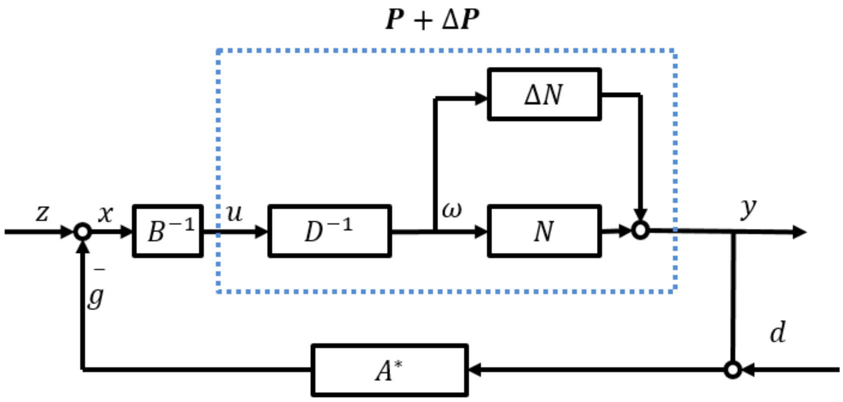

Figure 5.1: Considered nonlinear feedback system

and $\mathfrak{X}$ are three vector spaces, provided that the following conditions are satisfied,

$$(1)\ \Phi(a+b,c) = \Phi(a,c) + \Phi(b,c)$$
$$(2)\ \Phi(ka,b) = k\Phi(a,b)$$
$$(3)\ \Phi(a,b+c) = \Phi(a,b) + \Phi(a,c)$$
$$(4)\ \Phi(a,kb) = k\Phi(a,b)$$

Φ is called to be a bilinear operator.

It is worth to mention the properties of bilinear map that is a function following $\Phi : \mathfrak{V} \times \mathfrak{W} \to \mathfrak{X}$

such that for any $\mathfrak{w} \in \mathfrak{W}$ the map

$\mathfrak{v} \mapsto \Phi(\mathfrak{v}, \mathfrak{w})$ is a linear map from $\mathfrak{V}$ to $\mathfrak{X}$,

and for any $\mathfrak{v} \in \mathfrak{V}$ the map

$\mathfrak{w} \mapsto \Phi(\mathfrak{v}, \mathfrak{w})$ is a linear map from $\mathfrak{W}$ to $\mathfrak{X}$.

In other words, when we hold the first entry of the bilinear map fixed while letting the second entry vary, the result is a linear operator, and similarly for when we hold the second entry fixed.

Note that, the proposed bilinear operator serves as a tool to describe the relationship between perturbation and disturbance. The introduced bilinear operator can provide two degree of freedom to deal with perturbation and disturbance. In the follows, we will give **Lemma 5.1** for showing existence of such one bilinear operator controller for developing main results of this section.

Lemma 5.1. *As for the nonlinear system with perturbation and disturbance shown in Figure 5.1, there exists a bilinear operator controller* $\Phi(y(t), d(t))$ *defined in the suitable spaces such that*

$$\Phi(y(t), d(t)) = d(t) \int_0^t K_1(y(\tau))d\tau + y(t) \int_0^t K_2(d(\tau))d\tau \tag{5.1}$$

where $K_1(y(t))$ *and* $K_2(d(t))$ *are two stable linear operator.*

Proof: According to the definition of bilinear operator, if the following two conditions is satisfied, then the lemma can be proved.

$$\begin{aligned}\Phi(y_1(t) + y_2(t), d(t)) = d(t) &\int_0^t K_1(y_1(\tau) + y_2(\tau))d\tau \\ &+ (y_1(t) + y_2(t)) \cdot \int_0^t K_2(d(\tau))d\tau \\ &= \{d(t) \int_0^t K_1(y_1(\tau))d\tau\} + y_1(t) \int_0^t K_2(d(\tau))d\tau \\ &+ \{d(t) \int_0^t K_1(y_2(\tau))d\tau\} + y_2(t) \int_0^t K_2(d(\tau))d\tau \\ &= \Phi(y_1(t), d(t)) + \Phi(y_2(t), d(t))\end{aligned} \tag{5.2}$$

$$\begin{aligned}\Phi(ky(t),d(t)) &= d(t)\int_0^t kK_1(y(\tau))d\tau + ky(t)\int_0^t K_2(d(\tau))d\tau \\ &= k\{d(t)\int_0^t K_1(y(\tau))d\tau + y(t)\int_0^t K_2(d(\tau))d\tau\} \\ &= k\cdot\Phi(y(t),d(t)) \end{aligned} \tag{5.3}$$

This lemma is completed.

In terms of the equation, it is proposed for separating the appearing perturbation and disturbance, which has two time varying input y(t) (output with perturbation) and d(t) (disturbance of the considered system). That is, this proposed bilinear operator controller is an operator mapping an input with y(t) and d(t) to an output. Due to existing perturbation and disturbance in the given nonlinear systems, firstly, robust stability of the uncertain nonlinear system is considered for the normal performance of system, which is important in the real systems.

After that, a controller is designed using the proposed bilinear operator for developing the result of guaranteeing robust stability based on right coprime factorization.

5.2.3 Robust stability and control

In this subsection, robust stability of the uncertain nonlinear system shown in Figure 5.1 is guaranteed based on the proposed bilinear operator controller.

Theorem 5.1. *As for the uncertain nonlinear system shown in Figure 5.1, a controller $A^*(y(t),d(t)) = \Phi(y(t),d(t))$ is designed to be stable, and let $m = \| A^* - AN \|$ and $n = \| M^{-1} \|$. Provided that the following condition is satisfied with $mn < 1$, then robust stability of the considered nonlinear system is guaranteed, where A^* is under the context of bilinear operator defined in* **Lemma 5.1**.

Proof: Suppose that $\tilde{M} = M + [A^* - AN] = [I + (A^* - AN)M^{-1}]M$. Since, $mn < 1$ we can get $(A^* - AN)M^{-1} \in Lip(U)$, $I + (A^* - AN)M^{-1}$ is invertible, where I is the identity operator.

Consequently, it follows that

$$\tilde{M}^{-1} = M^{-1}[I + (A^* - AN)M^{-1}]^{-1}$$

Meanwhile, from $\tilde{M} = M + [A^* - AN]$, $(A^* - AN)M^{-1} \in Lip(U)$, and $M \in \mathcal{U}(W, U)$, obtain

$$\tilde{M} \in \mathcal{U}(W, U)$$

in the context of that the systems shown in Fig.2 is well-posed.

As a result, for any $u \in U$ we have $\omega = \tilde{M}^{-1}u \in W$. Further, combining $y(t) = (N + \Delta N)(\omega(t)), x(t) = BD(\omega(t))$, and $g(t) = A^*(d(t), y(t))$, stability of $A^*, B, N, \Delta N$ and D implies that $y \in Y$, $x \in U$ and $g \in U$. Then, the nonlinear system with perturbation and disturbance shown in Figure 5.1 is robust stability. This completes the theorem proof.

