Operator based robust nonlinear control design for an L-shaped arm driven by linear motor

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ABSTRACT

In this dissertation, the operator-based robust nonlinear vibration control is proposed for an L-shaped arm driven by a linear pulse motor. The aim of this study is to allow the motor move fast and reduce the arm vibration by controlling the motion of the motor and the behaviour of the piezoelectric actuator simultaneously.

Much manipulating devices in the industrial system are constructed with flexible arms and driven by motors. When the system works, the vibration of the arm will degrade the system performance. To control the vibrations, there are mainly two active ways. One way is controlling the motor motion such that the vibrations are reduced. Another way uses smart materials as actuator on the flexible arms to suppress the vibrations. In this dissertation, the L-shaped arm pasted with a piezoelectric actuator is driven by a linear pulse motor. It is difficult to control the linear motor and the piezoelectric actuator at same time to meet all the requirements, while the system has uncertainties and hysteresis nonlinearities. The operator-based nonlinear control approach is effective and easy implemented for these nonlinear systems. Therefore, two robust nonlinear controls based on the nonlinear operator control theory are designed to control the motor motion and the behavior of the piezoelectric actuator simultaneously, such that the motor moves fast while the arm vibration is reduced as much as possible.

First, for the L-shaped arm without load, its forced vibration caused by the linear motor is modelled by assuming the arm as two interacted can-
tilever Euler-Bernoulli beams. The first three modes of the arm vibration are considered in the control design, other higher modes are considered as uncertainties which are compensated in the control design. Prandtl-Ishlinskii model is utilized to model the behaviour of the piezoelectric actuator. The model is divided into two parts, one part is linear to be combined into the operator among the control design; the residual part including the hysteresis is to be compensated in the tracking controller such that the stability of the system is guaranteed. Based on the operator-based nonlinear control approach, two controllers are designed to control the system in parallel. One controller controls the motor motion in optimal trajectory while reducing the arm vibration. Another one controls the behaviour of the piezoelectric actuator such that the arm vibration is further reduced. The hysteresis non-linearity of the actuator is compensated in a tracking controller. Simulations are conducted in Matlab comparing with the Proportional-Integral (PI) control to confirm the effectiveness of the proposed control design. The results illustrate that the operator-based control systems designed in this dissertation is effective to reduce the arm vibration while control the motor motion in less time and can guarantee robust stability of the system.

Second, for the arm with unknown load, its forced vibration is modelled as a whole two dimensional Euler-Bernoulli beam. The relationship between the arm vibration, motor motion and the load mass is obtained. By integrating an on-line discrete wavelet transform (DWT) with the operator-based robust control approach, the proposed control for the system is designed. The wavelet transform is utilized to process the real-time arm vibration. By
decomposition, processing and reconstruction, the undesired disturbances in
the arm vibration signal are removed before it is served for the operator-based controller. The operator-based right coprime factorization method is utilized to guarantee the robust stability of the motor-arm system. The piezoelectric actuator is controlled to further reduce the arm vibration. With a modified compensator, the hysteresis of the piezoelectric actuator is compensated in the control design by using a Prandtl-Ishlinskii hysteresis model. The load is estimated by DWT and fast Fourier transform (FFT) method based on the relationship between vibration model of the L-shaped arm and the mass of load, such that the main parameters of the system dynamic is determined. The DWT is used to decompose the arm vibration signal and extract the first mode of the arm vibration, FFT is then used to transform the arm vibration signal from time domain into frequency domain. Simulation results comparing with previous control are demonstrated to validate performance of the proposed control design. The results show that the on-line DWT is effective to remove the influence of some uncertainties and improve the performance of the operator-based control; the load estimation method is workable.

In addition, the main parameters of the L-shaped arm vibration experimental system are identified; the proposed two control designs are programmed into C++ code to test their performances in experiments. For the arm without load, experiments are conducted comparing with the conditional PI control and minimum time control, the results indicate that the proposed control design is effective. For the L-shaped arm with load, experiments are performed comparing with the previous operator-based control.
The results show that the arm vibration is reduced more effectively when the on-line DWT is involved, the performance of the operator-based control is improved and the load mass is estimated.

To sum up, this dissertation proposes two kinds of robust nonlinear system control design for an L-shaped arm driven by a linear pulse motor in two different situations. The operator-based nonlinear optimal vibration control focuses on the finite time motor motion control and arm vibration control for the arm without load, where the nonlinearity of piezoelectric actuator is compensated in the controller. Beside these control tasks, another control design also addresses the situation when the arm is loaded with an unknown mass; the on-line DWT is involved to remove the influence of some unwanted disturbances and estimate the load mass. Both control designs are validated by simulations and experiments, the results show that the proposed control designs meet all the requirements.
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Chapter 1

Introduction

1.1 Background

For the flexible beams or arms, when they are driven or manipulated with speed change, their vibrations are inevitable. In much industrial plants, warehouses ports and transfer robotic systems, the carrying system used for handling objects are designed into different structures to meet different requirements and various occasions. The vibration will influence the manipulation and degrade the system performance \([1, 2]\). Some manipulating devices are designed into two-dimensional or more complicated structures, which aggravates the vibration complexity. To reduce the arm vibration is always a challenging issue in engineering. The countermeasure to reduce arm vibration mainly falls into two categories: passive vibration control and active vibration control. The passive vibration control relies on the optimal structural design and material selection considering the damping features; the well-known measures are using various vibration dampers and absorbers in the system structure. The active vibration control relies on some sen-
sors to feed back the real time vibration states and reducing the unwanted vibration by the anti-force generated by powered actuators [3–6].

The traditional actuating components usually use pneumatic devices, linear motors, electromagnetic devices etc. in vibration control. Thanks to the reciprocal physical characteristic of smart materials including magnetostrictive materials, shape memory alloys and piezoelectric materials etc., they are much commonly used in modern engineering to suppress the structural vibrations. Among them, piezoelectric material is one kind of such materials that produces strain and stress when voltages are applied, and vice versa. Therefore, it has been utilized as sensors and actuators and being studied by much researchers [7–9].

A flexible arm is usually modelled as an infinite dimensional plant, if it is under forced vibration with uncertainties, the dynamics will be a nonlinear plant. It’s difficult to design a stable and robust controller for the nonlinear arm-motor model combined with the nonlinear piezoelectric actuator using common approaches. An operator theory has been studied by much researches [10–15], which can guarantee the bounded-input, bounded-output (BIBO) stability of a nonlinear feedback control system based on robust right coprime factorization. To ensure the robust stability of linear systems, a small gain theorem based simple adaptive control scheme was proposed in [16]. A robust control approach using passivity-based robust right coprime factorization was proposed in [17]. Deng et al. [18] have developed the conditions for the output tracking problem with disturbance, then with continuous extensions, operator-based nonlinear control approach
has become more comprehensive and effective. It has been used in many practical applications [19–24].

The wavelet transform can convert the signal into time-scale domain, has attracted increasing attentions for its ability to extract signal features [25–30]. Wavelet transform has been extended to civil engineering, machining condition monitoring and detection [31–33]. However, most approaches are off-line precess or need the whole length of signals, which limits its on-line applications for real-time control. Some researchers propose on-line or real-time segmented wavelet transform applying for wheel system, rotor and other rotational machines [34–38].

1.2 Current development of arm vibration control

For the arm driven by a motor, there are mainly two active ways to reduce the arm vibration during operation. One way is controlling the motor motion, another way is using smart materials as actuator. In the motor motion control, the system is considered as an underactuated plant, by searching for optimal trajectories of the motor, the arm results in small vibration. Researchers have proposed different approaches for this kind of trajectory planning problem, especially for the overhead crane systems [2, 39–52]. Sun et al. propose a amplitude-saturated output feedback control approach for underactuated crane systems exhibiting double-pendulum effects [75]. For one kind of underactuated systems with rotational-translational actuator, Sun et al. propose a nonlinear continuous control to globally stabilize both
the rotation and translation of the rotational-translational actuator systems influenced by nonvanishing matched disturbances [76]. In this study, the whole system is a Multi-Input Multi-Output (MIMO) plant. For the MIMO nonlinear plants, some operator-based approaches are proposed for tackling the coupling problem in [53–56]. However, these plants have the same number of inputs as outputs, not suitable for the plants with unequal inputs and outputs.

Another way to reduce the arm vibration is using smart materials as actuators. Different approaches have been proposed by researchers in this field. Among them, using piezoelectric material as actuator has been proved to be an effective method for the small size flexible arm [7, 8]. Recently, different control methods have been investigated for the active vibration control with piezoelectric material, such as Proportional-Integral-Differential (PID) control, sliding mode control, optimal control [57–59]. Khorshidi et al [60] studied the active vibration control of plates coupled with piezoelectric layers by using Linear-Quadratic Regulator (LQR) and Fuzzy Logic Controller (FLC). With these methods, by controlling the input voltages, the piezoelectric material acts as a actuator generating opposite moment to suppress the arm vibration. However, the piezoelectric actuator has a nonlinear property called hysteresis, the output error will lower the performance of the piezoelectric actuator. To represent the hysteresis behavior and control the piezoelectric actuator accurately, some models have been proposed, including Duhem model, Dahl model, Bouc-Wen model, Preisach model and Prandtl-Ishlinskii model [9, 61–73, 81]. The Prandtl-Ishlinskii model is commonly
1.2. Current development of arm vibration control

utilized to describe hysteresis nonlinearity using stop hysteresis operators or play hysteresis operators because of its simplicity, accuracy and ease of implementation. However, these operators are symmetric, which will still result in compensation error. Al Janaideh et al [82] proposed an analytic inverse of a generalized Prandtl-Ishlinskii model, it can be conveniently implemented as a real-time feed-forward compensator to compensate for hysteresis nonlinearities. A non-symmetric Prandtl-Ishlinskii hysteresis model with unknown slopes was given to describe the hysteresis in [73], the nonlinear compensator based on this model can compensate the hysteresis more effectively.

An operator-based nonlinear control method has been given to control the vibration of a flexible arm using the piezoelectric actuator [86]. However that paper only studied the free vibration of a simple uniform clamped-free beam and without considering the hysteresis of actuator. For controlling the free vibration of an aircraft-tail-like plate using the piezoelectric actuator, an operator-based nonlinear system control technique has been given by Katsurayama et al. [74], the lower order modes of vibration are considered. However, that study only considered the free vibration the plate without driving source control.

The L-shaped arm considered in this dissertation is forced vibration and vibrating in two dimensions which is more complicated and difficult to be controlled. Moreover, the linear motor control system has two outputs and only one control input, namely, it is an underactuated nonlinear MIMO plant. To deal with this kind of problem, many control strategies have been proposed, especially for the overhead crane systems, such as the adaptive control,
sliding-mode control, fuzzy control and optimal control. Wen et al. [52] used data-based support vector machine (SVM) to estimate the swing angle, found an optimal trajectory to reduce the payload swing using operator-based control. However the control object in that paper is a payload driven by a linear motor with cable, the swaying of the payload is much more slowly and quite different from the arm vibration. As the L-shaped arm has a more complicated vibration dynamics, it is difficult for the SVM to learn. Therefore the control method in that paper is not suitable for this L-shaped arm vibration.

1.3 Motivations of the dissertation

Scaled down from a real industrial transported system, a flexible L-shaped arm driven by a linear pulse motor is studied in this dissertation. When the system is working, vibration of the arm is inevitable. It is necessary to seek appropriate methods to control the motor motion while reduce the arm vibration at the same time.

Motivated by the optimal motor motion control for the underactuated system, this dissertation intend to set up an optimal control method for the linear motor. The motor is controlled with optimal trajectory resulting in less time consumption and less arm vibration. It means that the arm vibration status will be measured by sensors and feed back into the motor motion control. The main difficulty is to meet two output requirements with one control input while keep the system stable and robust.

Motivated by the superior advantages of the piezoelectric material, this dissertation intend to use piezoelectric material as sensors and actuator, to
1.3. Motivations of the dissertation

measure and suppress the arm vibration. Prandtl-Ishlinskii model is used to model the hysteresis of the piezoelectric actuator and modify it according to the control design method, its nonlinearity will be considered to be compensated to improve the control performance.

Motivated by the superior advantages of the operator-based nonlinear control method, this dissertation intend to use the method to factorize the system model and design optimal controllers to control the linear motor motion and reduce the L-shaped arm vibration at the same time. The system has two control inputs including driving force of linear pulse motor and voltage for the piezoelectric actuator, three outputs including the moving distance of linear motor and the vibrations of two parts of arm. In the above mentioned papers, the arm was simple cantilever type without load. In practice, the arm manipulates different loads, which will determine the parameters in the controller relating to the plant dynamics. It is inconvenient to measure the load mass in advance. Therefore, an automatic load mass estimation combined with the controller is necessary. In addition, the operator-based control uses the outputs of system directly as feedback signals for the controller, the system performance is influenced by the quality of the signal. The disturbances and uncertainties will lower the control performance. For this end, the output signal should be processed by appropriate measures, wavelet transform can undertake such tasks. An on-line DWT is to be proposed to use in the operator-based nonlinear control, working together for the nonlinear L-shaped arm vibration system. Operator-based right coprime factorization method is used to guarantee the stability and robustness of the
system. The on-line DWT is constructed to estimate the unknown load, remove the influence of some uncertainties and improve the performance of the operator-based control.

In summary, this dissertation intend to use operator-based nonlinear control approach and on-line DWT in control design for actively control the L-shaped arm vibration system, and validate the control design in simulation and experiment. The aim of this research is to allow the motor move fast and reduce the arm vibration as much as possible while keep the system stable and robust.

1.4 Contributions of the dissertation

For the L-shaped arm experiment system control design, the difficulty is controlling the linear motor and the piezoelectric actuator at same time to meet all the requirements, while the system has uncertainties and hysteresis nonlinearities. We design two controllers based on the nonlinear operator control theory to accomplish it. One controller allows the fast movement to destination while reducing the vibration of the arm, the other one controls the piezoelectric actuator to further reduce the vibration. The main contributions of this study are summarized as follows.

(1) The vibration of the Motor-Arm system with and without load is modelled.

For the L-shaped arm without load, its vibration with linear motor is modelled by assuming the arm as two interacted cantilever Euler-Bernoulli beam. For the arm with unknown load, the arm is modelled as a two dimen-
1.5. Organization of the dissertation

sional Euler beam by mechanical analysis based on Euler-Bernoulli theory, and the relationship between the load and the vibration mode is given. By introducing a evaluation index, the dominant modes of the arm vibration are considered in the control design, other higher modes are considered as uncertainties compensated in the control design.

(2) The load estimation method is given by using DWT and FFT.

Based on the vibration model of the L-shaped arm with load, the relationship between the load mass and the natural frequency of the arm vibration is obtained. When the arm vibration is measured in time domain signal, after decomposing by the DWT, the first mode vibration is isolated and transformed into frequency domain by FFT. The load is estimated using the obtained relationship, such that the main parameters of the system dynamic is determined.

(3) Two operator-based nonlinear controllers for motor motion and arm vibration are designed employing a short-symmetrical on-line DWT.

One operator-based robust nonlinear control is proposed for the linear pulse motor to be fast while reducing the arm vibration. Another operator-based robust nonlinear control with hysteresis compensation is proposed for the piezoelectric actuator to further reduce the arm vibration. When the L-shaped arm has unknown load, a short-symmetrical on-line DWT is used in the control design so as to reduce the impact of disturbances and uncertainties and to estimate the unknown load mass.
1.5 Organization of the dissertation

Beginning with theoretical preliminaries and problem statement, this dissertation is organized as follows.

In Chapter 2, some fundamental definitions and the theoretical basis are provided for the system modelling and control design in this dissertation. Euler-Bernoulli beam theory is utilized to model the flexible arm vibration; Prandtl-Ishlinskii model is used to model the hysteresis of the piezoelectric actuator. Some fundamental definitions of operator theory are introduced, the operator-based nonlinear control approach and the discrete wavelet transform are given. Based on the background and these theories, the problem to be studied in this dissertation is stated.