From **Theorem 5.1**, robust stability of the uncertain nonlinear system is guaranteed using the proposed bilinear operator controller. The merit of the proposed method lies in that it utilizes the characteristic of bilinear operator to design two stable integral controller such that the disturbance can be reduced and meantime output maintains. That is, this proposed bilinear operator controller for separating the appearing perturbation and disturbance, which has two time varying input $y(t)$ and $d(t)$. Meantime, by using the designed integral controller to realize reducing the adverse effect by disturbance. Next, we will prove insensitivity property for the uncertain nonlinear system from the input-output view of point.

5.3 Insensitivity analysis for perturbed nonlinear system

In this section, insensitivity property of the uncertain nonlinear system is proved based on the proposed design scheme. When the considered normal plant varies to the perturbation plant, we said that this operator based nonlinear control system is insensitive to the bounded perturbation of the normal plant if the output corresponding to a given input will not blow up. More details on sensitivity definition can be founded in [85], [86]. For simplicity, as for each operator $\mathcal{H} \in \mathcal{N}(J_s, V)$, define $\mu_{\mathcal{H}} = inf\{\langle \mathcal{H}x - \mathcal{H}x_0, x - x_0 \rangle \parallel x - x_0 \parallel^{-2}: x, x_0 \in U, x \neq x_0\} > -\infty$.

5.3.1 Insensitivity property of perturbed system

Further, we consider the general input-output system from the view of the dependence of the output on small variations of the plant due to uncertainties when the input is fixed. Essentially, we call this system insensitive. In the following, the definition on insensitivity will be introduced.

Defination 5.2. Let $[T, P]$ be to describe an input-output system, where J_s is denoted as the set of states,

$$T : W \times U \to \mathcal{N}(J_s, U)$$

and

$$P : W \times U \to \mathcal{N}(J_s, V)$$

and let $\Theta_0 \in W$.

(i) The input-output system $[T, P]$ is called insensitive with respect to $\Theta_0 + \mathcal{P}$ if there exist numbers $r_z > 0$ and $a_z \geq 0$ such that

$$\parallel y - y_0 \parallel \leq a_z \parallel \Theta - \Theta_0 \parallel \tag{5.4}$$

whenever $x, x_0 \in J_s$, $y, y_o \in V$, $\Theta \in \Theta_0 + \mathcal{P}$, $\parallel \Theta - \Theta_0 \parallel \leq r_z$.

(ii) The input-output system $[T, P]$ is called uniformly insensitive with respect to $\Theta_0 + \mathcal{P}$ if there exist fixed constants $r > 0$ and α, $\beta \geq 0$ such that

$$\parallel y - y_0 \parallel \leq (\alpha + \beta \parallel z \parallel) \parallel \Theta - \Theta_0 \parallel \tag{5.5}$$

whenever $z \in U$, $x, x_0 \in J_s$, $y, y_o \in V$, $\Theta \in \Theta_0 + \mathcal{P}$, $\| \Theta - \Theta_0 \| \leq r$.

Furthermore, no matter (i) or (ii), the validity of the equations

$$T(\Theta_0, z)x_0 = 0, \qquad y_0 = P(\Theta_0, z)x_0 \tag{5.6}$$

$$T(\Theta, z)x = 0, \qquad y = P(\Theta, z)x \tag{5.7}$$

all hold, which is accordance with the initial condition of right coprime factorization.

After that, the following lemma is proposed to develop main result on insensitivity of the nonlinear systems with perturbation and disturbance.

Lemma 5.2. Let $T : W \times U \to \mathcal{N}(J_s, U)$, $\mathcal{P} : W \times U \to \mathcal{N}(J_s, V)$ and $\Theta_0 \in W$ be fixed.

If the input-output system can be denoted in the form of $T(\Theta, z)x = T(\Theta)x - z$, also, satisfying the below assumptions:

(i) there exist $v > 0$ and $T(\Theta_0), K \in Lip(J_s, U)$ such that

$$\langle T(\Theta_0)x - T(\Theta_0)x_0, Kx - Kx_0 \rangle \geq v \| x - x_0 \|^2 \tag{5.8}$$

for all $x, x_0 \in J_s$.

(ii) there exists $\lambda > 0$ so that $T(\Theta) - T(\Theta_0) \in Lip(U, U)$ and

$$\| T(\Theta) - T(\Theta_0) \|_0 \leq \lambda \| \Theta - \Theta_0 \| \tag{5.9}$$

whenever $\Theta \in \Theta_0 + \mathcal{P}$.

(iii) there exist $\rho > 0$ and $T(\Theta_0) \in Lip(J_s, U)$ such that

$$\| T(\Theta_0)x - T(\Theta_0)x_0 \| \geq \rho \| x - x_0 \| \tag{5.10}$$

for all $x, x_0 \in J_s$.

Then the system $[T, I]$ is uniformly insensitive with respect to $\Theta_0 + \mathcal{P}$.

Theorem 5.2. *Let* $S^* = (N + \Delta N)D^{-1}B^{-1}$*,* $S_0^* = ND^{-1}B^{-1}$*. and* $x \in U$*,* $S_0^* \in Lip(U)$*,* $S^* \subset S_0^* + \mathcal{P}$*. If* $\mu_{s_0^*} > n$ *and* $\mu_{s_0^*}\mu_{A^*} > -mn$*, whenever*

$T(S^*, z) = (I + A^*(S^*)) - z$, then the uncertain nonlinear systems shown in Figure 5.1 is uniformly insensitive with respect to $S_0 + \mathcal{P}$.

Proof. First from Figure 5.1, see that the input z, x, and output y are related to the equations $x = z - A^*S^*(x)$ and $y = S^*(x)$. Thus, referring to sensitivity definition, the systems can be redescribed as

$$T(S^*, z) = (I + A^*S^*)x - z, \qquad P(S^*, z) = S^* \tag{5.11}$$

and let $T(S^*) = I + A^*S^*$. Referring to **Lemma 5.2**, put $K = S_0^*$, as for x, $x_0 \in U$.

Then, obtain

$$\begin{aligned}
&\langle T(S_0^*)x - T(S_0^*)x_0, Kx - Kx_0\rangle \\
&= \langle (I + A^*S_0^*)x - (I + A^*S_0^*)x_0, S_0^*x - S_0^*x_0\rangle \\
&= \langle x - x_0, S_0^*x - S_0^*x_0\rangle + \langle A^*S_0^*x - A^*S_0^*x_0, S_0^*x - S_0^*x_0\rangle \\
&\geq \mu_{S_0^*} \parallel x - x_0 \parallel^2 + \mu_{A^*} \parallel S_0^*x - S_0^*x_0 \parallel^2 \\
&\geq n(1 + mn) \parallel x - x_0 \parallel^2
\end{aligned} \tag{5.12}$$

where since $\parallel S_0^*x - S_0^*x_0 \parallel \geq \mu_{S_0^*} \parallel x - x_0 \parallel$, $\mu_{s_0^*} > n > 0$ and $\mu_{s_0^*}\mu_{A^*} > -mn > -1$. Hence, based on the definition of sensitivity is satisfied with $n(1 + mn) > 0$.

Also, for each $x \in U$, we have

$$\begin{aligned}
\parallel [T(S^*) - T(S_0^*)]x \parallel &= \parallel [(I + A^*S^*)x - (I + A^*S_0^*)]x \parallel \\
&= \parallel A^*(S^* - S_0^*)x \parallel \\
&\leq \parallel A^* \parallel_0 \parallel S^* - S_0^* \parallel \parallel x \parallel
\end{aligned} \tag{5.13}$$

thus, $\parallel T(S^*) - T(S_0^*) \parallel \leq \parallel A^* \parallel \parallel S^* - S_0^* \parallel$. Hence, the second condition of definition of sensitivity also holds, with $\lambda = \parallel A^* \parallel$. Then the systems $[T, I]$ is insensitive with respect to $S_0^* + \mathcal{P}$.

From Equation (5.5), we can get

$$\| y - y_0 \| = \| x - x_0 \| \leq (\alpha + \beta \| z \|) \| S^* - S_0^* \| \tag{5.14}$$

Based on $P(S^*, z) = S^*$, we can get

$$\begin{aligned}
\| y - y_0 \| &= \| (P + \Delta P)(z(t)) - P(z(t)) \| \\
&= \| P(S^*, z)x - P(S_0^*, z)x_0 \| \\
&\leq \| [P(S^*, z) - P(S_0^*, z)]x \| \\
&\quad + \| P(S_0^*, z)x - P(S_0^*, z)x_0 \| \\
&\leq \| S^* - S_0^* \| \{\| x - x_0 \| + \| x_0 \|\} \\
&\quad + \| S_0^* \|_0 \| x - x_0 \| \\
&\leq (\gamma + \| S_0^* \|)(\alpha + \beta \| z \|) \| S^* - S_0^* \| \\
&\quad + (\sigma + \omega \| z \|) \| S^* - S_0^* \| \\
&\leq (\hat{\alpha} + \hat{\beta} \| z \|) \| S^* - S_0^* \|
\end{aligned} \tag{5.15}$$

where $\hat{\alpha} + \hat{\beta} \| z \| = max\{\sigma + \rho \| z \|, (\gamma + \| S_0^* \|)(\alpha + \beta \| z \|)\}$, and σ, ρ are constants large than zero. By the definition of uniformly insensitivity, the system $[T, P]$ is uniformly insensitive with respect to $S_0^* + \mathcal{P}$. The proof is completed.

In this subsection, insensitivity property of the given nonlinear systems is proved. In terms of the proposed design scheme, we can not only guarantee robust stability for the uncertain nonlinear systems, but also prove insensitivity to the existing perturbation and disturbance. That is, the desired tracking performance can be provided.

5.3.2 Tracking performance

As mentioned above, the uncertain nonlinear systems have been stabilized based on proposed bilinear operator combining robust right coprime factorization and insensitivity property is proved as well. In this subsection, tracking performance will be addressed using the bilinear operator-based right coprime factorization method for the nonlinear systems with perturbation and disturbance by designing a feedforward controller.

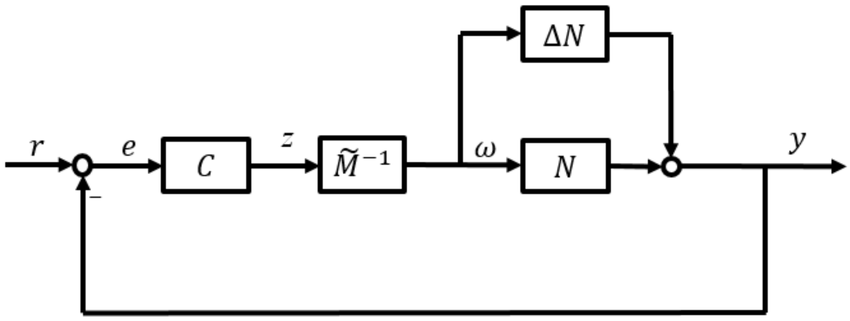

Figure 5.2: Transformed nonlinear feedback control system

After that, the nonlinear systems with perturbation and disturbance is transformed to Figure 5.2. The objective of realizing tracking performance is to design the compensator C. Firstly, denoting $\tilde{P} = N\tilde{M}^{-1}$ and $\tilde{R} = \Delta N\tilde{M^{-1}}$, the nonlinear systems can be expressed as:

$$\begin{cases} e(t) = r(t) - y(t) \\ y(t) = \tilde{P}C(e(t)) + \tilde{R}(z(t)) \end{cases} \tag{5.16}$$

where r denotes the reference input signal, e the error signal between the reference signal r and y, ω denotes the perturbation signal respects to the perturbed operator ΔN, and the output is denoted as y, where y is required to follow the given reference signal r.

It is then easy to see that

$$e(t) = (I + N\tilde{M}^{-1}C)^{-1}(r(t) - \tilde{R}(z(t))) \tag{5.17}$$

In what follows, the inverse of $I + N\tilde{M}^{-1}C$ will be discussed. Moreover, considering the inverse of $I + N\tilde{M}^{-1}C$ is difficult to calculated, we will propose a design scheme by designing a simple and effective operator to transform calculation of inverse in the following theorem.

Theorem 5.3. *In terms of Equation (5.17), if $\| N\tilde{M}^{-1}C \| < 1$, there exists a control operator Q as shown in Equation (5.18), whenever t tends to be infinite such that output tracking performance can be realized for the nonlinear systems with perturbation and disturbance.*

Proof. First, from $\| N\tilde{M}^{-1}C \| < 1$, the inverse of $I + N\tilde{M}^{-1}C$ can be obtained. For any generalized Lipschitz operator $Q \in Lip(V)$ satisfying $FQ \in Lip(V)$ with $\| FQ \| < \frac{1}{2}$, $(I - FQ)^{-1}$ exists and is also in $Lip(V)$, then define $C := Q(I - FQ)^{-1}$, where $F = N\tilde{M}^{-1}$.

Therefore,

$$\begin{aligned} \| N\tilde{M}^{-1}C \| &= \| N\tilde{M}^{-1}Q(I - FQ)^{-1} \| \\ &\leq \| N\tilde{M}^{-1}Q \| \| (I - FQ)^{-1} \| < 1 \end{aligned}$$

which implies that $\| N\tilde{M}^{-1}C \| < 1$ is satisfied. Hence, we observe that $(I + N\tilde{M}^{-1}C)^{-1} = I - FQ$.

Furthermore, the nonlinear operator Q with the time-varying gain is designed as follows,

$$Q(\eta(t)) = \eta(t)E \int_0^t 2e^{-\eta(\tau)} \cdot \sin(\eta(\tau))) \cdot \dot{\eta}(\tau) d\tau \tag{5.18}$$

where $\eta(t)$ is a variable, and $FE = I$.