In Chapter 3, the forced vibration of the L-shaped arm driven by a linear pulse motor is modelled by considering it as two connected Euler-Bernoulli beams. Based on the operator-based nonlinear control approach, two controllers are designed to control the system in parallel. One controller aims to control the motor motion in optimal trajectory while reducing the arm vibration. Another one aims to control the behaviour of the piezoelectric actuator to further reduce the arm vibration. The hysteresis nonlinearity of the actuator is compensated in a tracking controller. Simulation is conducted to confirm the effectiveness of the proposed control design.

In Chapter 4, the L-shaped arm is modelled as a two dimensional Euler-Bernoulli beam and an unknown load is considered in the model. An on-line DWT is constructed to remove the influence of some uncertainties and im-
prove the performance of the operator-based control. Involving the on-line DWT, two operator-based controllers are proposed. One is to control the motor motion resulting in less arm vibration. Another one is to further reduce the arm vibration by using a piezoelectric actuator. Simulations comparing with previous control are demonstrated to validate performance of the proposed control design. The load estimation method using FFT and DWT is given based on the system model.

In Chapter 5, the structure of the L-shaped arm vibration experimental system is introduced, the main parameters of the devices are identified. Using the operator-based nonlinear optimal control proposed in Chapter 3, experiments are conducted comparing with the conditional PI control to test the experimental performance of the control design. Using the operator-based nonlinear control with on-line DWT proposed in Chapter 4, comparative experiments are conducted for the L-shaped arm with unknown load to validate the load estimation method and test the performances of the control designs correspondingly.

In Chapter 6, the main work of this dissertation is summarized, the results are discussed and the problem stated in Chapter 2 is concluded. Based on the performance of the proposed control design, the main contributions of this dissertation are conformed and reiterated in detail. The limitations and future works of this dissertation are provided for the probable researches in this field.
Chapter 2

Preliminaries and problem statement

2.1 Introduction

This chapter provides the mathematical and theoretical basis for the system modelling and control design in the following chapters of this dissertation. It also specify the aims and objectives of this research.

In Section 2.2, the dynamics of flexible arm vibration is introduced, and its modelling method based on Euler-Bernoulli beam theory is provided.

In Section 2.3, the dynamics of the piezoelectric actuator is introduced, the hysteresis of the piezoelectric actuator is modelled using a Prandtl-Ishlinskii hysteresis model.

In Section 2.4, the fundamental definitions of operator theory are introduced, the operator-based nonlinear control approach is outlined.

In Section 2.5, the theoretical basis of wavelet transform is given, some related definitions of DWT is introduced.

In Section 2.6, the problem to be studied in this dissertation is stated.
CHAPTER 2. PRELIMINARIES AND PROBLEM STATEMENT

Primarily, the framework of modelling the L-shaped arm vibration and hysteresis of the piezoelectric actuator, operator-based control design utilizing DWT is outlined.

2.2 Flexible arm vibration model

If a thin uniform arm has external distributed transverse force $q(x, t)$ on it, shown as in Fig. 2.1(a), take any element of the beam with length $dx$ as object, its free-body diagram is shown in Fig. 2.1(b), where $V(x, t)$ is the shear force, $M(x, t)$ is the bending moment, $\rho$ is the mass density.

Figure 2.1: Transverse vibration of a uniform thin arm

Neglecting the torsional vibration, based on the Euler-Bernoulli theory
2.3 Model of piezoelectric actuator

[77], force and moment equilibrium, the forced transverse vibration of the uniform arm is obtained as:

\[ E_a I \frac{\partial^4 w}{\partial x^4} + c_s I \frac{\partial^2 y(x, t)}{\partial x^4 \partial t} + \rho S S \frac{\partial^2 w}{\partial t^2} = q(x, t) \quad (2.1) \]

where \( q(x, t) \) is the external distributed forces on the arm, including the linear motor driving force and the piezoelectric actuator moment. \( w \), standing for \( w(x, t) \), is the transverse displacement along the neutral axis of the arm. \( E_a, I, \rho, S, c_s \) are the Young’s modulus, moment of inertia, density, cross-sectional area and strain-rate damping coefficient of the arm, respectively.

The equation (2.1) can be solved using the boundary conditions and the arm’s initial conditions.

### 2.3 Model of piezoelectric actuator

For the piezoelectric actuator pasted on an arm, when voltages are applied, the actuator will generate moments and deform the arm in a certain way. By controlling input voltage, the actuator is utilized to reduce the arm vibration. However, the relationship between the output moment and the control input is a nonlinear process, because the piezoelectric actuator has a hysteresis nonlinear property. To represent the hysteresis behavior and control the piezoelectric actuator accurately, some models have been proposed, including Duhem model, Dahl model, Bouc-Wen model, Preisach model and Prandtl-Ishlinskii model. The Prandtl-Ishlinskii model is commonly utilized to describe hysteresis nonlinearity using stop hysteresis operators or play hysteresis operators because of its simplicity, accuracy and ease of imple-
CHAPTER 2. PRELIMINARIES AND PROBLEM STATEMENT

mentation. To consider this effect in the system model, in this dissertation, the Prandtl-Ishlinskii hysteresis model based on the play hysteresis operator is utilized, represented as follows.

\[ M_p(t) = D_{PI}(u(t)) + \Delta_{PI}(u(t)) \] (2.2)

The output of piezoelectric actuator is represented as two terms. The first term \( D_{PI} \) is an invertible operator for certain parameters, the residual term \( \Delta_{PI} \) is the nonlinear part of the model, which will change with the input \( u(t) \) and be influenced by the design parameters, it needs to be compensated by the controller. The details of \( D_{PI} \) and \( \Delta_{PI} \) are shown as follows.

\[ D_{PI}(u(t)) = Ku(t) = u(t) \int_0^H p(h)dh, \] (2.3)

\[ \Delta_{PI}(u(t)) = -\int_0^{h_x} S_n p(h)dh + \int_{h_x}^H p(h)[F_h(u(t_i)) - u(t)]dh, \] (2.4)

\[ S_n = \text{sign}(u(t) - F_h(u(t_i))) \]

\[ F_h(u(t)) = \begin{cases} 
    u(t) + h & u(t) \leq F_h(u(t_i)) - h \\
    F_h(u(t_i)) - h < u(t) - F_h(u(t_i)) < h & u(t) - h \\
    u(t) \geq F_h(u(t_i)) + h & \end{cases} \]

\[ t_i < t \leq t_{i+1}, \ 0 \leq i \leq N - 1 \]

\[ 0 = t_0 < t_1 < \cdots < t_N = t_E, \ [0, t_E]. \] (2.5)

where, \( u(t) \) and \( h \) are the input voltage and the threshold of play hysteresis operator, respectively. The initial condition is given by \( F_h(u(0)) = \max(u(0) - h, \min(u(0) + h, (u_1)^*)) \). \( h_x \) is the upper limit of \( h \) that satisfies \( h \leq |u(t) - F_h(u(t_i))| \). \( p(h) \) is the density function satisfying \( p(h) \geq 0 \) with \( \int_0^\infty hp(h)dh < \infty \) to be determined through experiments.
2.4 Operator-based nonlinear control approach

2.4.1 Definitions of spaces

In this dissertation, the operator theory is based on several spaces in mathematics, which are defined as follows.

**Normed linear space:**

Denote a space $X$ of time functions, it is said to be a vector space if it is closed under addition and scalar multiplication. It is said to be *normed* if each element $x$ in $X$ is endowed with norm $\| \cdot \|_X$, satisfying the following conditions:

1) $\|x\| > 0$, if $x \neq 0$.

2) $\|ax\| = |a|\|x\|$.

3) $\|x_1 + x_2\| \leq \|x_1\| + \|x_2\|$.

**Banach space:**

A Banach space is a vector space $X$ over the real or complex numbers with a norm $\| \cdot \|$ such that every Cauchy sequence (with respect to the metric $d(x, y) = \|x - y\|$) in $X$ has a limit in $X$. Many spaces of sequences or functions are infinite dimensional Banach spaces.

**Extended linear space:**

Let $Z$ be the family of real-valued measurable functions defined on $[0, \infty)$, which is a linear space. For each constant $T \in [0, \infty)$, let $P_T$ be the Pro-
jection operator mapping from $Z$ to another linear space, $Z_T$, of measurable functions such that
\[
f_T(t) := P_T(f)(t) = \begin{cases} 
  f(t), & t \leq T \\
  0, & t > T
\end{cases}
\] (2.6)
where, $f_T(t) \in Z_T$ is called the truncation of $f(t)$ with respect to $T$. Then, for any given Banach space $X$ of measurable functions, if
\[
X^e = \{ f \in Z : \| f_T \|_X < \infty, \text{for all } T < \infty \},
\] (2.7)
the space $X^e$ is called the extended linear space associated with the Banach space $X$.

This dissertation uses the extended linear space because the control signals are finite time-duration in experiments.

### 2.4.2 Definitions of operators

Let $U$ and $Y$ be linear spaces over the field of real numbers, and let $U_s$ and $Y_s$ be normed linear subspaces, called the stable subspaces of $U$ and $Y$, respectively.

**Operator:**

An operator $Q : U \rightarrow Y$ is a mapping defined from input space $U$ to the output space $Y$. The operator $Q$ can be expressed as $y(t) = Q(u)(t)$ where $u(t)$ is the element of $U$ and $y(t)$ is the element of $Y$.

**Invertible:**

An operator $Q$ is said to be invertible if there exists an operator $P$ such that
\[
Q \circ P = P \circ Q = I.
\] (2.8)
2.4. Operator-based nonlinear control approach

$P$ is called the inverse of $Q$ and is denoted by $Q^{-1}$, where, $I$ is identity operator, and $Q \circ P$ (or simply $Q(P(\cdot))$ or $QP$) is an operation satisfying

$$\mathcal{D}(Q \circ P) = P^{-1}(\mathcal{R}(P) \cap \mathcal{D}(Q)). \quad (2.9)$$

**Unimodular operator:**

Let $\mathcal{S}(U, Y)$ be the set of stable operators mapping from $U$ to $Y$. Then, $\mathcal{S}(U, Y)$ contains a subset defined by

$$\mathcal{U}(U, Y) = \{ M : M \in \mathcal{S}(U, Y), \ M \text{ is invertible with } M^{-1} \in \mathcal{S}(Y, U) \}. \quad (2.10)$$

Elements of $\mathcal{U}(U, Y)$ are called unimodular operators.

**Lipschitz operator:**

For any subset $D \subseteq U$, let $\mathcal{F}(D, Y)$ be the family of nonlinear operators $Q$ such that $\mathcal{D}(Q) = D$ and $\mathcal{R}(Q) \subseteq Y$. Introduce a (semi)-norm into (a subset of) $\mathcal{F}(D, Y)$ by

$$\|Q\| := \sup_{\substack{x, \tilde{x} \in D \\text{ s.t. } x \neq \tilde{x}}} \frac{\|Q(x) - Q(\tilde{x})\|_Y}{\|x - \tilde{x}\|_U}$$

if it is finite. In general, it is a semi-norm in the sense that $\|Q\| = 0$ does not necessarily imply $Q = 0$. In fact, it can be easily seen that $\|Q\| = 0$ if $Q$ is a constant operator (need not to be zero) that maps all elements from $D$ to the same element in $Y$.

Let $\text{Lip}(D, Y)$ be the subset of $\mathcal{F}(D, Y)$ with its all elements $Q$ satisfying $\|Q\| < \infty$. Each $Q \in \text{Lip}(D, Y)$ is called a Lipschitz operator mapping from $D$ to $Y$, and the number $\|Q\|$ is called the Lipschitz semi-norm of the
operator $Q$ on $D$. A Lipschitz operator is bounded and continuous on its own domain.

**Generalized Lipschitz operator:**

Let $U^e$ and $Y^e$ be extended linear spaces associating respectively with two given Banach spaces $U$ and $Y$ of measurable functions defined on the time domain $[0, \infty)$, and let $D$ be a subset of $U^e$. A nonlinear operator $Q : D \to Y^e$ is called a generalized Lipschitz operator on $D$ if there exists a constant $L$ such that

$$
\|[Q(x)]_T - [Q(\tilde{x})]_T\|_Y \leq L\|x_T - \tilde{x}_T\|_U \quad (2.11)
$$

for all $x, \tilde{x} \in D$ and for all $T \in [0, \infty)$. Note that the least such constant $L$ is given by the norm of $Q$ with

$$
\|Q\|_{\text{Lip}} := \|Q(x_0)\|_Y + \|Q\| = \|Q(x_0)\|_Y + \sup_{T \in [0, \infty)} \sup_{x, \tilde{x} \in D, x_T \neq \tilde{x}_T} \frac{\|[Q(x)]_T - [Q(\tilde{x})]_T\|_Y}{\|x_T - \tilde{x}_T\|_U} \quad (2.12)
$$

for any fixed $x_0 \in D$.

Based on (2.12), it follows immediately that for any $T \in [0, \infty)$

$$
\|[Q(x)]_T - [Q(\tilde{x})]_T\|_Y \leq \|Q\|\|x_T - \tilde{x}_T\|_U \leq \|Q\|_{\text{Lip}}\|x_T - \tilde{x}_T\|_U. \quad (2.13)
$$

**Lemma 2.1** Let $U^e$ and $Y^e$ be extended linear spaces associating respectively with two given Banach spaces $U$ and $Y$, respectively, and let $D$ be a subset
2.4. Operator-based nonlinear control approach

of $U^e$. The following family of Lipschitz operators is a Banach space:

$$\text{Lip}(D, Y^e) = \left\{ Q : D \to Y^e \left| \|Q\|_{\text{Lip}} < \infty \right. \right\}.$$  \hfill (2.14)

**Bounded input bounded output (BIBO) stability:**

Let $Q$ be a nonlinear operator with its domain $\mathcal{D}(Q) \subseteq U^e$ and range $\mathcal{R}(Q) \subseteq Y^e$. If $Q(U) \subseteq Y$, $Q$ is said to be input output stable. If $Q$ maps all input functions from $U_s$ into the output space $Y_s$, that is $Q(U_s) \subseteq Y_s$, then operator $Q$ is said to be bounded input bounded output (BIBO) stable or simply, stable. Otherwise, if $Q$ maps some inputs from $U_s$ to the set $Y^e \setminus Y_s$ (if not empty), then $Q$ is said to be unstable. For any stable operators defined following in this dissertation stands for BIBO stable.

### 2.4.3 Right coprime factorization

Represent a nonlinear time varying system with uncertainties as operator $P + \Delta P : U \to Y$. Where $P$ is the nominal plant, $\Delta P$ stands for the uncertainties, $U$ and $Y$ denote the input and output space of the plant.

**Right factorization:**

By introducing an intermediate variable $\omega \in W$, $W$ is called a quasi-state space of $P$, the input and output of the operator $P$ are expressed as $y = N(\omega)$ and $u = D(\omega)$. If $D$ is invertible, $\omega(t) = D^{-1}(u)(t)$, then $P(u)(t) = N(\omega)(t) = ND^{-1}(u)(t)$; if further $N$ and $D$ are two stable operators, the operator $P$ is said to have a right factorization, as shown in Fig. 2.2.
CHAPTER 2. PRELIMINARIES AND PROBLEM STATEMENT

**Right coprime factorization:**

After right factorization of a plant $P$ into $(N, D)$, if two operators $A$ and $B$ satisfy the following Bezout identity, the factorization is said to be right coprime factorization.

$$AN + BD = M$$  \hspace{10cm} (2.15)

Where $B$ is invertible and $M \in \mathcal{U}(W,U)$ is a unimodular operator. The block diagram of the right coprime factorization of a nonlinear system $P$ is shown in Fig. 2.3. Fig. 2.3 is therefore to be said as a operator-based feedback control for the nonlinear plant $P$, and the operators $A$ and $B$ serve as controllers.

It is worth to mention that the initial state should also be considered, namely, $AN(w_0,t_0) + BD(w_0,t_0) = M(w_0,t_0)$ should be satisfied. In this
dissertation, \( t_0 = 0 \) and \( w_0 = w_0(t_0) \) are selected.

**Well-posedness:**

The feedback control system shown in Fig. 2.3 is said to be well-posed, if for every input signal \( r \in U \), all signals in the system (i.e., \( e, u, w, b \) and \( y \)) are uniquely determined.

**Overall stable:**

The feedback control system shown in Fig. 2.3 is said to be overall stable, if \( r \in U_s \), implies that \( u \in U_s, y \in V_s, w \in W_s, e \in U_s \) and \( b \in U_s \).

**Lemma 2.2** Assume that the system shown in Fig. 2.3 is well-posed. If the system has a right factorization \( P = ND^{-1} \), then the system is overall stable if and only if the operator \( M \) in (2.15) is a unimodular operator.

**Robustness:**

For a nonlinear plant \( \tilde{P} \), it is represented as a nominal plant \( P \) and bounded uncertainty \( \Delta P \), and \( \tilde{P} = P + \Delta P \). The right factorization of the nominal plan \( P \) and the overall plant \( \tilde{P} \) are

\[
P = ND^{-1}
\]  

(2.16)

and

\[
P + \Delta P = (N + \Delta N)D^{-1}
\]  

(2.17)

where \( N, \Delta N, \) and \( D \) are stable operators, \( D \) is invertible, \( \Delta N \) is unknown but the upper and lower bounds are known. According to **Lemma 2.2**, if
the following Bezout identity is satisfied,

$$A(N + \Delta N) + BD = \tilde{M} \quad (2.18)$$

and $\tilde{M}$ is a unimodular operator, the the nonlinear feedback control system is said to be BIBO stable.

With the determined operators $A$ and $B$, if they further satisfy the following condition,

$$\| [A(N + \Delta N) - AN] M^{-1} \|_{\text{Lip}} < 1 \quad (2.19)$$

then the robustness of the uncertain system is guaranteed, where $\| \cdot \|_{\text{Lip}}$ is a Lipschitz operator norm. The robust feedback control system is shown in Fig. 2.4.

Figure 2.4: Nonlinear feedback control system with uncertainties

Lemma 2.3 Let $U^e_s$ be a linear subspace of the extended linear space $U^e$ associated with a given Banach space $U_B$, and let $(A(N - \Delta N) + AN) M^{-1} \in \text{Lip}(U^e)$). With the Bezout identity of the nominal plant and the overall plant $AN + BD = M \in U(W, U)$, $A(N + \Delta N) + BD = \tilde{M}$, respectively. If

$$\| (A(N + \Delta N) - AN) M^{-1} \| < 1 \quad (2.20)$$

then the system shown in Fig. 2.4 is said to be robust stable.

For more details about the operator theory, please refer to [15, 18, 21, 24].

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2.5 Fundamental theories on discrete wavelet transform

2.5.1 Wavelet transform

Wavelet transform is performed by using wavelet functions for time domain signal, the wavelet function is usually expressed in the following form.

\[ \Psi_{a,b} = \frac{1}{\sqrt{b}}\psi\left(\frac{t-a}{b}\right) \]  

(2.21)

Where \( a \) is the shifting parameter and \( b \) is the scaling parameter.

For a time domain signal \( x(t) \), the one-dimensional continuous wavelet transform (CWT) is a convolution of \( x(t) \) and the complex conjugate of a wavelet function, which can be expressed as follows.

\[ \text{cwt}(a,b) = \frac{1}{\sqrt{b}} \int x(t)\psi^*(\frac{t-a}{b}) \]  

(2.22)

where \( \psi^*(\cdot) \) is the complex conjugate of a wavelet function \( \psi(\cdot) \).

2.5.2 Discrete wavelet transform

CWT need a large amount of computation and resources, which is not suit for the control in practice. It is needed to transit from CWT to discrete wavelet transform (DWT). By insinuate the parameters \( a \) and \( b \) with a dyadic scales, the DWT is changed in the follow form.

\[ \psi_{m,k}(t) = 2^{-m/2}\psi(2^{-m}t - k) \]  

(2.23)

where \( m \) is the scale parameter and \( k \) is the shift parameter, both which are integers. The filter bank includes a cascading low-pass filter \( g \) and a high-
pass filter $h$. A sampled signal $x$ is decomposed by passing the filters. At one certain decompose level, the outputs giving the detail coefficients (from the high-pass filter) and approximation coefficients (from the low-pass) as follows.

\[ y_{\text{low}}[n] = \sum_n x[k]g[2n - k] \]

\[ y_{\text{high}}[n] = \sum_n x[k]h[2n - k] \] (2.24)

The number of decomposition levels are decided by the frequency band of every level and the frequency frequency of desired signal. The frequency band for the approximation $AC_j$ and the detail $DC_j$ at level $j$ are given by

\[ 0 \leq f_{AC_j} \leq \frac{f_s}{2^{j+1}} \]

\[ \frac{f_s}{2^{j+1}} \leq f_{DC_j} \leq \frac{f_s}{2^j} \] (2.25)

where $j$ is the wavelet decomposition level, $f_s$ is the signal sampling frequency. If the reconstruction decomposition level, $f_s$ is the signal sampling frequency. If the reconstruction level is determined by (2.25), by selecting the gains $K_{a1}, K_{dj}$ ($j = 1, 2, \cdots, L$), the processed signal is reconstructed, as shown in Fig. 2.5.

Figure 2.5: DWT processing scheme
2.6 Problem statement

In this dissertation, the L-shaped arm pasted with a piezoelectric actuator is driven by a linear pulse motor. The motor is required to move to destination with less time consuming, while the arm vibration is required to be reduced as much as possible. The difficulty is controlling the linear motor and the piezoelectric actuator at same time to meet all the requirements, while the system has uncertainties and hysteresis nonlinearities. Moreover, if the arm has different loads, the mass of load will impact the system dynamics, therefore, the load mass is required to be determined before control.

Firstly, the L-shaped arm is driven by a linear pulse motor, motivated by the optimal motor motion control for the underactuated system, this dissertation intend to find control method for the linear motor. The motor is controlled with optimal trajectory resulting in less time consumption and less arm vibration. It means that the arm vibration status will be measured by sensors and feed back to the motor motion control. The flexible arm is usually modelled as an infinite dimensional plant, however it cannot be fully considered in the control design in practice; the common method is to select the first several dominant modes, other higher modes are considered as uncertainties. When the arm is driven by a motor, the vibration and disturbances of the motor will influence the arm vibration, the whole plant is nonlinear for the control system. It’s difficult to design a stable and robust controller for the nonlinear forced arm vibration. The vibration of L-shaped arm existing in two dimensions is more complicated and difficult to be controlled. There-
fore, one of the objectives is how to design an optimal controller to control this underactuated system such that the motor move fast in a optimal trajectory, the arm results in less vibration, while keep the system stable and robust.

Secondly, motivated by the superior advantages of the piezoelectric material, this dissertation intend to use piezoelectric material as sensors and actuator, to measure and suppress the arm vibration. Prandtl-Ishlinskii model is used to model the hysteresis of the piezoelectric actuator and modify it according to the control design method. The main difficulty is how to use the model and design controller to compensate the hysteresis of the piezoelectric actuator.

Thirdly, motivated by the superior advantages of the operator-based nonlinear control method, this dissertation intend to utilize the method to factorize the system model and design optimal controllers to control the linear motor motion and reduce the L-shaped arm vibration at the same time. The system has two control inputs including driving force of linear pulse motor and voltage for the piezoelectric actuator, three outputs including the moving distance of linear motor and the vibrations of two parts of arm. The main difficulty is how to design the two controllers working together to meet the system requirements.

Moreover, when the arm has different loads on it, it is difficult to measure the load mass in advance. Therefore, an automatic load mass estimation combined with the controller is necessary. The disturbances and uncertainties will lower the control performance. For this end, the output signal should
be processed by appropriate measures, wavelet transform can undertake such tasks. An on-line DWT is to be proposed to use in the operator-based nonlinear control, working together for the nonlinear L-shaped arm vibration system. Operator-based right coprime factorization method is used to guarantee the robust stability of the system. The on-line DWT is constructed to estimate the unknown load, remove some uncertainties and improve the performance of the operator-based control.

In summary, this dissertation intend to use operator-based nonlinear control approach and on-line DWT in the control design for actively control the L-shaped arm vibration system, and validate the control design in simulation and experiment. The aim of this research is to allow the motor move fast and reduce the arm vibration as much as possible while keeping the system to be robust stable.

2.7 Conclusion

In this chapter, the dynamics of the flexible arm and the piezoelectric actuator, the theoretical foundation of operator-based control and discrete wavelet transform are introduced. In addition, the problem to be tackled in this dissertation is stated, which provides framework of this study.
Chapter 3

Operator-based control design for the L-shaped arm without load

3.1 Introduction

To address the problems mentioned in Chapter 2, in this chapter, the dynamics on L-shaped arm vibration involving linear motor and piezoelectric actuator is modelled, controls for the motor motion and arm vibration are designed.

In Section 3.2, the vibration of the L-shaped arm is modelled by considering it as two connected Euler-Bernoulli beams, the relationship between the arm vibration and linear motor, piezoelectric actuator is given.

In Section 3.3, based on the operator-based nonlinear control approach, two controllers are designed to control the system in parallel. One controller is designed to allow the motor move fast to destination while reducing the arm vibration. Another one is designed to control the input of the piezoelectric...
actuator to further reduce the arm vibration. The hysteresis nonlinearity of
the actuator is modelled using a Prandtl-Ishlinskii hysteresis model, and the
nonlinear part is compensated in the tracking controller.

In Section 3.4, simulation is conducted using the experimental data
and comparing with the PI control, the results are shown to confirm the
effectiveness of the proposed control design.

In Section 3.5, the conclusion of this chapter is given.

3.2 Model of the L-shaped arm vibration

In this dissertation, the L-shaped arm driven by a linear pulse motor which
running along the horizontal motor guide is studied, the schematic diagram
of the system is shown in Fig. 3.1. When the motor starts running, the arm
will unavoidably vibrate.

![Schematic diagram of L-shaped arm system](image)

Figure 3.1: Schematic diagram of L-shaped arm system

For easy modelling, the arm is considered as two parts, Arm 1 for vertical
part and Arm 2 for horizontal part. The arm is connected with the motor, when the motor moves, the acceleration or deceleration will cause vibration of the arm. In addition, the transient vibration of motor and the uneven friction between the motor and guide will cause the arm additional vibration. In this chapter, the influences beside the acceleration of the motor will be considered as uncertainties or disturbances of the plant. The transverse vibration of Arm 1 is subjected to the excitation of linear pulse motor, and the Arm 2 subjected to the excitation from Arm 1, both are seen as clamped-free Euler-Bernoulli beams [77]. When modelling the vibration of Arm 1, the Arm 2 is considered as a tip mass at the free end. The excitation on Arm 2 depends on the relative transverse vibrations of Arm 1. Neglecting the external damping factor, only considering the strain-rate damping, the vibration of the arm is represented as

\[
pS \frac{\partial^2 y_i(x, t)}{\partial t^2} + E_a I \frac{\partial^4 y_i(x, t)}{\partial x^4} + c_s I \frac{\partial^5 y_i(x, t)}{\partial x^5 \partial t} = q_i(x, t) \quad (3.1)
\]

where \(y_i(x, t)\) is the transverse displacement relative to the clamped end of the arm along neutral axis, \(q_i(x, t)\) is external distributed force on the arm, \(i(i = 1, 2)\) is the order number representing for Arm 1 and Arm 2. \(\rho, S, E_a, I\) and \(c_s\) are density, cross-sectional area, Young’s modulus, moment of inertia and strain-rate damping coefficient of the arm, respectively. The external force on Arm 1 is represented as

\[
q_1(x, t) = - \left[ \rho S + m_2 \delta(x - L_1) \right] \frac{F_{01}(t)}{m_1 + m_2} \quad (3.2)
\]

where \(\delta(\cdot)\) is the Dirac delta function, \(F_{01}(t)\) is the force from the linear motor to drive arm, \(m_1, m_2\) are the masses of Arm 1 and Arm 2, respectively.
Remark 3.1: In Equation (3.2), the linear motor driving force $F_{01}$ is an equivalent term, because the linear motor used in the experimental system is controlled using the speed mode. The certain level friction between slider and stator does not influence the motor motion, unless it exceeds the output range of the motor.

The external forces on Arm 2 include the force from Arm 1 and the moment from the piezoelectric actuator, it is represented as

$$q_2(x, t) = -\frac{F_{12}(t)}{L_2} + M_p \frac{\partial^2}{\partial x^2} [H(x - x_{p2}) - H(x - x_{p1})]$$  \hspace{1cm} (3.3)

where $F_{12}(t)$ is the equivalent force from Arm 1 driving Arm 2, $M_p$ is the moment generated by the piezoelectric actuator. $H(\cdot)$ is a Heaviside function, $x_{p1}$, $x_{p2}$ are positions of the piezoelectric actuator on Arm 2.

Considering the boundary conditions of the arm, based on the expansion theorem, the solutions of Equation (3.1) are obtained as

$$y_1(x, t) = \sum_{m=1}^{\infty} J_1^m(x) \int_0^t [e^{-\alpha_1^m(t-\tau)} \sin \omega_1^m(t-\tau)f_1^m u_1(\tau)] d\tau, \hspace{1cm} (3.4)$$

$$y_2(x, t) = \sum_{m=1}^{\infty} J_2^m(x) \int_0^t [e^{-\alpha_2^m(t-\tau)} \sin \omega_2^m(t-\tau)(f_2^m F_{12}(\tau) + f_3^m \ddot{u}_2(\tau))] d\tau. \hspace{1cm} (3.5)$$

where $m(m = 1, 2, 3, \cdots)$ is the vibration mode order, $u_1(t) = F_{01}(t)$ is the control input for the linear motor, $\ddot{u}_2(t) = M_p(u_2)(t)$ is the moment from the
3.3. Proposed robust nonlinear control design

Piezoelectric actuator. \( f_i^m \) are coefficients relative to the external forces.

\[
\begin{align*}
  f_1^m &= -\frac{1}{m_1 + m_2} \left[ \rho S \int_0^{l_1} \Phi_1^m(x) dx + m_2 \Phi_1^m(l_1) \right] \\
  f_2^m &= -\frac{1}{L_2} \int_0^{L_2} \Phi_2^m(x) dx \\
  f_3^m &= \frac{\partial \Phi_2^m(x_{p2})}{\partial x} - \frac{\partial \Phi_2^m(x_{p1})}{\partial x} \\
  F_{12} &= m_2 \frac{\partial^2 y_1^2(l_1, t)}{\partial t^2}
\end{align*}
\]

Other parameters in Equations (3.4) and (3.5) are expressed as follows.

\[
\begin{align*}
  J_i^m(x) &= \frac{\Phi_i^m(x)}{\omega_{id}^m}, \\
  \zeta_i^m &= c_s \omega_i^m, \\
  \alpha_i^m &= \zeta_i^m \omega_i^m, \\
  \omega_i^m &= \frac{\lambda_i^m}{L_i} \sqrt{\frac{E_a I}{\rho S}}, \\
  \omega_{id}^m &= \omega_i^m \sqrt{1 - c_s^m}, \\
  c_s &= c_m E_a.
\end{align*}
\]

where \( \Phi_i^m(x) \) is the mass normalized eigenfunction of the clamped-free arm for the \( m \)-th mode [77]. \( \omega_i^m \) is the undamped natural frequency of the \( m \)-th mode, \( \omega_{id}^m \) is the damped natural frequency, \( \alpha_i^m \) is the damping ratio. The dimensionless frequency parameter of the \( m \)-th mode \( \lambda_i^m \) could be obtained from the corresponding characteristic equations. The details about the vibration model arm with load are shown in Appendix A.