Denote $e(t) = \Phi(t)$, and $\eta(t) = r - \tilde{R}(z(t))$, according to the defined C, $\Phi(t)$ is shown as follows,

$$\begin{aligned} \Phi(t) &= (I + N\tilde{M}^{-1}C)^{-1}(\eta(t)) \\ &= (I - FQ)(\eta(t)) \\ &= \eta(t) - FQ(\eta(t)) \end{aligned} \tag{5.19}$$

Then, it has that

$$\Phi(t) = \eta(t) - \eta(t)\int_0^t 2e^{-\eta(\tau)} \cdot \sin(\eta(\tau)) \cdot \dot{\eta}(\tau)d(\tau) \tag{5.20}$$

as $t \to \infty$,

$$\int_0^t 2e^{-\eta(\tau)} \cdot \sin(\eta(\tau)) \cdot \dot{\eta}(\tau)d(\tau) \to 1. \tag{5.21}$$

under Equation (5.16),

$$\eta(t) - \eta(t)\int_0^t 2e^{-\eta(\tau)} \cdot \sin(\eta(\tau)) \cdot \dot{\eta}(\tau)d(\tau) \to 0 \tag{5.22}$$

Based on the above analysis, get $e(t) \to 0$, i.e. $y(t) \to r(t)$. The proof is completed.

In this section, bilinear operator-based right coprime factorization method is proposed for the nonlinear systems with perturbation and disturbance. Based on the proposed design scheme, robust stability is guaranteed and insensitivity property is obtained. Meanwhile, output tracking performance of the considered system is realized by using the proposed designed controller C. In next section, a simulation example is given to show effectiveness of the proposed design scheme.

5.3.3 Simulation example

In this section, a numerical example is given to show the effectiveness of the proposed method. As for the plant, assume that $X_B = L_\infty$ is the standard Banach space of real-valued measurable functions defined on $[0, \infty)$, with the associated extended linear space $X^e = L^e_\infty$. Suppose that the nominal plant P without uncertainties is given by the following unstable, time-varying system.

$$P(u(t)) = 2e^t(2t+1)^{-1}u(t) - t(2t+1)^{-2}u(t) - 3$$

Based on the given system, a right factorization $D(\omega(t))$ and $N(\omega(t))$ for $P(u(t))$ are obtained as follows:

$$N(\omega(t)) = 2\omega(t) - \frac{t}{2t+1}e^{-t}\omega(t) - 3$$

$$D(\omega(t)) = (2t+1)e^{-t}\omega(t)$$

In terms of $D(\omega(t))$, $N(\omega(t))$, stability can be verified. Moreover, the inverse operator of $D(\omega(t))$ is unstable.

Next, for establishing a Bezout identity, we pick a stable controller A such that the $I - AN$ is invertible and the two controllers A and B are designed as follows,

$$A(y(t)) = \frac{1}{2}y(t)$$

$$B(u(t)) = \frac{t}{2(2t+1)^2}u(t) + \frac{3}{2}$$

According to the designed controllers, it can be verified that A and B satisfy the Bezout identity. Indeed, we have

$$(AN + BD)(\omega(t)) = I(\omega(t)) \tag{5.23}$$

After that, the perturbations, disturbance, the proposed bilinear operator controller and robust right factorization of the overall systems are given as follows, where the perturbations $\Delta N(\omega(t))$ and $d(t)$ are chosen as $\Delta N(\omega(t)) = \frac{2t}{2t+1}e^{-t}\omega(t)$, $d(t) = (t+1)e^{-2t}$ for confirming the effectiveness of proposed design scheme. The robust right factorization are given as follows,

$$(P + \Delta P)(u(t)) = \frac{2e^t}{2t+1}u(t) + \frac{t}{(2t+1)^2}u(t) - 3$$

$$(N + \Delta N)(\omega(t)) = (2 + \Delta N)\omega(t) - \frac{t}{2t+1}e^{-t}\omega(t) - 3$$

$$D(\omega(t)) = (2t+1)e^{-t}\omega(t)$$

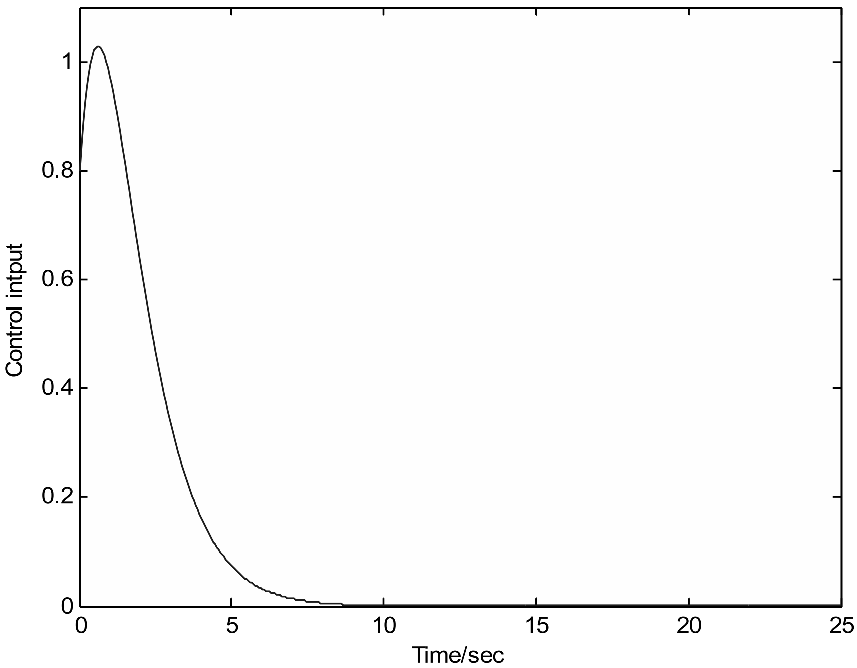

Figure 5.3: Control input of the considered nonlinear plant with uncertainty

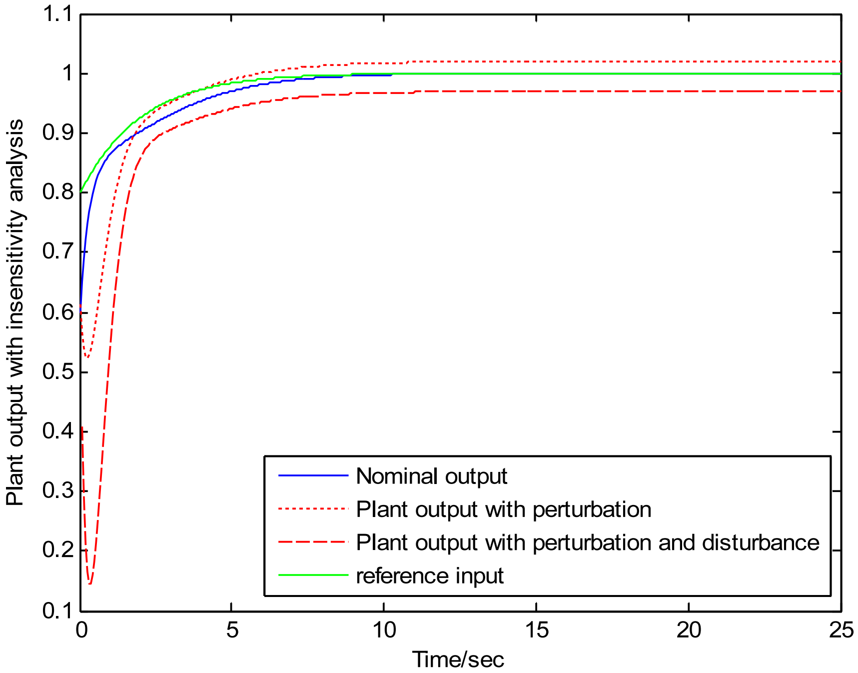

Figure 5.4: Output of the considered plant with perturbation and disturbance by former method