**Remark 3.2:** In the experimental system, the Arm 2 is not lumped mass and the fixed mechanism of the arm will cause the model error. The vibration mode of the arm can be measured by sensors; the results could be used to estimate the equivalent parameters of the arm so as to correct the error.
3.3 Proposed robust nonlinear control design

3.3.1 Control scheme for the Arm-Motor system

In the whole plant of the L-shaped arm system, there are three outputs and two inputs. By controlling the linear pulse motor and the piezoelectric actuator, Arm 1 and Arm 2 track the reference trajectories. In addition, the piezoelectric actuator and the arm vibration coupling with unknown disturbances from the motor are both nonlinear systems. To guarantee the robust stability of the system, in this dissertation, we design two controllers based on operator theory, the control scheme is shown in Fig. 3.2.

Because the output of piezoelectric actuator is smaller enough comparing with the motor force, the piezoelectric actuator’s impact on linear motor is neglected. The Controller 1 is designed to control the linear motor motion, the feedback operator is designed to keep the system stable and asymptotically convergent, such that the linear motor be faster and result in smaller arm vibration. The Controller 2 is designed to control the behaviour of the piezoelectric actuator to further reduce the vibration of the arm. The plant...
3.3. Proposed robust nonlinear control design

outputs $y_1$ and $y_2$ are measured by the two piezoelectric sensors.

In detail, the relationship between the two control loops is shown as in Fig. 3.3. For the linear motor control, the inner loop is designed using operator theory to keep the plant stable and asymptotically convergence. The outer control loop is designed using Proportional-Integral (PI) controller combined with the feedback signals generated by the inner control loop, so as to control the linear motor motion with less time consuming and resulting in smaller arm vibration.

To reduce the measurement noise, filters are designed before the signals feedback. The appropriate filter type and structure depends on the noise feature. In this dissertation, we use an IIR low-pass filter and a notch filter [92–98], which will be discussed later. For simplicity, the filters are not shown in the following control designs, the feedback signals default to filtered signals from the sensors. The flowchart of the proposed control is shown in Fig. 3.4.

### 3.3.2 Operator-base system representation

Assuming that the uncertainties of the two plants are additive uncertainties, represented as $\Delta P_i$. Using operator theory mentioned above, the two plants
CHAPTER 3. CONTROL DESIGN FOR ARM WITHOUT LOAD

![Control Flowchart]

Figure 3.4: The proposed control flowchart

of the control object including linear pulse motor and piezoelectric actuator
are expressed as follows.

\[ [P_1 + \Delta P_1] (u_1)(t) = (1 + \Delta_1) \sum_{n=1}^{3} J_1^n f_1^n \int_0^t \left[ e^{-\alpha_n(t-\tau)} \right. \left. \cdot \sin \omega_{dn}(t-\tau) u_1(\tau) \right] d\tau, \]

\[ (3.6) \]

\[ [P_2 + \Delta P_2] (u_2)(t) = (1 + \Delta_2) \sum_{n=1}^{3} J_2^n \int_0^t \left[ e^{-\alpha_n(t-\tau)} \right. \left. \cdot \sin \omega_{dn}(t-\tau) u_2(\tau) \right] d\tau. \]

\[ (3.7) \]

where \( P_1, P_2 \) represent for the plants of Arm 1 and Arm 2 vibration, respectively. \( u_2^* (t) = f_2^n F_{12}(t) + f_3^n \tilde{u}_2(t) \) is the input for plant 2. \( \Delta_i \) are uncertainties of the plants including modelling errors, transient vibration of motor, and
other influencing factors. The first three modes of arm vibration are considered in the plant; the other modes are regarded as uncertainty included in $\Delta P_i$.

According to robust right coprime factorization, plant $P_i + \Delta P_i$ is factorized as follows.

\[
D_1(\delta_1)(t) = e^{-\hat{\alpha}_1 t} \delta_1(t),
\]

\[
[N_1 + \Delta N_1](\delta_1)(t) = (1 + \Delta_1) \sum_{n=1}^{3} J_1^n J_1^n e^{-\alpha_n t} \int_0^t e^{-\hat{\alpha}_n \tau} \sin \omega_d n(t - \tau) \delta_1(\tau) d\tau;
\]

\[
D_2(\delta_2)(t) = e^{-\hat{\alpha}_2 t} \delta_2(t),
\]

\[
[N_2 + \Delta N_2](\delta_2)(t) = (1 + \Delta_2) \sum_{n=1}^{3} J_2^n e^{-\alpha_n t} \int_0^t e^{-\hat{\alpha}_n \tau} \sin \omega_d n(t - \tau) \delta_2(\tau) d\tau.
\]

where $\hat{\alpha} = \sum_{n=1}^{3} \alpha_n$, $\hat{\alpha}_n = \hat{\alpha} - \alpha_n$. (3.8) and (3.10) can be written as $\delta_1(t) = D_1^{-1}(u_1)(t) = e^{\hat{\alpha}_1 t} u_1(t)$ and $\delta_2(t) = D_2^{-1}(u_2^*)(t) = e^{\hat{\alpha}_2 t} u_2^*(t)$. Then we can obtain that $P_1 = N_1 D_1^{-1}$ and $P_2 = N_2 D_2^{-1}$.

### 3.3.3 Optimal control for the linear pulse motor

The vibration source of arm includes the linear pulse motor’s acceleration and its transient vibration. In this dissertation, we only consider the first factor, the transient vibration of motor is seen as uncertainty. It is an underactuated nonlinear plant with two outputs and only one input. Because linear pulse
motor can be seen as a linear system and its acceleration or deceleration decide the arm vibration, the arm vibration displacement can be reduced by controlling the motor motion. However, reducing the acceleration of linear motor will consume more time to get to the destination, there must be a trade-off between the duration of linear motor and vibrating displacement of arm. Therefore, an optimal trajectory for the linear motor motion that consume short travel time with less arm vibration displacement is needed. A cost function is defined as follows.

\[
J = t_f - t_0 + \frac{\sigma}{t_f} \int_{t_0}^{t_f + \Delta t} y_1^2(t)dt
\]  

(3.12)

where \( J \) is the cost index, \( t_0 \) and \( t_f \) are the starting time and final time, respectively. \( \sigma \) is a weighting factor for tuning the weight of arm vibration in the cost index. To reflect the vibration during and after motion, the vibration in a period \( \Delta t \) after the motion stop is considered in the function. The optimal problem is to minimize the cost index \( J \).

To make the motor move for a distance \( r_0 \) in finite time, we firstly design a Proportional-Integral (PI) tracking controller \( C_0, b_1 \) from the operator \( A_1 \) is considered as compensation for motor control to reduce the arm vibration. Then, for the plant of Arm 1, operator-based controllers \( A_1 \) and \( B_1 \) keep it stable. The control scheme for linear motor control considering arm vibration is shown as in Fig. 3.5. Where \( y_0, y_1, r_0 \) and \( C_0 \) are the moving distance of linear motor and the relative displacement of Arm 1, tracking references and tracking controller for the linear motor, respectively. \( M \) stands for the linear motor, \( B_0 \) is an operator to satisfy the stable conditions. As motor M is
Proposed robust nonlinear control design

a linear system and the output is proportional to its real input, so we can combine it in the right coprime factorization, namely, \( B_1(u(t)) = B_0(u(t)) \).

\[
C_0(e_0(t)) = K_{I1} \int_0^t e_0(\tau)d\tau + K_{P1}e_0(t) \tag{3.13}
\]

where \( K_{I1} \) and \( K_{P1} \) are design parameters, \( e_0(t) \) is the error between the output \( y_0(t) \) and the target value \( r_0(t) \).

![Figure 3.5: Operator-based control system for linear motor](image)

With certain final time \( t_f \) and weighting factor \( \sigma \), the cost index \( J \) depends on the gain parameters in operator-based controller and \( K_{I1} \) or \( K_{P1} \). Therefore, by selecting a weighting factor \( \sigma \), the appropriate control parameters can be determined by the minimum cost index \( J \). The linear pulse motor used in this system has speed and acceleration constraints in practical experiments. Under the limits of linear motor, the minimum consuming time \( t_{f\min} \) can be determined, starting from \( t_{f\min} \), the different cost index \( J \) can be obtained by iterative algorithm.

Using right coprime factorization, the operator-based controllers \( A_1 \) and
CHAPTER 3. CONTROL DESIGN FOR ARM WITHOUT LOAD

$B_1$ for the plant $P_1$ can be obtained as

$$A_1(y_1)(t) = b_1(t) = e^{-\alpha_1 t} - \frac{K_1}{J_1 \omega_{1d}} \eta_1(t),$$  \hspace{1cm} (3.14)

$$\eta_1(t) = \ddot{y}_1 + 2\alpha_1 \dot{y}_1 + (\alpha_1^2 + \omega_{1d}^2)y_1,$$  \hspace{1cm} (3.15)

$$B_1(u_1)(t) = K_1 u_1(t).$$  \hspace{1cm} (3.16)

where $K_1$ is a design parameter for tuning the feedback signal from operator $A_1$. The controllers $A_1$ and $B_1$ guarantee the control system to be BIBO stable and robust, the PI tracking controller $C_0$ with optimization makes the linear motor track the target value $r_0$. Therefore the motor moves to the desired position with less arm vibration during and after its travel. The operator based optimal control ensures the arm vibration to be stable, and asymptotically converge to zero.

3.3.4 Control system for Arm 2 vibration with piezoelectric actuator

Considering the advantages of piezoelectric materials, in this dissertation, we use piezoelectric sensors and actuator to control the vibration of arm. Because of the hysteresis nonlinear property, the relationship between the moment output $M_p$ of the piezoelectric actuator and the control input is a nonlinear process. In this dissertation, the hysteresis model is represented as follows.

$$M_p(t) = D_{PI}(u)(t) + \Delta_{PI}(u)(t)$$  \hspace{1cm} (3.17)

where $D_{PI}$ and $\Delta_{PI}$ are represented by the Prandtl-Ishlinskii model using play hysteresis operators, for the details, please refer to [73, 74]. $D_{PI}$ is an
invertible operator, $\Delta P_I$ is the residual part stands for the nonlinear part of the model, it changes with the input $u(t)$ and is influenced by design parameters, so it needs to be compensated in the control.

By controlling the motor motion, the first controller can only reduce the vibration of arm at a certain degree but never eliminate it. For Arm 2, we use piezoelectric actuator to suppress the rest vibration. The control input $u_2$ is the voltage applied to piezoelectric actuator, the output $y_2$ is the displacement of Arm 2. The control target is to eliminate the vibration of Arm 2, namely, $r_2 = 0$. The control design is shown in Fig. 3.6.

For a certain piezoelectric actuator, $D_{PI}$ is a constant. It is combined into the plant $P_2$, and denote that $\tilde{D} = D_{PI}^{-1} D$. The residual part $\Delta P_I$ is a bounded uncertainty, will be compensated by a compensator $T_c$. Under the new equivalent plant $\tilde{D}$, based on Equation (2.15), the operators $A_2$ and $B_2$ are required to satisfy Bezout identity $A_2N_2 + B_2\tilde{D}_2 = \tilde{U}$, then $A_2$ and $B_2$
are obtained as follows.

\[ A_2(y_2)(t) = \frac{e^{-\alpha_2 t} - K_2 K_1^{-1}}{J_2 \omega_{2d}} \eta_2(t), \quad (3.18) \]

\[ \eta_2(t) = \ddot{y}_2 + 2\alpha_2 \dot{y}_2 + (\alpha_2^2 + \omega_{2d}^2)y_2, \]

\[ B_2(u_2)(t) = K_2 K(u_2)(t). \quad (3.19) \]

where \( K_2 \) is a design parameter. According to the robust right coprime factorization approach, with operators \( A_2 \) and \( B_2 \), the control system is guaranteed to be BIBO stable. Moreover, if robust condition \((2.19)\) is satisfied, the designed control system is said to be robust.

**Remark 3.3:** In Equations \((3.14)\) and \((3.18)\), the output signals \( y_1 \) and \( y_2 \) are ideal displacement of Arm 1 and Arm 2, respectively. In the experimental system, they may include measuring errors caused by disturbances. Therefore, appropriate filters should be designed, and use the filtered signal in the controllers \( A_1 \) and \( A_2 \).

![Figure 3.7: Equivalent control system with hysteresis compensator](image)

The compensator \( T_c \) is designed to compensate \( \Delta P_I \) and the external force \( F_{12} \) on Arm 2. Therefore, an equivalent diagram of control system in Fig. 3.6 is shown as in Fig. 3.7, and the equivalent plant output is expressed as follows.

\[ y_2(t) = (N_2 + \Delta N_2)\bar{U}^{-1}(r^*(t) + B_2 D_{PI}^{-1}(\Delta P_I + F_{12})) \quad (3.20) \]
3.4 Simulation results and discussion

From Fig. 3.7, we can find that the dis-invertible part \( \Delta P_I \) and external force \( F_{12} \) can be compensated in the tracking controller \( C_2 \), and the control pant is kept BIBO stable and tracks the reference.

Based on the above two operator-based control designs, the robust stability of the whole plant is guaranteed, the three outputs track the reference values. It means that the linear motor runs to destination in finite time, while the vibrations of Arm 1 and Arm 2 are reduced as small as possible.

### 3.4 Simulation results and discussion

To demonstrate the effectiveness of the proposed design scheme, simulations were conducted by using MATLAB. Parameters of the L-shaped arm and the linear pulse motor are shown in Table 3.1 and Table 3.2, respectively.

Table 3.1: Some parameters of the L-shaped Arm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>Length of OA</td>
<td>0.5</td>
<td>m</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>Length of AB</td>
<td>0.3</td>
<td>m</td>
</tr>
<tr>
<td>( w_a )</td>
<td>Width of arm</td>
<td>0.02</td>
<td>m</td>
</tr>
<tr>
<td>( t_a )</td>
<td>Thickness of arm</td>
<td>0.003</td>
<td>m</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density of arm</td>
<td>8030</td>
<td>kg/m³</td>
</tr>
<tr>
<td>( E_a )</td>
<td>Young’s modulus</td>
<td>( 197 \times 10^9 )</td>
<td>N/m²</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>Damping modulus</td>
<td>( 5 \times 10^{-4} )</td>
<td>–</td>
</tr>
<tr>
<td>( x_{p1} )</td>
<td>Left end of PZT on AB</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>( x_{p2} )</td>
<td>Right end of PZT on AB</td>
<td>0.09</td>
<td>m</td>
</tr>
</tbody>
</table>

In the simulation, the sampling interval was 0.01 s, the output \( y_0 \) was controlled to move in one direction for a distance of 0.5 m. The aim of linear
motor motion control is to reduce the tip displacement of Arm 1 while the motor running as faster as possible with an optimal trajectory.

To be consistent with the experimental conditions, the speed and acceleration of linear motor were limited within a scope. For the piezoelectric actuator control, the voltage was applied when $t > 0$ s, output $y_2$ was controlled to track zero.

For comparison, firstly we conduct a simulation of the linear motor with minimal time feed-forward control, namely, the motor runs under the maximum speed and acceleration. The vibration of Arm 1 during and after the linear motor motion is shown as the dashed line in Fig. 3.8. As can be seen from the results, the linear motor stops with minimum time consumption at 3.4 s, which leads to biggest arm vibrations, especially at the moment the linear motor starts and stops, which was caused by its biggest accelerations and decelerations, respectively.