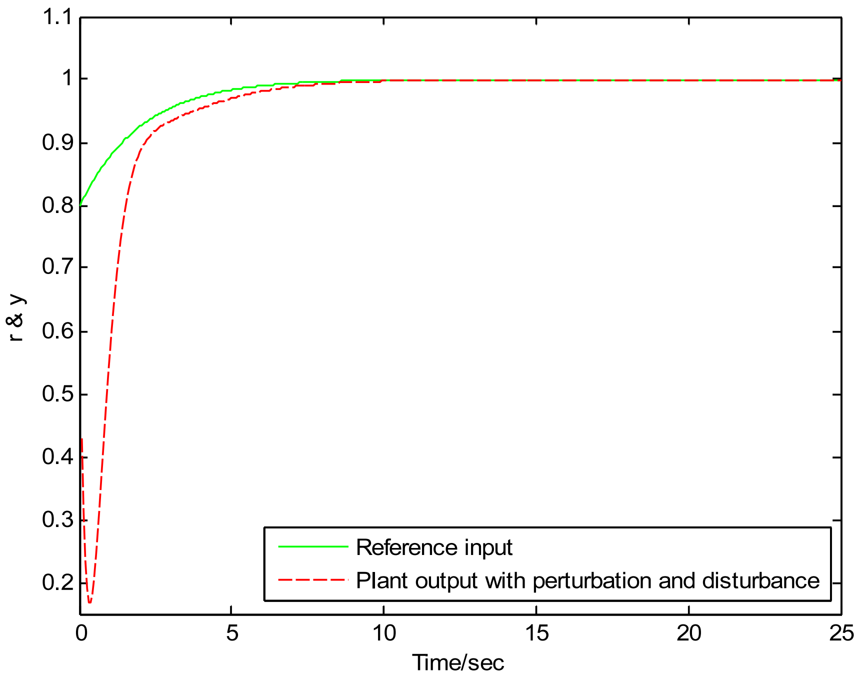

Figure 5.5: Tracking performance of the considered plant with the proposed controller

In this example, the bilinear operator controller is designed show in (5.24), where $K_1(y(t)) = y(t)(t^2+1)^{-\frac{3}{2}}$, $K_2(d(t)) = d(t)2(t+1)^{-1}e^{2t-t^2}$

$$
\begin{aligned}
A^*(y(t), d(t)) = &(te^{-2t}) \int_0^t y(\tau)(\tau^2+1)^{-\frac{3}{2}} d\tau \\
&+ y(t) \int_0^t 2e^{-\tau^2} d\tau
\end{aligned}
\tag{5.24}
$$

The control input of the considered nonlinear plant with uncertainty is shown in Figure 5.3. Based on the design scheme for the proposed nonlinear system, the output of the nominal plant and the output with perturbation and disturbance are shown respectively in Figure 5.4, in which the solid curve show the output of the nonimal plant without any uncertainties, the dotted curve shown that the stability has been realized in the nonlinear system with perturbation by using the robust right coprime factorization method as well as the dashed curve shown that the considered nonlinear system is much more insensitive to the effect of bounded disturbance when the conditions are satisfied. Then the tracking performance is shown in Figure 5.5, where dashed curve shows output of the considered nonlinear plant, solid curve shows the reference input of the considered nonlinear plant chosen as $r(t) = 1 - 0.2e^{-0.5t}$. It is easy to find that the proposed design scheme is effective in reducing the uncertainties, while the system output can asympotically track to the reference input while the system output is stable. Therefore, the simulation results demonstrate the fact that the effectively and feasibility of the proposed design scheme.

5.4 Reset control and robust stability analysis

In this section, the operator-based reset control for nonlinear systems with unknown bounded disturbance is addressed. That is, in the context of operator-based right coprime factorization, reset control is realized and robust stability of nonlinear systems with unknown bounded disturbance is guaranteed.

Firstly, the reset control under the framework of operator-based right coprime factorization for the considered nonlinear systems is presented for developing main results.

The motivated idea of this section lies in how to realize the reset control for the perturbed nonlinear systems shown in Fig. 5.6 based on the right coprime factorization method from the view of point of input-output. In this section, we consider the general nonlinear feedback system by using operator-based reset control combining with right coprime factorization method. Therefore, reset elements including a reset law and a reset operator controller are addressed for the nonlinear system with unknown bounded disturbance satisfying right coprime factorization, whose framework is shown in Fig. 5.6.

Considering that the disturbance is under the framework of robust right coprime factorization. That is, one necessary condition is that the unknown disturbance is supposed to be bounded. As for the detailed information for the assumptions on the disturbance, it can be founded in [35], [37] and [61].

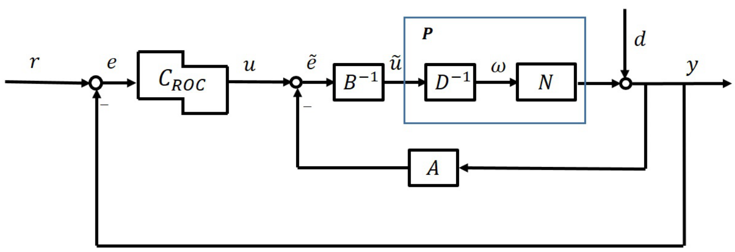

Figure 5.6: The proposed nonlinear system with unknown bounded disturbance

As to the nominal system P, there exist two controllers A and B satisfying with Bezout identity, which guarantees BIBO stability of the nominal system. When disturbance happens in the nominal system, the established Bezout identity cannot hold due to that the feedback signal including adverse effect resulting from the appearing disturbance. For dealing with this issue, a reset controller is proposed to define the triggering reset condition as follows,

$$M = \{Q \in \mathcal{S}(U, Y) : AN + BD \in \mathcal{U}(U, Y)\}$$

Based on the proposed reset controller and operator-based right coprime factorization, the reset operator controller is designed as follows, where K is a stable operator controller.

$$C_{ROC} = \begin{cases} K + & A(y(t)) - AN(\omega(t)) \\ & \text{if AN+BD} \notin \mathcal{U}(U, Y) \\ r(t) & \\ & \text{if AN+BD} \in \mathcal{U}(U, Y) \end{cases} \tag{5.25}$$

Mention that right coprime factorization method based on operator theory is established based on the input-output relationship. Therefore, as for the design of controller K, the feedback signal of $A(y(t))$ is employed. For the design of (5.25), we merely need the feedback signal through $A(N(\omega) + d)$, namely, the signal y, which can be measured by sensor, and without requiring to estimate the unknown bounded disturbance. This is one merit of operator-based right coprime factorization method.