Then, by using PI controller without feedback signal from the vibration
3.4. Simulation results and discussion

![Graph showing displacement vs time for minimal time control and PI control.]

Figure 3.8: Displacement of Arm 1 with and without feedback control

of arm, the motor position and speed is shown in Fig. 3.9. The corresponding vibration of Arm 1 with PI control is compared with the result under minimal time control, as shown in Fig. 3.8, the solid line is displacement under PI control. It illustrates that with PI control, the arm vibration was reduced, but the travel time of the linear motor was longer, arriving at the destination within 4.5 s.

For the optimization problem as shown in Equation (3.12), under a certain weighting factor $\sigma = 1.5 \times 10^7$, and the after stop duration was setted as 0.15 s, with different design parameters, the corresponding cost index $J$ is shown in Fig. 3.10. It illustrates that, by selecting appropriate parameters PI gains $K_{P1}$ or $K_{I1}$ and the operator feedback factor $K_1$, the cost value can be
minimized, namely, the linear motor motion and arm vibration are balanced.

The corresponding simulation results using the operator-based optimal control for motor motion is shown in Fig. 3.11. The displacement of Arm 1 with the two controllers are shown in Fig. 3.12, the dashed line is displacement with PI control, the solid line is displacement with operator-based optimal control. The results show that the linear motor with operator-based control moves to the desired position within 4.5 s, same as PI control. However, the displacement of Arm 1 was smaller than it under PI control. It illustrates that with the optimal feedback signal of arm vibration, the controller using operator theory can reduce the vibration of arm, especially, the after stop arm vibration was reduced significantly, and the plant was kept to
3.4. Simulation results and discussion

Figure 3.10: Cost indexes under optimal control with different parameters

be stable.

The vibration of Arm 2 without actuator, with actuator no hysteresis compensation and with hysteresis compensation were simulated for comparison. Displacement of Arm 2 without the piezoelectric actuator control and with control but without considering hysteresis are shown in Fig. 3.13, the dashed line is vibration of Arm 2 without control, the solid line is vibration of Arm 2 with actuator control, which indicates that the piezoelectric actuator can reduce the arm vibration with the proposed control. The corresponding control input is shown in Fig. 3.14.

Furthermore, by using the hysteresis compensation tracking controller $T_C$, the output of Arm 2 is shown in Fig. 3.15 together with the results under control without compensation. The vibration result without compensation
shown in dashed line, the solid line the result using the compensator. The corresponding control input is shown in Fig. 3.16. Fig. 3.15 shows that the proposed controller is effective to further reduce the vibration of arm, which means that both the model of hysteresis and the designed hysteresis compensator are effective.

### 3.5 Conclusion

In this chapter, an L-shaped arm driven by a linear pulse motor was considered to find approaches to control the forced vibration of arm. By considering the L-shaped arm as two connected Euler-Bernoulli beams, the dynamics on arm vibration involving linear motor and piezoelectric actuator was modelled. Two operator-based robust nonlinear control systems were proposed
3.5. Conclusion

in this chapter, the first one was designed to make the motor not only move to destination in certain time but also reduce the vibration of arm. Another one was designed to control the input of the piezoelectric actuator, the hysteresis nonlinearity of the actuator was modelled using a Prandtl-Ishlinskii hysteresis model, and the nonlinear part was compensated in the tracking controller. Finally, to confirm the effectiveness of the proposed control design, simulations were conducted comparing with the PI control, the results illustrate that the operator-based control systems designed in this dissertation are more effective and can guarantee the system robust stability.
CHAPTER 3. CONTROL DESIGN FOR ARM WITHOUT LOAD

Figure 3.13: Displacement of Arm 2 without and with actuator control (without hysteresis compensation)

Figure 3.14: Control input for actuator without hysteresis compensation
3.5. Conclusion

Figure 3.15: Displacement of Arm 2 without and with hysteresis compensation

Figure 3.16: Control input for actuator with hysteresis compensation
Chapter 4

Operator-based control design for the L-shaped arm with unknown load

4.1 Introduction

In the above Chapter, the L-shaped arm considered is load-free, and the arm vibration is modelled by assuming the arm as two separated segments. In practice, the arm manipulates different loads, which determines the parameters in the controller relating to the plant dynamics. It is inconvenient to measure the load mass in advance. Therefore, an automatic load mass estimation combined with the controller is necessary. In addition, the operator-based control uses the outputs of system directly as feedback signals for the controller, the system performance is influenced by the quality of the signal. The disturbances and uncertainties will lower the control performance. For this end, the output signal should be processed by appropriate measures, wavelet transform can undertake such tasks.
In Section 4.2, different from last Chapter, the L-shaped arm is modelled as a whole, the load is considered in it. The load estimation method is given based on the model.

In Section 4.3, base on the DWT theory, a short-symmetrical on-line DWT is constructed to use it in the operator-based control design.

In Section 4.4, based on the right coprime factorization method, two operator-based controllers are proposed by using the on-line DWT in it. One controls the motor motion resulting in less arm vibration. Another one further reduces the arm vibration by using a piezoelectric actuator.

In Section 4.5, simulations comparing with previous control are demonstrated to validate performance of the proposed control design.

In Section 4.6, the main contends of this Chapter is summarized.

4.2 Modelling of the system with load

4.2.1 Model of arm vibration with load

In this chapter, the uniform steel L-shaped arm with load is considered, the Motor-Arm structure is shown in Fig. 4.1. The arm is driven by a linear pulse motor to the destination along the motor guide in y direction.

To analyse the arm vibration dynamics easily and illustrate it clearly, the L-shaped armed is marked with two segments OA and AB, as shown in Fig. 4.2. Two piezoelectric sensors are pasted on one side of the arm to measure the relative vibration displacements of the arm, one at the OA segment, another at AB segment. A piezoelectric actuator is mounted on the
4.2. Modelling of the system with load

opposite side of AB segment to suppress the arm vibration. The linear motor
runs in $y$ direction along the frame, which will result in the arm vibration.
Denoting the vibration displacement with time as $w(x,t)$, neglecting the
torsional vibration, being seen as an Euler-Bernoulli beam [77], the forced
transverse arm vibration is approximated as

$$E_a I \frac{\partial^4 w}{\partial x^4} + c_s I \frac{\partial^5 y(x,t)}{\partial x^4 \partial t} + \rho S \frac{\partial^2 w}{\partial t^2} = q(x,t)$$  \hspace{1cm} (4.1)

where $q(x,t)$ is the external distributed forces on the arm, including the
linear motor driving force and the piezoelectric actuator moment. $w$, stands
for $w(x,t)$, is the transverse displacement along the neutral axis of the arm.
$E_a, I, \rho, S$ are the Young’s modulus, moment of inertia, density, and cross-
sectional area of the arm, respectively. The arm vibrations at two segments OA and AB can be determined by variables separation method based on the boundary conditions and the initial conditions of the system, the details about the vibration model arm with load are shown in Appendix A.

We name the OA segment of the arm as Arm 1 and the AB segment as Arm 2, the relative vibration along Arm 1 as \(y_1\), and vibration along Arm 2 as \(y_2\), they are expressed as follows.

\[
y_1(x_1, t) = \sum_{n=1}^{\infty} J_1^n(x_1) \int_0^t \left[ e^{-\alpha_n(t-\tau)} \sin \omega_{dn}(t-\tau) f_1^n u_1(\tau) \right] d\tau, \quad (4.2)
\]
\[
y_2(x_2, t) = \sum_{n=1}^{\infty} J_2^n(x_2) \int_0^t \left[ e^{-\alpha_n(t-\tau)} \sin \omega_{dn}(t-\tau)(f_2^n u_2(\tau) + f_3^n) \right] d\tau. \quad (4.3)
\]

where, \(n(n = 1, 2, 3, \ldots)\) is the vibration mode order, \(u_1\) is the motor force acted on the arm, \(u_2 = M_p\) is the moment generated by the piezoelectric actuator. \(J_1^n = \Phi_1^n(x_1)/\omega_{dn}\), \(J_2^n = \Phi_2^n(x_2)/\omega_{dn}\), \(\alpha_n = c_1 \omega_n^2\) is the damping factor of the arm vibration. \(f_i^n\) is the coefficients relative to the external
4.2. Modelling of the system with load

forces, shown as follows.

\[
\begin{align*}
    f_1^n &= -\frac{\rho S}{m_s} \int_0^{l_1} \Phi_1^n(x_1)dx_1, \\
    f_2^n &= \frac{\partial \Phi_2^n(x_2)}{\partial x_2} - \frac{\partial \Phi_2^n(x_{p1})}{\partial x_2}, \\
    f_3^n &= -\frac{\rho S}{m_s} \int_0^{l_2} \Phi_2^n(x_2)dx_2 - m_t \Phi_2^n(l_2).
\end{align*}
\]

where \(m_s = m_a + m_t\) is the total weight of the arm with load, \(m_a\) is the mass of the arm. \(x_{p1}, x_{p2}\) are the positions of the both end of the actuator on AB section as shown in the arm sketch Fig. 4.2.

### 4.2.2 Load estimation method

As shown in the Appendix A, the mass of the load will impact the arm vibration dynamics, especially the important parameters in the vibration model, such as the natural frequency of every mode, the frequency decreases with the increasing of the load mass. According to Eq. (A.24), it obtains that

\[
\beta_n = \sqrt{\frac{\rho S (2\pi f_n)^2}{EaI}} \tag{4.4}
\]

From the Eq. (A.21) shown in Appendix A, it yields

\[
m_t = \frac{P_I \rho S}{P_{II} \beta} \tag{4.5}
\]

where

\[
P_I = \sin \beta l^- \sinh \beta l^+ - \sinh \beta l^- \sin \beta l^+ - 2 \cos \beta l_2 \cosh \beta l_2 - 2 \cos \beta l_1 \cosh \beta l_1 - 2 \cos \beta l^+ \cosh \beta l^+ - 2
\]

\[
P_{II} = 2 \cos \beta l^+ \sinh \beta l^- - 2 \cos \beta l^+ \sin \beta l^- + 2 \cos \beta l_2 \sinh \beta l_2 - \cosh \beta l_2 \sin \beta l_2 + \cosh \beta l^- \sin \beta l^+ - \sin \beta l^- \cosh \beta l^+ - \sin \beta l^- \cosh \beta l^+
\]
According to Eq. (4.5), the relationship between the first mode frequency and load mass is shown in Fig. 4.3.

![Figure 4.3: The relationship between the first mode frequency and load mass](image)

If we can measure the natural frequency of one mode of the arm vibration in experiment, such as $f_1$ (Hz), according to the relationship in Eq. (4.4), $\beta_1$ is obtained, substitute it into the frequency equation (4.5), the mass of load could be estimated.

The load mass estimation flow is given as: initial vibration $\rightarrow y_1(t) \rightarrow$ DWT $\rightarrow$ reconstructed of the approximation of the first frequency band $\rightarrow$ FFT $\rightarrow f_1 \rightarrow$ equation (4.4) $\rightarrow \beta_1 \rightarrow$ equation (4.5)$\rightarrow$ load mass $\hat{m}_t$. 
4.3 On-line wavelet transform

The DWT decomposes a time domain signal into an orthogonal set of wavelets, presented in time-frequency domain, which is useful for different purposes. However, most DWT applications are off-line needing the entire signal. These approaches are not suitable for the real-time vibration control. To solve the problem and apply wavelet transforms to arm vibration controls, a real-time wavelet approach is implemented in this section.

The on-line DWT usually utilizes a moving window. When the control loop starts, the on-line DWT must wait for the available signal with length equal to \( l_n \). The larger the \( l_n \), the more significant time delay. On the other hand, with smaller \( l_n \), the DWT results in lower accuracy. In this dissertation, the size of the moving window \( l_n \) depends on the sampling frequency and the vibration frequency band considered in the controller. For a certain \( f_s \), the considered frequency band decides the wavelet decomposition levels \( L \), then the size of the moving window \( l_n \) is decided.

To deal with the boundary effects, one of the methods is artificially extend the signals at boundaries before processing. Symmetric extension is usually adopted to keep the continuity. However, the whole symmetrization will increase the computation load, probably causes time delay for the control and lower the performance. Therefore, we extend the data stream using short-symmetrical, the length of extension is denoted as \( l_t \), \( l_w = l_n + l_t \), \( l_w \) is the length of signal for on-line DWT, requiring \( l_w \geq 2^L \). The moving window
and $\hat{y}$ are defined as follows.

$$W_i = \begin{cases} \text{none,} & i < l_n \\ y(i - l_n + 1), \ldots, y(i), y(\text{extension}), & i \geq l_n \end{cases}$$

(4.6)

$$y(\text{extension}) = [y(i), \ldots, y(i - l_t + 1)]$$

$$\hat{y}(i) = \begin{cases} y(i), & i < l_n \\ y_{wti}(l_n), & i \geq l_n \end{cases}$$

(4.7)

where $y_{wti} = DWT(W_i)$ is the output of the on-line DWT for the moving window $W_i$. The on-line DWT process is shown in Fig. 4.4.

---

**Figure 4.4: On-line DWT processing flow**
4.4 Operator-based control design with DWT

For some nonlinear systems, the disturbances are complicated and unknown. It is difficult to satisfy the factorization conditions and robustness condition as shown in (2.19). In addition, if the disturbances include high frequency signal, the system output will fluctuate severely; the performances of the system will degrade. Therefore, we use an on-line DWT in the operator-based control, the control scheme is shown in Fig. 4.5. A DWT processor is added between the system output $y_a$ and operator $A$ to decompose the output $y_a$ into time-frequency domain. According to the characteristics of the system dynamic, the necessary signal components are extracted and reconstructed as $\tilde{y}$. If the unwanted disturbances are fully removed, namely $\tilde{y} \approx y$, the coprime condition (2.18) approximates the condition (2.15), then the desired operators $A$ and $B$ could be obtained more easily.

![Figure 4.5: Nonlinear operator-based robust control with DWT](image)

4.4.1 Control scheme for the Arm-Motor system

Based on the discrete wavelet transform (DWT) and the operator-based nonlinear control, we propose a control scheme as shown in Fig. 4.6. It includes
two wavelet-operator-based controllers, the Controller 1 is designed to control the linear motor motion; the Controller 2 is designed to control the behaviour of piezoelectric actuator. They work together to satisfy all of the requirements. $ref_1$ and $y_0$ are the target position and real-time position of the linear motor respectively. The control flow is shown in Fig. 4.7, where $y_{a1}$ and $y_{a2}$ are displacements of the arm vibration at OA and AB segments, respectively. $u_0$, $u_1$ and $u_2$ are the control inputs for the linear motor, Arm 1 and Arm 2, respectively. $K_P$, $K_I$, $K_1$ and $K_2$ are the tuning parameters for the designed controllers. Through the Controller 1, the linear motor is controlled to move to the destination fast while reducing the vibration of the arm. The Controller 2 further reduce the vibration of arm by controlling the piezoelectric actuator output. The system outputs are measured by the two piezoelectric sensors; the sampled signals contain noise and disturbances. Therefore, two wavelet transform processor works together with the operator-based controllers, remove the undesired uncertainties and disturbances, keep the system stable and robust.

Figure 4.6: Proposed control scheme for the arm with load
4.4. Operator-based control design with DWT

As shown in the dynamics of the arm vibration (4.2) and (4.3), they are superposition of infinite modes vibration. It is difficult to design the operator-based controllers with thus models. Therefore, the dominant vibration modes should be selected without loss of the system characteristic. According to the contributions of different modes in the whole vibration, we propose a

Figure 4.7: Proposed control flow chart
CHAPTER 4. CONTROL DESIGN FOR ARM WITH LOAD

threshold and the dominant vibration modes are determined as follows.

\[ r_{\text{mode}}(n_c) := \frac{J_n^c(l_2)}{\sum_{n=1}^{n_c} J_n^c(l_2)} \geq r_{\text{th}} \]

\[ r_{\text{mode}}(n_c + 1) < r_{\text{th}} \]

where \( r_{\text{mode}}(n_c) \) stands for the contributions of the \( n_c \) mode in the first \( n_c \) modes vibration, \( r_{\text{th}} \) is the threshold. It means that in the whole displacement, the proportion of the modes higher than \( n_c + 1 \) is small enough, they are neglected. In the controlled plant, we only consider the first \( n_c \) modes of arm vibration.

Remark 4.1: \( r_{\text{mode}} \) is derived from the relative vibration dynamics of Arm 2 (4.3). Considering an impulse force is applied on the arm, without control, the \( n \) mode response at the tip of the arm is denoted as \( y_2^n(l_2, t) \), then

\[ r_{\text{mode}}(n_c) := \frac{|y_2^n(l_2, t)|_{\text{max}}}{|\sum_{n=1}^{n_c} y_2^n(l_2, t)|_{\text{max}}} \approx \frac{J_n^c(l_2)}{\sum_{n=1}^{n_c} J_n^c(l_2)} \]

4.4.2 Control design for the linear pulse motor

As shown in the control scheme Fig. 4.6, the linear motor control has two output \( y_0 \) and \( y_1 \) with only one input \( u_0 \). The control target is to allow the linear motor to the destination as fast as possible while reducing the arm vibration.

By using the right coprime factorization method, plant \( P_1 + \Delta P_1 \) is factorized as follows.

\[ D_1(\delta_1)(t) = e^{-\alpha t}z_1(t), \]

\[ [N_1 + \Delta N_1](\delta_1)(t) = (1 + \Delta_1) \sum_{n=1}^{N_1} f_1^n J_1^n e^{-\alpha_n t} \int_0^t e^{-\alpha_n \tau} \sin \omega_{dn}(t - \tau) \delta_1(\tau) d\tau, \]

(4.10)
where $\ddot{\alpha} = \sum_{n=1}^{N_d} \alpha_n$, $\dot{\alpha}_n = \ddot{\alpha} - \alpha_n$.

It is necessary to mention that the dynamics of the arm vibration are superposition of infinite modes. It is difficult to design an operator-based controllers with all models. Therefore, the dominant vibration modes should be determined without loss of the system characteristic. It depends on the sampling interval and vibration dynamics in experiments. To allow the linear motor to the destination $r_0$ in finite time, a PI tracking controller $C_0$ is used. To reduce the arm vibration, a wavelet-based operator is involved, working as a feedback compensator. the compensator depends on the arm vibration.

The control scheme for linear motor control considering arm vibration is shown in Fig. 4.8, where LM represents for the linear motor, $A_1$, $B_0$ are the designed operators to satisfy the stable conditions. The real output $y_{a1}$ includes disturbances and uncertainties, which will feedback an undesired fluctuation signal, it will cause the linear motor unnecessary motion resulting in performance degradation. Therefore a DWT is utilized to work with the operator $A_1$, to remove the undesired disturbances in the signal. The tracking controller $C_0$ is shown as follows.

$$C_0 := K_{I1} \int_0^t e_0(\tau) d\tau + K_{P1} e_0(t)$$ (4.11)
where $K_{I1}$ and $K_{P1}$ are design parameters, $e_0(t)$ is the error between the output $y_0(t)$ and the target value $r_0(t)$. To balance the speed of the linear motor and the arm vibration, a cost index $J$ proposed in last chapter is used. By tuning the gain parameters $K_{I1}$ or $K_{P1}$ and the parameter for the operator, a desired motor motion is obtained.

Based on the operator theory, according to the factorization as in (4.9) and (4.10), we design operators $A_1$ and $B_1$ to satisfy the stable condition.

\begin{align*}
A_1(\dot{y}_1)(t) &= b_1(t) = \frac{e^{\alpha_1 t} - K_1}{J_{c1}\omega_{d1}} \eta_1(t), \quad (4.12) \\
\eta_1(t) &= \ddot{y}_1 + 2\alpha_1 \dot{y}_1 + (\alpha_1^2 + \omega_{d1}^2)\dot{y}_1, \quad (4.13) \\
B_1(u_1)(t) &= K_1 u_1(t). \quad (4.14)
\end{align*}

where $J_{c1} = C f_1^1 J_1^1$, $C$ is a parameter decided by $\omega_n$; $K_1$ is a tuning parameter for control the feedback from operator $A_1$. Then the force input of the motor is shown as follows.

\begin{equation}
 u_1(t) = K_1^{-1} [C_0(e_0)(t) - b_1(t)] \quad (4.15)
\end{equation}

Operators $A_1$ and $B_1$ works together with the DWT processor to guarantee the robust stability of the system. Operator $C_0$ along with the operator feedback allow the linear motor to move fast while resulting in less arm vibration.

### 4.4.3 Control design for vibration of Arm 2 with the piezoelectric actuator

At the same time with the motor motion control, the piezoelectric actuator actuates to further reduce the arm vibration. By using the right coprime
4.4. Operator-based control design with DWT

factorization method, plant $P_2 + \Delta P_2$ is factorized as follows.

$$D_2(\delta_2)(t) = e^{-\hat{\alpha}_t} \delta_2(t),$$  \hspace{1cm} (4.16)$$

$$[N_2 + \Delta N_2](\delta_2)(t) = (1 + \Delta_2) \sum_{n=1}^{N_c} J^n_2 e^{-\alpha_n t} \int_0^t e^{-\hat{\alpha}_n \tau} \sin \omega_{dn}(t - \tau) \delta_2(\tau) d\tau.$$  \hspace{1cm} (4.17)

Applying a control voltage $u_2$, the piezoelectric actuator outputs a moment $M_p$ represented as follows.

$$M_p(t) = D_{P1}(u_2)(t) + \Delta_{P1}(u_2)(t)$$  \hspace{1cm} (4.18)$$

$$D_{P1}(u_2(t)) = K u_2(t) = \int_0^{H_c} p(h) dhu_2(t)$$  \hspace{1cm} (4.19)$$

The first term $D_{P1}$ is an invertible operator, another term $\Delta_{P1}$ represents for residual uncertain part including the hysteresis, it needs to be compensated by the controller. For the details of the model, please refer to the reference [24].

Without loss of the system stability, $D_{P1}$ is combined with the operator $D_2$, and denote as $\tilde{D}_2 = D_{P1}^{-1} D_2$. $\Delta_{P1}$ and the driving force from the motor $F_{02}$ are equivalently compensated before the BIBO stable loop. The $\dot{y}_2$ is obtained through the wavelet transform. Then, by using the above mentioned robust right coprime factorization method, operators $A_2$ and $B_2$ for the new plant $\tilde{P}_2 = N_2 \tilde{D}_2^{-1}$ are obtained as follows.

$$A_2(\dot{y}_2)(t) = \frac{e^{\hat{\alpha}_2 t} - K_2 K^{-1}}{J_{\omega^2_{d2}}} \eta_2(t),$$  \hspace{1cm} (4.20)$$

$$\eta_2(t) = \ddot{y}_2 + 2 \alpha \dot{y}_2 + (\alpha^2_2 + \omega^2_{d2}) \dot{y}_2,$$

$$B_2(u_2)(t) = K_2 K(u_2)(t).$$  \hspace{1cm} (4.21)$$
where $K_2$ is a tuning parameter working for the operator $B$.

Fig. 4.9 is the control design for Arm 2 with a modified hysteresis compensation, which is different from it in Chapter 3. The BIBO stability of the system is guaranteed by the operators $A_2$, $B_2$ and the DWT processor. Moreover, the most uncertainties have been removed by the wavelet transform, therefore, the designed control is said to be robust. The plant output is deduced as follows.

$$y_{a2}(t) = (N_2 + \Delta N_2)\ddot{U}^{-1}[\ddot{r}_2(t) + B_2(\Delta_{PI} + D_{PI}^{-1}F_{02})]$$  \hspace{1cm} (4.22)

As shown in Fig. 4.9, the external force $F_{02}$ and the non-invertible part $\Delta_{PI}$ are compensated by the controller $C_2$ as follows.

$$C_2 := K_{I2} \int_0^t e_2(\tau)d\tau + K_{P2}e_2(t) - B_2(\Delta_{PI} + D_{PI}^{-1}F_{02})$$

where $e_2(t)$ represents for the difference between the output $y_{a2}$ and the reference $r_2$; $K_{I2}$ and $K_{P2}$ are the PI gains. The first two terms work for tracking the target. The last term works as a feed-forward compensator, compensating the $\Delta_{PI}$. Therefore, the tracking compensator $C_2$ working
along with $A_2$, $B_2$ and the DWT processor, further reduce the arm vibration and keep the system stable and robust.

With the above two operator-based control design, the arm vibration is theoretically reduced as small as possible while the linear motor is allow to move fast. The whole system is guaranteed to be stable and robust.

4.5 Numerical simulations and discussion

To verify the effectiveness of the proposed design, simulations were conducted under Matlab. The parameters of the L-shaped arm mentioned above are listed in Table 3.1.

4.5.1 The modified control without DWT

To validate the new model of the arm vibration and the modified hysteresis compensation, we performed the simulations based on the proposed control design as shown in Fig. 4.8 and Fig. 4.9 without DWT and compared with the control design in Chapter 3. The results are shown as follows.

Fig. 4.10 is the result under the proposed control in Fig. 4.8 without DWT using the new arm vibration model. It shows that with the new developed arm model in this chapter, the arm vibration is reduced. Fig. 4.11 is the result under the proposed control comparing with the result using the original model in last chapter, which illustrates the better effectiveness of the new model.

Fig. 4.12 and Fig. 4.13 are the results using the modified arm model in the control in Fig. 4.9 without hysteresis compensation and DWT, these results
show that the new model is also effective in the control of Arm 2 vibration.

Fig. 4.14 and Fig. 4.15 are the results using the modified hysteresis compensation in the control in Fig. 4.9 without DWT, the results indicate that the modified hysteresis compensation is effective to compensate the nonlinearity of the piezoelectric actuator, with the compensation, the vibration of the arm is further reduced.

### 4.5.2 Operator-based control with DWT

To validate the effectiveness of the proposed on-line DWT in the operator-based control design, we conducted simulations including load estimation, vibration control of Arm 1 and Arm 2, the results are shown as follows.

Using these parameters, under free vibration, the first natural frequencies
4.5. Numerical simulations and discussion

Figure 4.11: Vibration of Arm 1 with new model

Table 4.1: Load mass estimation in simulation

<table>
<thead>
<tr>
<th>$m_t$ (kg)</th>
<th>$\dot{m}_t$ (kg)</th>
<th>$\omega_1$ (rad/s)</th>
<th>$\dot{\omega}_1$ (rad/s)</th>
<th>errors (%)</th>
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<td>0.0497</td>
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<tr>
<td>0.08</td>
<td>0.0808</td>
<td>22.133</td>
<td>22.0893</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
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<td>21.1424</td>
<td>208621</td>
<td>1.33</td>
</tr>
<tr>
<td>0.15</td>
<td>0.1536</td>
<td>19.144</td>
<td>19.0214</td>
<td>0.64</td>
</tr>
<tr>
<td>0.2</td>
<td>0.1936</td>
<td>17.6177</td>
<td>17.7942</td>
<td>1</td>
</tr>
</tbody>
</table>

of the arm with different loads is estimated by using DWT and FFT method, the results are listed in Table 4.1. As shown in the table, the estimation error is less than 0.2%; the result indicates that the load estimation method proposed in thin dissertation is effective. The error depends on the sample frequency and the calculation accuracy of WDT and FFT by Matlab.

To determine the dominant modes considered in the control plant, we
set the threshold $r_{th} = 0.01$. Substituting the vibration parameters into the modes decision equation (4.8), it was determined $n_c = 3$, namely, the first three modes of arm vibrations are considered in the the plants. For a certain load, according to the first three modes frequencies and the sample frequency, the decompose level $L$ and the reconstruction gains were determined; other parameters in the plant and controller were calculated accordingly.

The forced vibration of the arm with load $m_t = 0.02$ kg was simulated by using the proposed method comparing with the previous operator-based method. The sampling interval in the simulation was $0.01$ s, the Daubechies 4 wavelet was used in the DWT processor. The size of the moving window was decided as $l_n = 50$, the extension length $l_t = 14$, thus $l_w = 64 = 2^6$. The decompose level was set as 5. The motion of the linear pulse motor is shown
4.5. Numerical simulations and discussion

Figure 4.13: Vibration of Arm 2 with new model

in Fig. 4.16. By comparing with the vibration results, we can find that, the motor stopped at time 4.56 s.

The vibrations of the segment OA and AB are shown respectively. The displacements of Arm 1 (segment OA) is shown in Fig. 4.17 and Fig. 4.18. The vibration result comparison between the proposed control and without control is shown in Fig. 4.17. The green dashed line is the arm vibration without control, the red solid line is the vibration with wavelet-operator control. We added an Gaussian white noise between 4.5 s and 5 s to simulate the unknown disturbances, and a sine wave signal $1.5 \times 10^{-5} \sin 240t$ after 5 s to simulate the higher modes vibrations of the arm. The result shows that, with the proposed wavelet-operator-based control the undesired disturbances are removed, the vibration of the arm is reduced and stabilized. Fig. 4.18
is the comparison between the proposed control and the previous operator-based control. The blue dashed line is the vibration of arm with the previous operator-based control without DWT. As seen from the result, with DWT processing the vibration of the arm is further reduced, and the stability of the system is more easily ensured. Considering the input constrain of the piezoelectric actuator, the voltages decided by control system were limited within $\pm 100$ V.

For the Arm 2 (segment AB) vibration, using piezoelectric actuator, we conduct two experiments to verify the proposed controller, including the operator control without and with wavelet transform.

For comparison, the results are shown together with the result without wavelet transform in Fig. 4.19 and Fig. 4.20. The vibration of Arm 2 with
Figure 4.15: Vibration of Arm 2 with modified compensation

and without control are shown in Fig. 4.19. The green dashed line is the result without control, the red solid line is vibration with wavelet-operator control. The result shows that with the proposed control, the vibration of the arm is reduced. Comparison between the proposed control with and without hysteresis compensation is shown in Fig. 4.20. The blue dashed line is the vibration of arm without hysteresis compensation, the red solid line with compensation. The result shows that the hysteresis compensator with the proposed control can reduce the arm vibration further.

The robustness evaluation for the proposed control is shown in Fig. 4.21. The robustness indexes in process are all less than 1, which confirms that the robust stability of the proposed control are ensured. Therefore, the proposed control scheme for the L-shaped arm in this dissertation is effective. With
In this Chapter, the vibration control of the L-shaped arm with unknown load is studied. The linear motor is controlled to be fast while reducing the vibration of the arm. Meanwhile, the piezoelectric actuator is controlled to further reduce the vibration of the arm. The hysteresis of the piezoelectric actuator is compensated based on the Prandtl-Ishlinskii model.

By considering the whole arm as a two dimensional Euler-Bernoulli beam, the vibration of the L-shaped arm with unknown load was modelled. With the new model, the operator-based robust control was designed by using a short-symmetrical on-line wavelet transformation. After being processed by the on-line DWT, the vibration signal of the arm has less disturbances, the
robust stability of the system are easily guaranteed. The load mass was estimated by wavelet based on the frequency equation.