Remarking that the reset law proposed in this section is derived from the preliminary knowledge of the nominal nonlinear systems. That is, based on the known Bezout identity of the nominal nonlinear system, the system is stable. As to reset law, it can be judged by on-line checking A and B to judge the reset law based on the feedback inner loop. When the systems slips away the stability zone, it means the Bezout identity become unsatisfied, which leads to the reset control working.

However, one of the main drawbacks of reset control lies in that stability property of the reset control system is not always guaranteed by the nominal systems, it is well known that the reset action can destabilize a stable feedback system to some extent. Thus, based on the proposed reset control, robust stability of the nonlinear systems with unknown bounded disturbance will be discussed in the following.

In the above analysis, the operator-based reset control is proposed for considering the nonlinear system with unknown bounded disturbance. Therefore, the main result of this section on robust stability properties of operator-based reset control for nonlinear systems with unknown bounded disturbance is proposed.

Theorem 5.4. As for the reset operator controller, if the K is unimodular, then the reset control system shown in Fig.5.6 is guaranteed to be robust stability.

Proof. For guaranteeing robust stability, the reset control system is supposed to guarantee the stable relationship between e and w. Therefore, as the cases where the nominal nonlinear system has disturbance, based on the proposed design scheme shown in Fig. 5.6, obtain

$$\begin{aligned} A(y(t)) + BD(\omega(t)) &= A(N(\omega(t) + d) + BD(\omega(t)) \\ A(N(\omega(t) + d) {+} BD(\omega(t)) &= K(e(t)) \\ &\quad + A(N(\omega(t)) + d) - AN(\omega(t)) \end{aligned} \tag{5.26}$$

From (5.26), derive $A(N(\omega(t)) + BD(\omega(t)) = K(e(t))$. Combining with $A(N(\omega(t)) + BD(\omega(t)) = M(\omega(t))$, obtain

$$K(e(t)) = M(\omega(t))$$

i.e.

$$e(t) = K^{-1}M(\omega(t))$$

Therefore, based on the above relationship, Fig.5.6 can be transformed to be Fig. 5.7, where $\bar{K}^{-1} = M^{-1}K$. Since N and $\bar{K}^{-1}$ are stable, as for the cases where the nominal nonlinear system has disturbance, the perturbed nonlinear system is robust stability.

The proof of the theorem is completed. Based on the given reset law

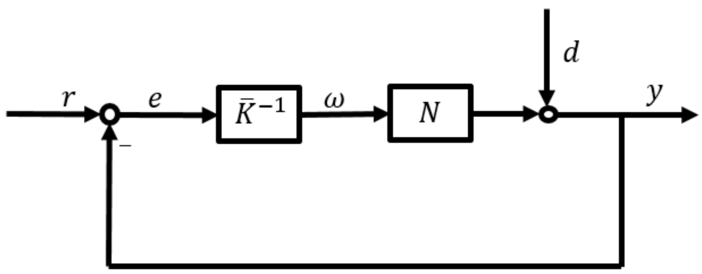

Figure 5.7: Equivalent system

and operator-based right coprime factorization method, the controller K is supposed to be unimodular for guaranteeing robust stability of the perturbed nonlinear systems shown in Fig. 5.6. Similar to former works [35], [37], and [61], we can pick one from the suitable set $S(K)= K : K \in S(U,\, Y),\ K$ is invertible with $K^{-1} \in S(Y, U)$.As to the real practice, the feedback signal through $A(N(\omega) + d)$, namely, the signal $A(y)$, which can be measured by sensor, and without requiring to estimate the unknown bounded disturbance

Mentioned that the robust stability is guaranteed by using the reset control, without using the Lipschitz norm inequation such as [37], [63] and [73]. The proposed method based on the reset control provides a convenient and feasible design scheme for guaranteeing robust stability for the nonlinear system with unknown bounded disturbance. Besides, compared to the former result, this part relax the assumption that disturbance is connected with the internal signal. That is, in this section, the unknown bounded disturbance is not necessary to have a interconnection with the signal $\omega(t)$.

5.4.1 Simulation example

In this section, we will show an example in order to illustrate the effectiveness of the proposed design scheme. Let $C_{[0,\infty)}$ be the space of continuous functions, and $C^1_{[0,\infty)}$ consists of all the functions having a continuous first derivative, both are defined on $[0,\infty)$. The nominal plant and right factorization are given as follows:

$$\begin{aligned} P(\tilde{u}(t)) &= 3(2t+1)\tilde{u}(t) \\ &\quad - \frac{1}{0.2t+1}\sin(t)\tilde{u}(t)\int_0^t (2\tau+1)e^{-\tau}d\tau \end{aligned} \tag{5.27}$$

Therefore, a right factorization is proposed as follows,

$$\begin{aligned} N(\omega(t)) &= 3\omega(t) \\ &\quad - \frac{1}{0.2t+1}\sin(t)\tilde{u}(t)\int_0^t (2\tau+1)e^{-\tau}d\tau \end{aligned} \tag{5.28}$$

$$D(\omega(t)) = \frac{1}{2t+1}\omega(t) \tag{5.29}$$

From the obtained N and D, $P(\tilde{u}(t)) = ND^{-1}(\tilde{u}(t))$ can be confirmed. Next, two controllers $A(y(t))$ and $B(\tilde{u}(t))$ are designed for guaranteeing Bezout identity.

$$A(y(t)) = \frac{1}{3}y(t) \tag{5.30}$$

Therefore, based on the designed controller $A(y(t))$, the feedforward controller $B(\tilde{u}(t))$ can be obtained as follows,

$$B(\tilde{u}(t)) = \frac{1}{3}\sin(t)\int_0^t \tilde{u}(t)e^{-\tau}d\tau \tag{5.31}$$

It can be verified that A and B are satisfied with the Bezout identity. The reference input is chosen as $r(t) = 0.5te^{-2t} + 0.01$. The disturbance is assumed to be $d(t) = te^{-2t}$.Considering the assumed disturbance in the example, and the obtained controllers A and B, obtain the reset operator controller C_{ROC} as follows.

$$C_{ROC} = \begin{cases} \frac{5}{2}r(t) + \frac{1}{3}te^{-2t} + \quad \frac{1}{2} & \\ & \text{if AN+BD} \notin \mathcal{U}(U, Y) \\ 0.5te^{-2t} + 0.01 & \\ & \text{if AN+BD} \in \mathcal{U}(U, Y) \end{cases} \tag{5.32}$$

In order to show effectiveness of the operator-based reset control, more explicitly, simulation results are given in Figure 5.8 and 5.9 as follows, where the reference input r is shown in Figure 5.8, plant output is shown in Figure 5.9. These two simulation results are given for confirming the effectiveness of the proposed controller. Therefore, we can find the proposed design scheme is effective in guaranteeing robust stability of nonlinear system with unknown disturbance.