Simulations were conducted to validate the new arm model, the modified hysteresis compensation and the operator-based nonlinear control with on-line DWT, the results illustrate that the proposed operator-based active control design using on-line DWT is effective and can guarantee the system robust stability. To validate the effectiveness of proposed control design, experiments will be conducted in the next chapter.
CHAPTER 4. CONTROL DESIGN FOR ARM WITH LOAD

Figure 4.18: Vibration control of Arm 1 with and without DWT

Figure 4.19: Vibration of Arm 2 with piezoelectric actuator
Figure 4.20: Vibration of Arm 2 with and without compensation

Figure 4.21: Robustness of the proposed control
Chapter 5

Experimental study on the control design

5.1 Introduction

The L-shaped arm vibration without and with load have been studied in Chapter 3 and Chapter 4 respectively. The proposed control designs have been validated by the corresponding simulation results. To further test the performances of these two control designs, we conduct the experiments correspondingly.

In Section 5.2, the structure of the L-shaped arm vibration experimental system is introduced, the main parameters of the devices are identified.

In Section 5.3, using the optimal linear motor motion control and the L-shaped arm vibration control with piezoelectric actuator proposed in Chapter 3, experiments are conducted comparing with the conditional PI control.

In Section 5.4, using the operator-based motor motion control and arm vibration control with on-line DWT proposed in Chapter 4, comparative experiments are conducted for the L-shaped arm with unknown load. Different
loads are estimated using the proposed estimation method.

In Section 5.5, the conclusion of this Chapter is drawn.

5.2 Experiment system structure

An uniform L-shaped arm vibration control experiment system is set up as shown in Fig. 5.1. The up end of the arm is clamped with a pulse linear motor; the linear motor drives the the arm to destination along the motor guide horizontally.

![Experimental device](image)

Figure 5.1: Experimental device

The linear pulse motor is HRM 0205, driven by a driver LD-300 (both made by THK CO., LTD). The motor driver communicates with the PC by using a PCI board SMC-4P (PCI) (made by CONTEC CO., LTD). The motor moves 0.05 mm for every pulse input. The non-zero minimum speed of the linear motor and minimum acceleration rate are decided by the PCI
5.2. Experiment system structure

Figure 5.2: Schematic diagram of the arm

board. The maximum speed and acceleration rate are limited for safety, the parameters of the linear motor are listed in Table 3.2.

Table 5.1: Parameters of the Piezoelectric Actuator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p$</td>
<td>Thickness</td>
<td>$0.5 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$w_p$</td>
<td>Width</td>
<td>$20 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>$l_p$</td>
<td>Length</td>
<td>0.05</td>
<td>m</td>
</tr>
<tr>
<td>$x_{p1}$</td>
<td>Location on Arm 2</td>
<td>0.04</td>
<td>m</td>
</tr>
<tr>
<td>$d_{31}$</td>
<td>Electric charge constant</td>
<td>$-2.1 \times 10^{-10}$</td>
<td>m/V</td>
</tr>
<tr>
<td>$E_p$</td>
<td>Young’s modulus</td>
<td>$6.2 \times 10^{10}$</td>
<td>N/m²</td>
</tr>
</tbody>
</table>

Two piezoelectric sensors are pasted on one side of the arm to measure the relative vibration displacements of the arm. One piezoelectric actuator is mounted on the other side of the arm to suppress its vibration, as shown
in Fig. 5.2. The actuator (C-6, made by FUJI Ceramics Corporation) and sensors (C-63, made by FUJI Ceramics Corporation) used in this dissertation both are PZT (Pb[Zr,Ti]O₃) ceramics type material. The parameters of the actuator are listed in Table 5.1. The proposed controllers in this dissertation are programmed in Microsoft Visual C++ under Windows XP operating system, the sensor and control input signals for piezoelectric actuator are processed by the PCI board (PCI-3521, made by Interface Corporation). A voltage amplifier (HOPS-0.3B10, made by Matsusada Precision) is used between the PCI board and the actuator to amplify the input voltages (restricted within ±100 V).

5.3 Experiments on system without load

The general aim of this study is to reduce the arm vibration while let the motor move fastly. To validate the effectiveness of the proposed design scheme, several experiments were conducted comparatively. The parameters of the linear pulse motor, the L-shaped arm and the piezoelectric actuator in experimental system are shown in Tables 3.2, 5.1 and 3.1. The sampling frequency in the experiments was 200 Hz; it was bigger than twice the third mode frequency of the arm vibration. The density function $p(h)$ of the hysteresis model (2.3) was identified by a characteristic experiment represented as the following equation, where $h \in [0, 100]$.

$$p(h) = 0.00032 \times e^{-0.00086(h-1)^2}. \hspace{1cm} (5.1)$$

The filters mentioned above were designed into a first-order infinite im-
5.3. Experiments on system without load

pulse response (IIR) low-pass filter combined with bi-quad IIR notch filter. The low-pass filter was used to attenuate the high frequencies noise signals, the notch filter was used to attenuate certain frequency signal caused by piezoelectric sensor’s measurement noise. The cut-off frequency of filters was determined by vibration experiments. The displacements of Arm 1 were measured by sensor 1.

![Figure 5.3: Vibration of Arm 1 with motion control (without piezoelectric actuator)](image)

To validate the proposed motor control, we conducted three experiments for comparison, including the proposed optimal operator control (the piezoelectric actuator on Arm 2 didn’t work), a conventional PI control and a minimum time control, the results are shown in Fig. 5.3. The dotted line in Fig. 5.3 represents for the vibration of Arm 1 with minimum time control. In
this situation, only considering the time consumption, the linear motor was controlled to run for a distance of 0.5 m at the maximum speed and acceleration, the minimum time consumption was 3.49 s. The results indicate that the maximum acceleration leaded to a maximum arm vibration especially at the moment the linear motor starts and stops.

In Fig. 5.3, the dashed line is the displacement with PI control, the solid line is the displacement with the proposed optimal operator feedback control. The time consumption of the linear motor were both tuned at 4.36 s for these two control methods. The result illustrates that with PI control, the arm vibration is reduced, but the travel time of the linear motor is longer than the minimum time control. The proposed optimal operator control further reduced the vibration of arm, especially after the motor stopped. It indicates that with the operated feedback signal, the motor was ensured to run at an optimal trajectory resulting in reduced arm vibration, while the plant was kept stable. The extra undesired vibration around 0.5 s and 3 s was presumably caused by the uneven contact and friction between linear motor and motor guide.

To validate the proposed control for Arm 2, we conducted two experiments including the operator control without and with hysteresis compensator. The input voltages for the piezoelectric actuator were limited within ±100 V for safety. For comparison, the results are shown together with the result without actuator in Fig. 5.4. The output of the plant without actuator is shown in dash-dot line, the dashed line is the result by using actuator without considering the hysteresis, while the displacement of Arm 2 with
5.3. Experiments on system without load

actuator with proposed hysteresis compensation. The result indicates that
the piezoelectric actuator can reduce the arm vibration with the proposed
control. With the proposed hysteresis compensation, the vibrations of arm
are further reduced at most of time during the motor motion.

Figure 5.4: Vibration of Arm 2 with and without actuator (linear motor with
operator control)

To show the effectiveness clearly, an evaluation index $\int_0^T y_2^2(t) dt$ derived
from the cost function (3.12) is used to evaluate the vibration power of the
arm, which is called cumulative vibration intensity. The vibration index of
Arm 2 without actuator, with actuator no compensation and actuator with
compensation are shown in Fig. 5.5 in dash-dot line, dashed line and solid
line, respectively. From which, we conclude that the arm vibration can be
reduced effectively with piezoelectric actuator, and with the proposed hys-
teresis compensation the arm vibration can be further reduced. The results illustrate that both the control design for actuator and the hysteresis compensator are effective.

![Graph](image)

Figure 5.5: Cumulative vibration intensity of Arm 2 with and without actuator

Therefore, the proposed control scheme for the piezoelectric actuator in this dissertation is effective to suppress the vibration of arm; with the hysteresis compensator, the plant outputs better results.

The robust stability evaluations for the proposed control are shown in Fig. 5.6. The values of y-coordinate were calculated by $\| [A_2(N_2 + \Delta N_2) - A_2N_2]M^{-1}\|_{Lip}$. It is evident that the value are much less than 1 during the experiment. According to the definition of robustness (2.19), the result indicates that the proposed control is robust stable. The robustness includes
5.4 Experiments on system with load

To test the performance of the proposed control design, we conducted several experiments comparing with the previous work. The proposed controller was performed by through Microsoft Visual C++ 2010 and run on a PC with Windows XP system. Limited by the performance of the PC in experiments, the sampling frequency was determined as 100 Hz.

According to the sampling frequency and the vibration dynamics of the arm, the dominant modes was determined as $N_d = 2$, namely, the first two modes of arm vibrations were considered in the plants. For a certain load, according to the first three modes frequencies and the sample frequency,
CHAPTER 5. EXPERIMENTAL STUDY

Table 5.2: Load mass estimation in experiment

<table>
<thead>
<tr>
<th>( m_t ) (kg)</th>
<th>( \hat{m}_t ) (kg)</th>
<th>( f_1 ) (Hz)</th>
<th>error (%)</th>
</tr>
</thead>
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<td>10.0</td>
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<td>0.0408</td>
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<td>0.087</td>
<td>0.0808</td>
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<td>7.1</td>
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<td>0.108</td>
<td>0.1072</td>
<td>3.307</td>
<td>9.0</td>
</tr>
<tr>
<td>0.152</td>
<td>0.1541</td>
<td>3.036</td>
<td>1.4</td>
</tr>
<tr>
<td>0.204</td>
<td>0.1937</td>
<td>2.787</td>
<td>5.0</td>
</tr>
</tbody>
</table>

the decompose level \( L \) and the reconstruction gains were determined; other parameters in the plant and controller were calculated accordingly. In the experiment, the load mass estimation flow is shown as:

Initial vibration \( \rightarrow y_1(t) \rightarrow \text{DWT} \rightarrow \text{reconstructed of the approximation} \) at the level with frequency band including \( f_1 \rightarrow \text{FFT} \rightarrow f_1 \rightarrow \text{equation (4.4)} \) \( \rightarrow \beta_1 \rightarrow \text{equation (4.5)} \rightarrow \text{load mass} \ \hat{m}_t. \)

Using this method, we tested with five different loads, the results are listed in Table 5.2. The first column \( m_t \) is the real load mass we applied; the second column \( \hat{m}_t \) is the estimated load mass; the third column \( f_1 \) is the tested vibration frequency of the first mode; the last column is the estimation error, less than 10%. Considering the impact of disturbances in experiments, the proposed load estimation method is said to be effective.

To test the proposed nonlinear control method, we conducted experiments with load \( m_t = 0.042 \ \text{kg} \) and compared with previous operator-based method (without on-line DWT). The Daubechies 4 wavelet was used in the DWT processor. The size of the moving window was decided as \( l_n = 24 \), the extension length \( l_t = 8 \), thus \( l_w = 32 = 2^5 \). The decompose level was set
as 4. For different situations the linear motor was controlled to arrive the destination within 4.36 s. The motion of the linear motor is shown in Fig. 5.7.

![Figure 5.7: Position and force input of the linear motor](image)

With the proposed controller, forced by the linear motor, the resulted displacements of Arm 1 (segment OA) are demonstrated in Fig. 5.8 and Fig. 5.9. Fig. 5.8 is the vibration result comparison with proposed control and without control. The arm vibration without control is shown in blue dashed line, the vibration with wavelet-operator control is shown in red solid line. As seen from the result, the vibration of the arm was reduced by using the proposed wavelet-operator control and the system was stabilized. Fig. 5.9 is the comparison between the proposed control and the previous operator-based control without DWT. The vibration of arm with the previous operator-based control is shown in blue dashed line, the proposed control in
red solid line. As seen from the result, with the DWT processor the vibration of the arm was further reduced, and the stability of the system was more easily ensured.

![Vibration of Arm 1 with and without control](image)

Figure 5.8: Vibration of Arm 1 with and without control

For the Arm 2 (segment AB) vibration control using piezoelectric actuator, we conducted experiments to verify the proposed controller. The vibration result with DWT is shown in red solid line in Fig. 5.10, the result without DWT is shown in blue dashed line. The result shows that the proposed control reduced the arm vibration more effectively. Fig. 5.11 shows the comparison between the proposed control with and without hysteresis compensation. The arm vibration with hysteresis compensation is shown in red solid line, the result without compensation in blue dashed line. The result shows that the hysteresis compensator with the proposed control can reduce
5.4. Experiments on system with load

Figure 5.9: Vibration of Arm 1 with and without DWT

To illustrate the control effect more clearly, we defined the root of mean square error (RMSE) of the results with time as

\[
RMSE(nT) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (y_2(i) - r_2)^2}
\]  

(5.2)

where \( T \) is the sampling interval, \( n \) is the sampling number. The RMSE of Arm 2 displacements with time for different situations are shown in Fig. 5.12. It can be seen from the results that the vibration of Arm 2 was reduced more effectively with the proposed control than the previous control; the arm vibration was further reduced by using the hysteresis compensation. Therefore, the proposed control design in this study could reduce the L-shaped arm vibration effectively. With the on-line DWT, the control performance was
5.5 Conclusion

In this chapter, we tested the control designs proposed in Chapter 3 and Chapter 4 by experiments.

For the L-shaped arm without load, comparing with the PI control and minimum time control, the operator-based optimal motion control was performed for the linear motor. Then, using the proposed vibration control of Arm 2 with piezoelectric actuator, the experiments were performed along with the PI control. The results indicate that the proposed control design is effective and can guarantee the system robust stability.

For L-shaped arm with load, we conducted several comparative experi-

Figure 5.10: Vibration of Arm 1 with and without control

improved.
5.5. Conclusion

Figure 5.11: Vibration of Arm 1 with and without DWT

ments to validate the proposed control design using the operator-based approach and the on-line DWT. The results illustrated that the arm vibration was reduced effectively, the operator-based right coprime factorization could guarantee the system robust stability and with the on-line DWT, the performance of the operator-based control was improved, the load mass was easily estimated.
Figure 5.12: Position and force input of the linear motor
Chapter 6

Conclusions

In this dissertation, an L-shaped arm driven by a linear pulse motor is studied to find approaches to control motor motion and the forced vibration of arm at the same time. The arm without load and with load were researched respectively. Two different modelling methods of the arm were employed and two different nonlinear control designs were proposed correspondingly. Simulations and experiments for different situations and different control design were conducted to validate their effectiveness.

In Chapter 2, the Euler-Bernoulli theory for the flexible arm vibration is introduced, which provides the theoretical basis for modelling the L-shaped arm in this dissertation. The Prandtl-Ishlinskii model provides the method to model the hysteresis of the piezoelectric actuator. Some fundamental definitions and the theoretical basis of operator-based nonlinear control theory establish the foundation for the control design in this dissertation. The DWT is the theoretical basis for uncertainties removing and the load estimation.

In Chapter 3, operator-based robust nonlinear control was proposed to
control the forced vibration of arm. By considering the L-shaped arm as two connected Euler-Bernoulli beams, the dynamics on arm vibration involving linear motor and piezoelectric actuator was modelled. Two operator-based robust nonlinear control systems were proposed in this chapter, the first one was designed to make the motor not only move to destination in certain time but also reduce the vibration of arm. Another one was designed to control the input of the piezoelectric actuator, the hysteresis nonlinearity of the actuator was modelled using a Prandtl-Ishlinskii hysteresis model, and the nonlinear part is compensated in the tracking controller. Finally, to confirm the effectiveness of the proposed control design, simulations were conducted comparing with the PI control, the results illustrate that the operator-based control systems designed in this dissertation are more effective and can guarantee the system to be stable and robust.

In Chapter 4, the vibration control of the L-shaped arm with unknown load was studied. The linear motor is required to be fast while reducing the vibration of the arm. Meanwhile, the piezoelectric actuator was utilized to further reduce the vibration of the arm. First, the L-shaped arm with unknown load was modelled by considering it as a whole two dimensional Euler-Bernoulli beam. The relationship between the arm vibration and the load mass was given. Second, the operator-based robust control was designed by using a short-symmetrical on-line wavelet transformation. After being processed by the on-line DWT, the vibration signal of the arm has less disturbances, the robust stability of the system are easily guaranteed. The load mass was estimated by wavelet based on the frequency equation.
The hysteresis of the piezoelectric actuator was compensated based on the Prandtl-Ishlinskii model. Finally, simulation was conducted under Matlab, the results illustrate that the proposed operator-based active control design using on-line DWT is effective and can guarantee the system robust stability.