5.5 Conclusion

In this section, robust stability, sensitivity and tracking performance of nonlinear systems with perturbation and disturbance are considered using bilinear operator-based right coprime factorization. Robust control design scheme was proposed for guaranteeing the nonlinear systems under the context of bilinear operator controller framework as well as under the reset control framework, sufficient condition is discussed for guaranteeing robust stability of the considered nonlinear system. Then, combining with the robust condition and insensitivity property of the considered system, the desired tracking performance was obtained.

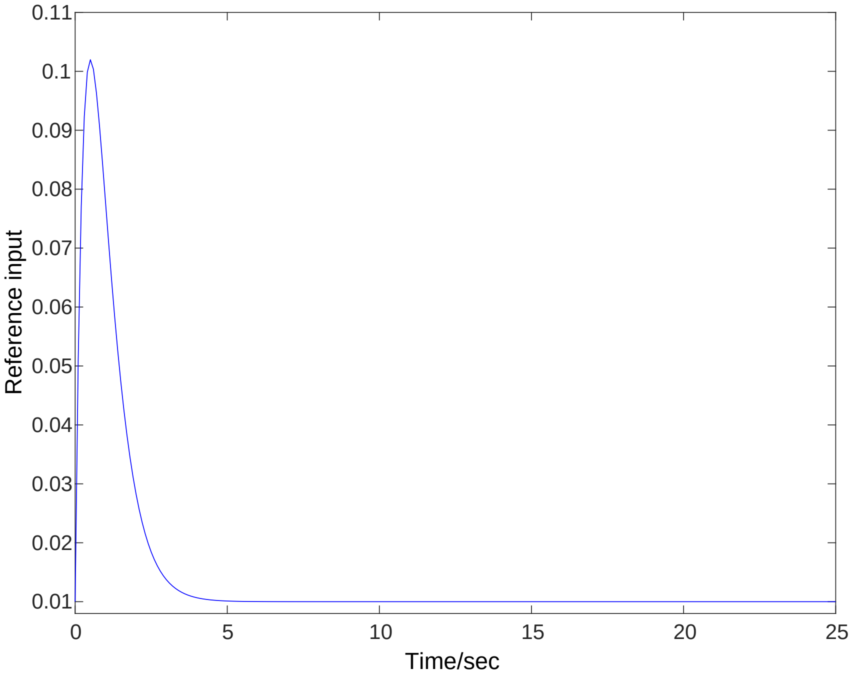

Figure 5.8: Reference input

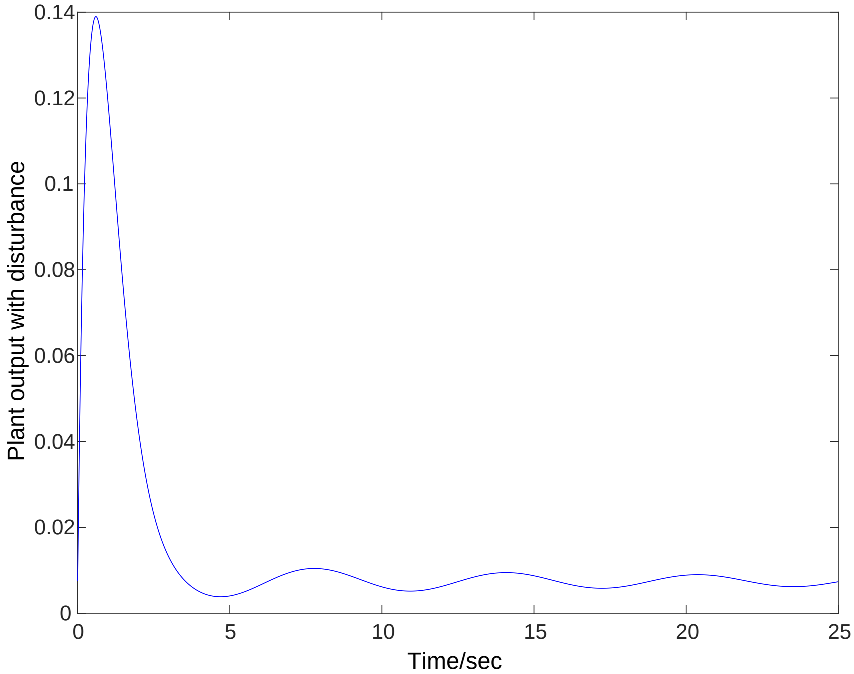

Figure 5.9: Reset control output

Chapter 6

Conclusions

In this dissertation, operator-based nonlinear system with perturbation and disturbance rejection using robust right coprime factorization has been discussed. This dissertation emphasizes analysis design rather than experiment, in the sense that mathematical theoretically, especially as regards construction method based on generalized Lipschitz operator with robust right coprime factorization theorem. The analysis that we proposed mainly embodies the quantitative characteristics and as such should open up a significant insights to assist in system design and analysis. The research essences that we have presented in detail address issues for instance robust stability, internal perturbation and external disturbance rejection, tracking performance and insensitivity analysis.

In Chapter 2, firstly, the mathematical preliminaries including the basic definitions and notations are introduced, which are necessary for developing main results of this dissertation. In details, such as the definition of extended linear spaces which play an foundation role the definition of generalized Lipschitz operators from a normed linear space to another normed linear space of complex-valued functions defined on the time domain serve as the fundamental entities for this dissertation, furthermore, that to be more useful for nonlinear systems control theory and engineering in the considerations of stability, robustness, and copriime factorization and so on. Next for considering nonlinear systems, the concept of right coprime factorization and robust right coprime factorization are described. Moreover, two main sufficient conditions are given in a fairly general operator setting for guaranteeing robust stability of the nonlinear systems with perturbations, which served as the tool of the theoretical basis for developing the main results in this dissertation. Finally, the concerned problems are also summarized in this chapter.

In Chapter 3, supported by some necessary fundamental developments in mathematics have been presented in the previous chapter, this chapter is devoted to study some general theories and strategies in qualitatively for now compensator design of the nonlinear system, in other words, to investigate an effective design scheme of combining right coprime factorization with a new nonlinear operator controller to deal with nonlinear systems with unknown disturbance for guaranteeing robust stability and reducing the adverse effects of unknown disturbance. That is, with the robust right coprime factorization method, the equivalent framework of nonlinear systems is obtained, which provides a convenient viewpoint to consider the above issue; then based on operator theory, for dealing with the unknown disturbance of nonlinear systems to reduce adverse effects on nonlinear systems a general constructive procedure for realizing the object has be discuss in a mathematical formulation viewpoint by providing an equivalent operator controller on the nonlinear systems.

In Chapter 4, nonlinear control systems with external disturbance and internal perturbation are considered by using operator-based robust right coprime factorization for guaranteeing robust stability, rejecting adverse effects resulting from the existing disturbance and perturbation quantitatively and meanwhile, realizing output tracking performance. In detail, firstly, robust stability is guaranteed based on a Lipschitz norm inequation using robust right coprime factorization. Secondly, based on the proposed design scheme, a convenient framework is obtained for discussing rejection issues for external disturbance and internal perturbation. Thirdly, from error signal point of view, the adverse effects resulting from external disturbance and internal perturbation of the nonlinear systems are removed by the designed nonlinear operator. Moreover, output tracking performance is realized using the proposed design scheme simultaneously. Finally, a simulation example is given to confirm effectiveness of the proposed design scheme.