In Chapter 5, we tested the control designs proposed in Chapter 3 and Chapter 4 by though experiments. For the L-shaped arm without load, comparing with the PI control and minimum time control, the operator-based optimal motion control was performed for the linear motor. Then, using the proposed vibration control of Arm 2 with piezoelectric actuator, the experiments were performed along with the PI control. The results indicate that the proposed control design is effective and can guarantee the system to be stable and robust.

For the L-shaped arm with unknown load, we conducted several comparative experiments to validate the proposed control design using the operator-based approach and the on-line DWT. The results illustrate that the arm vibration is reduced effectively, the operator-based right coprime factorization could guarantee the system robust stability and with the on-line DWT, the performance of the operator-based control is improved, the load mass is easily estimated.

In conclusion, this dissertation provides a nonlinear forced vibration control design method for the underactuated systems with multiple outputs and less control inputs, it can guarantee the system robust stability. The proposed method in this dissertation can also be used to reduced the impact of the uncertainties and disturbances of the system. Moreover, the on-line
DWT has potential in processing the complicated system signal in time and frequency domain and estimating some unknown parameters.

However, some limitations of this study are worth noting. The models of the arm vibration and the hysteresis of the actuator were simplified to some extent; more accurate models could probably improve the control effect. The linear motor control was limited by the motor characteristic and the interface board, the real time command sent by the controller are not fully executed. The sampling intervals and computing speed were limited by the PC’s capacity especially for the on-line DWT program, which requiring high computational performance. Further work will consider the impact of the uncertainties in detail especially the transient vibration of linear motor and use a more accurate system model and high-performance devices to improve the control effectiveness.
Bibliography


Appendix A

Model analysis of the L-shaped arm with load

A.1 Free vibration of the arm

For the thin uniform arm as shown in Fig. 2.1 in Chapter 2, without considering the damping factor, its free transverse vibration is expressed as follows.

\[ E_a I \frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = 0 \]  \hspace{1cm} (A.1)

where \( E_a, I, \rho, S \) are the Young’s modulus, moment of inertia, density, and cross-sectional area of the arm, respectively. Using the method of separation of variables, the solution can be expressed as

\[ w(x, t) = \Phi(x)T(t) \]  \hspace{1cm} (A.2)

Substituting Eq. (A.2) into Eq. (A.1) and rearranging it yields

\[ \frac{E_a I \Phi(x)}{\rho S \Phi(x)} \frac{d^4 \Phi(x)}{dx^4} = -\frac{1}{T(t)} \frac{d^2 T(t)}{dt^2} = \omega^2 \]  \hspace{1cm} (A.3)
APPENDIX A. MODEL ANALYSIS OF THE ARM

it can be rewritten as two equations:

\[
\frac{d^4\Phi(x)}{dx^4} - \beta^4\Phi(x) = 0 \tag{A.4}
\]

\[
\frac{d^2T(t)}{dt^2} + \omega^2T(t) = 0 \tag{A.5}
\]

where

\[
\beta^4 = \frac{\rho S \omega^2}{E_a I} \tag{A.6}
\]

The solution of Eq. (A.5) is given by

\[
T(t) = X \cos \omega t + Y \sin \omega t \tag{A.7}
\]

where \(X\) and \(Y\) are constants can be obtained from the initial conditions.

The solution of Eq. (A.4) is expressed as

\[
\Phi(x) = C_1 \cos \beta x + C_2 \sin \beta x + C_3 \cosh \beta x + C_4 \sinh \beta x \tag{A.8}
\]

where \(C_1, C_2, C_3,\) and \(C_4\) are constants; the function \(\Phi(x)\) is known as the normal mode of the arm. The natural frequencies of the arm can be determined from Eq. (A.6) as

\[
\omega = 2\pi f = \beta^2 \sqrt{\frac{E_a I}{\rho S}} \tag{A.9}
\]

The unknown constants \(C_1\) to \(C_4\) and the value of \(\beta\) can be determined from the known boundary conditions of the arm.

For any arm, there is an infinite number of normal modes with one natural frequency corresponding to each normal mode. If the \(n\)th natural frequency is denoted as \(\omega_n\) and the corresponding normal mode as \(\Phi_n(x)\), the total
free vibration response of the beam can be found by superposing the normal modes as

\[ w(x, t) = \sum_{n=1}^{\infty} \Phi_n(x)(X_n \cos \omega_n t + Y_n \sin \omega_n t) \]  

(A.10)

where the constants \(X_n\) and \(Y_n\) can be determined from the initial conditions of the beam.

**A.2 Free vibration of the L-shaped arm**

The following analytical modal analysis is given for the transverse vibrations of the L-shaped arm with a tip mass rigidly attached at the free end as shown in Fig. A.1.

![Figure A.1: Transverse vibration of the L-shaped arm](image)

Denoting \(w\) as the transverse displacement in \(y\) direction along the neutral axis of the L-shaped arm. The forced transverse arm vibration without damping is expressed as

\[ E_a I \frac{\partial^4 w}{\partial x^4} + \rho S \frac{\partial^2 w}{\partial t^2} = q(x, t) \]  

(A.11)
where \( q(x, t) \) is the external distributed forces on the arm, including the linear motor driving force and the piezoelectric actuator moment.

For easy analyse the arm vibration, we denote the displacement of segments OA and AB as \( w(x_1, t) \) and \( w(x_2, t) \), respectively, \( 0 \leq x_1 \leq l_1, 0 \leq x_2 \leq l_2 \). They are determined using the same method mentioned in last section as follows.

\[
\begin{align*}
  w(x_1, t) &= \Phi_1(x_1)(X \cos \omega t + Y \sin \omega t) \\
  w(x_2, t) &= \Phi_2(x_2)(X \cos \omega t + Y \sin \omega t)
\end{align*}
\]

where

\[
\begin{align*}
  \Phi_1(x_1) &= C_1^1 \cos \beta x_1 + C_2^1 \sin \beta x_1 + C_3^1 \cosh \beta x_1 + C_4^1 \sinh \beta x_1 \\
  \Phi_2(x_2) &= C_1^2 \cos \beta x_2 + C_2^2 \sin \beta x_2 + C_3^2 \cosh \beta x_2 + C_4^2 \sinh \beta x_2
\end{align*}
\]

For OA segment, the boundary conditions can be stated as

\[
\begin{align*}
  \Phi_1(0) &= 0, & M_A &= E_a I \frac{d^2 \Phi_1}{dx_1^2}
  & \bigg|_{x_1=l_1} \\
  \frac{d\Phi_1}{dx_1}
  & \bigg|_{x_1=0} = 0, & V_A &= E_a I \frac{d^3 \Phi_1}{dx_1^3}
  & \bigg|_{x_1=l_1}
\end{align*}
\]

For AB segment, the boundary conditions can be stated as

\[
\begin{align*}
  \Phi_2(0) &= \Phi_1(l_1) & M'_A &= E_a I \frac{d^2 \Phi_2}{dx_2^2}
  & \bigg|_{x_2=0} \\
  M'_A &= M_A, & V'_A &= E_a I \frac{d^3 \Phi_2}{dx_2^3}
  & \bigg|_{x_2=l_2} \\
  & = m_t \frac{\partial^2 w(l_2, t)}{\partial t^2}, & V_A &= V_A
\end{align*}
\]
Using all the boundary conditions yields

\[ C_1^1 (\cos \beta l_1 - \cosh \beta l_1) + C_2^1 (\sin \beta l_1 - \sinh \beta l_1) - C_1^2 - C_3^2 = 0 \]  
\[ C_1^1 (\cos \beta l_1 + \cosh \beta l_1) + C_2^1 (\sin \beta l_1 + \sinh \beta l_1) - C_1^2 + C_3^2 = 0 \]  
\[ C_1^1 (\sin \beta l_1 - \sinh \beta l_1) - C_2^1 (\cos \beta l_1 + \cosh \beta l_1) + C_2^2 = 0 \]  
\[ C_1^2 \cos \beta l_2 + C_2^2 (\sin \beta l_2 + \sinh \beta l_2) - C_2^2 \cosh \beta l_2 = 0 \]  
\[ C_1^2 (\sin \beta l_2 + \lambda \cos \beta l_2) - C_2^2 [\cos \beta l_2 + \cosh \beta l_2 - \lambda (\sin \beta l_2 - \sinh \beta l_2)] \]

\[ + C_3^2 (\sinh \beta l_2 + \cosh \beta l_2) = 0 \]  

(A.16)  
(A.17)  
(A.18)  
(A.19)  
(A.20)

where \( \lambda = \omega^2 m_t / (E_0 I \beta^3) \). Setting the determinant of the coefficient matrix Eqs. (A.16) to (A.20) yields the frequency equation of the arm vibration as

\[
\sin \beta l^- \sinh \beta l^+ - \sinh \beta l^- \sin \beta l^+ - 2 \cos \beta l_2 \cosh \beta l_2 - 2 \cos \beta l_1 \cosh \beta l_1 \\
- 2 \cos \beta l^+ \cosh \beta l^+ - 2 - \frac{m_t \beta}{\rho S} (2 \cos \beta l^+ \sinh \beta l^+ - 2 \cosh \beta l^+ \sin \beta l^+ \\
+ 2 \cos \beta l_2 \sinh \beta l_2 - \cosh \beta l_2 \sin \beta l_2 + \cos \beta l^- \sinh \beta l^- - \cos \beta l^- \sin \beta l^- \\
- \sin \beta l^- \cosh \beta l^+ - \sin \beta l^- \cos \beta l^+ = 0
\]

(A.21)

where \( l^+ = l_1 + l_2, l^- = l_1 - l_2 \), \( m_t \) is the mass of load.

For the given system parameters \( l_1, l_2, m_t, \rho \) and \( S \), one can solve for the roots of Eq. (A.21) obtaining \( \beta \); it has infinite positive eigenvalues for the infinite natural modes of vibration. Hence the \( n \)th mode shape of the arm at segments OA and AB can be expressed as

\[ \Phi_1^n(x_1) = C_1^n [ \cos \beta_n x_1 - \cosh \beta_n x_1 + \sigma_1^n (\sin \beta_n x_1 - \sinh \beta_n x_1)] \]  
\[ \Phi_2^n(x_2) = C_2^n (\sigma_2^n \cos \beta_n x_2 + \sigma_3^n \cosh \beta_n x_2 + \sin \beta_n x_2 - \sinh \beta_n x_2) \]

(A.22)  
(A.23)
APPENDIX A. MODEL ANALYSIS OF THE ARM

where \( C_1^n \) and \( C_2^n \) are the unknown constants for the \( n \)th mode vibration, could be obtained from the orthogonality conditions and the boundary conditions. \( \sigma_1^n, \sigma_2^n, \sigma_3^n \) are coefficients depending on the vibration mode, which are listed as follows.

\[
\sigma_1^n = \frac{\sin \beta_n l_1 \sin \beta_n l_2 + \sin \beta_n l_1 \sinh \beta_n l_2 - \sin \beta_n l_2 \sinh \beta_n l_1 - \sigma_{a1}}{\cos \beta_n l_1 \sin \beta_n l_2 + \cos \beta_n l_1 \sinh \beta_n l_2 + \cosh \beta_n l_1 \sin \beta_n l_2 + \sigma_{b1}}
\]

\[
\sigma_2^n = \frac{\cos \beta_n l_2 \cosh \beta_n l_2 - \sin \beta_n l_2 \sinh \beta_n l_2 - 2 m \beta_n \cosh \beta_n l_2 \sin \beta_n l_2 + 1}{\cos \beta_n l_2 \sin \beta_n l_2 + \cosh \beta_n l_2 \sin \beta_n l_2 + \frac{2 m \beta_n}{\rho S} \cos \beta_n l_2 \cosh \beta_n l_2}
\]

\[
\sigma_3^n = \frac{\cos \beta_n l_2 \sinh \beta_n l_2 + \sin \beta_n l_2 \sinh \beta_n l_2 + \frac{2 m \beta_n}{\rho S} \cos \beta_n l_2 \sinh \beta_n l_2 + 1}{\cos \beta_n l_2 \sin \beta_n l_2 + \cosh \beta_n l_2 \sin \beta_n l_2 + \frac{2 m \beta_n}{\rho S} \cos \beta_n l_2 \cosh \beta_n l_2}
\]

\[
\sigma_{a1} = \sinh \beta_n l_1 \sin \beta_n l_2 + 2 \cos \beta_n l_1 \cos \beta_n l_2 + 2 \cosh \beta_n l_1 \cosh \beta_n l_2
\]

\[
\sigma_{b1} = \cosh \beta_n l_1 \sinh \beta_n l_2 + 2 \cos \beta_n l_2 \sinh \beta_n l_1 + 2 \cosh \beta_n l_2 \sinh \beta_n l_1
\]

The undamped natural frequency of free vibration for the \( n \)th mode is obtained from Eq. (A.9) as

\[
\omega_n = 2 \pi f_n = \beta_n^2 \sqrt{\frac{E_a I}{\rho S}} \hspace{1cm} (A.24)
\]

Therefore, the free vibration of the L-shaped arm can be as a linear combination of all the natural modes motion of the arm.

\[
w(x_1, t) = \sum_{n=1}^{\infty} \Phi_1^n(x_1)(X_n \cos \omega_n t + Y_n \sin \omega_n t) \hspace{1cm} (A.25)
\]

\[
w(x_2, t) = \sum_{n=1}^{\infty} \Phi_2^n(x_2)(X_n \cos \omega_n t + Y_n \sin \omega_n t) \hspace{1cm} (A.26)
\]

where the \( \Phi_1^n(x_1) \) and \( \Phi_2^n(x_2) \) are the \( i \)th natural modes and \( X_n \cos \omega_n t + Y_n \sin \omega_n t \) is a time-dependent function to be determined.
A.3 Forced vibration due to initial conditions

Considering the strain-rate damping, the forced vibration of the L-shaped arm is represented as

\[ E_a I \frac{\partial^4 w}{\partial x^4} + c_s I \frac{\partial^2 y(x, t)}{\partial x^4 \partial t} + \rho S \frac{\partial^2 w}{\partial t^2} = q(x, t) \quad (A.27) \]

In view of orthogonality of the eigenfunctions, using Eq. (A.24) and Duhamel integral, the solution of Eq. (A.27) for zero initial conditions is given as

\[ w(x_1, t) = \sum_{n=1}^{\infty} \frac{\Phi_1^n(x_1)}{\omega_{dn}} \int_0^t Q_1^n(\tau)e^{-\zeta_n \omega_n(t-\tau)} \sin \omega_{dn}(t - \tau) d\tau \quad (A.28) \]

\[ w(x_2, t) = \sum_{n=1}^{\infty} \frac{\Phi_2^n(x_2)}{\omega_{dn}} \int_0^t Q_2^n(\tau)e^{-\zeta_n \omega_n(t-\tau)} \sin \omega_{dn}(t - \tau) d\tau \quad (A.29) \]

where \( Q_1^n(t) \) and \( Q_2^n(t) \) are the generalized forces corresponding to the \( n \)th mode given by

\[ Q_1^n(t) = \int_0^l \Phi_1^n(x_1)q_1(x, t)dx \quad (A.30) \]

\[ Q_2^n(t) = \int_0^l \Phi_2^n(x_2)q_2(x, t)dx \quad (A.31) \]
Appendix B

Publications

Journal papers


Proceedings papers
APPENDIX B. PUBLICATIONS


**Other papers**