In Chapter 5, nonlinear systems with perturbation and disturbance are discussed by using bilinear operator-based right coprime factorization method. In detail, firstly, for separating the appearing perturbation and disturbance in the system, the bilinear operator controller based on right coprime factorization for nonlinear systems with perturbation and disturbance is established in a special way. we propose a general design scheme for the bilinear operator controller, which can consider quantitatively adverse effect resulting from perturbation and disturbance, respectively, as well as making the proposed design scheme could have a more freedom to be satisfied with practical requirement. Secondly, based on the proposed method, a feasible framework is established for considering robust control, which not only separates perturbation and disturbance, but also provide a fundamental base to design controllers for the considered system. Meantime, sensitivity analysis is given to obtain desired tracking performance. Thirdly, tracking performance is realized by designing a compensator under the proposed framework. as well as simulation examples are given to confirm the effectiveness of the proposed design scheme. Finlay, operator-based reset control for nonlinear systems with unknown bounded disturbance is addressed. That is, in the context of operator-based right coprime factorization, reset control is realized and robust stability of nonlinear systems with unknown bounded disturbance is guaranteed.

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Appendix A

Proof

A.1 Proof of Lemma 2.1

Sufficiency: suppose that S is causal defined in $U_e \to U_e$. Then according to its definition we can get that $P_T S P_T = P_T S$, then, for any $x_1, x_2 \in U_e$ and $T \in [0, \infty)$, $x_{1T} = x_{2T}$, such that

$$\begin{aligned}[S(x_1)]_T = P_T S(x_1) = P_T S P_T(x_1) = P_T S(x_{1T}) = P_T S(x_{2T}) \\ = P_T S P_T(x_2) = P_T S(x_2) = [S(x_2)]_T \end{aligned} \tag{A.1}$$

Necessity: suppose that for any $x_1, x_2 \in U_e$ and $T \in [0, \infty)$, having $x_{1T} = x_{2T}$ implies $[S(x_1)]_T = [S(x_2)]_T$. Fixed a $T \in [0, \infty)$, for any $x_1 \in U_e$, let $x_2 = x_{1T}$, so that $x_{1T} = x_{2T}$, such that $[S(x_1)]_T = [S(x_2)]_T$. Consequently, we have that

$$\begin{aligned}P_T S P_T(x_1) = P_T S(x_{1T}) = P_T S(x_2) \\ = [S(x_2)]_T = [S(x_1)]_T = P_T[S(x_1)] \end{aligned} \tag{A.2}$$

Since $x_1 \in U_e$ and $T \in [0, \infty)$ are arbitrary, it follows that $P_T S P_T = P_T S$ for all $T \in [0, \infty)$, which implies that S is causal.

A.2 Proof of Lemma 2.2

First, note a sufficient condition that for all $x_1, x_2 \in U_e$ and all $T \in [0, \infty)$

$$\| [S(x_1)]_T - [S(x_2)]_T \| \leqslant \| S \|_{Lip} \| x_{1T} - x_{2T} \| \tag{A.3}$$

Hence, $x_{1T} = x_{2T}$ implies that $[S(x_1)]_T = [S(x_2)]_T$ for all $x_1, x_2 \in U_e$ and all $T \in [0, \infty)$.

A.3 Proof of Lemma 2.4

Sufficiency: Since $M \in \mathcal{U}(W, U)$, for any $r \in U_e$, we have

$$r(t) = (AN + BD)w(t)$$

in the sence that $r(t) = M(\omega(t)) \in W_e$. Moreover, since $y(t) = N(w(t))$, $e(t) = BD(w(t))$, and $b(t) = A(y(t)) = AN(w(t))$, as well the stability of A, B, N and D implies that $y \in Y_e$, $e \in U_e$ and $b \in U_e$. Thus, the system is overall stable.

Necessity: First, it drives by the well-posedness and through the path of N and A that $M : W \to U$ is invertible. Then, it can be verified that both M and M^{-1} are stable. As a result, $M \in \mathcal{U}(W, U)$.

A.4 Proof of Lemma 2.5

On account of M is unimodular, so as to it is invertible. Based on Bezout identity

$$AN + BD = M \tag{A.4}$$

$$A(N + \Delta N) + BD = \tilde{M} \tag{A.5}$$

we can get

$$\begin{aligned}\tilde{M} &= M + A(N + \Delta N) - AN \\ &= [I + (A(N + \Delta N) - AN)M^{-1}]M \end{aligned} \tag{A.6}$$

due to $(A(N+\Delta N)-AN)M^{-1} \in Lip(D_e)$, then $I+(A(N+\Delta N)-AN)M^{-1}$ is invertible, where I is the identity operator. Consequently,

$$\tilde{M}^{-1} = M^{-1}(I + A(N + \Delta N)M^{-1} - ANM^{-1})^{-1} \tag{A.7}$$

Meanwhile, since $(A(N + \Delta N) - AN)M^{-1} \in Lip(D^e)$ and $M \in \mathcal{U}(U, Y)$, then $\tilde{M} \in \mathcal{U}(U, Y)$ provided that the system is well-posed. As a result, for any $r \in U_e$, $w(t) = \tilde{M}^{-1}(r(t)) \in W_e$. Further, since $y(t) = (N + \Delta N)(w(t))$, $e(t) = BD(w(t))$ and $b(t) = A(N + \Delta N)(w(t))$, the stability of $A,\ B,\ N,\ D$ and ΔN implies that $y \in Y_e$,$e \in U_e$ and $b \in U_e$. Then, the system is overall stable.

Appendix B

Publications

Journal papers

1. **M. Li** and M. Deng, Operator-Based Nonlinear Control with Unknown Disturbance Rejection, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, Vol. E100-A, No. 4, pp. 982-988, 2017. (DOI: 10.1587/transfun.E100.A.982)

2. **M. Li** and M. Deng, Operator-Based External Disturbance Rejection of Perturbed Nonlinear Systems by Using Robust Right Coprime Factorization, *Transactions of Institute of Measurement and Control*, DOI: 10.1177/0142331217718899, 2017.

3. **M. Li** and M. Deng, Operator-Based Reset Control for Nonlinear System with Unknown Disturbance, *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, 2017. (Accepted)

Proceedings papers

1. **M. Li** and M. Deng, Robust Control and Perturbations Rejection for Nonlinear Systems using Operator-Based Right Coprime Factoriza-

tion, *Proceedings of the 2017 SICE Annual Conference*, pp. 1368-1373, September 19-22, 2017, Kanazawa, Japan.

2. **M. Li** and M. Deng, Sensitivity on Operator-Based Nonlinear Control Design of Uncertain System with Disturbance, *Proceedings of the 2017 International Conference on Advanced Mechatronic Systems*, pp. 423-428, December 6-9, 2017, Xiamen, China.

Other papers

1. **M. Li** and M. Deng, Operator-Based Unknown Disturbance Rejection of Nonlinear Feedback Systems, *IEE Japan Technical Meeting on Control*, pp. 11-16, June 27, 2015, Hiroshima, Japan.